Pricing and Performance of Mutual Funds: Lookback versus Interest Rate Guarantees

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Summary

The aim of this paper is to compare pricing and performance of mutual funds with two types of guarantees: a lookback guarantee and an interest rate guarantee. In a simulation analysis of different portfolios based on stock, bond, real estate, and money market indices, we first calibrate guarantee costs to be the same for both investment guarantee funds. Second, their performance is contrasted, measured with the Sharpe ratio, Omega, and Sortino ratio, and a test with respect to first, second, and third order stochastic dominance is provided. We further investigate the impact of the underlying fund's strategy, first looking at a conventional fund having a constant average rate of return and standard deviation over the contract term, and then at a Constant Proportion Portfolio Insurance managed fund. This analysis is intended to provide insights for investors with different risk-return preferences regarding the interaction of guarantee costs and the performance of different mutual funds with embedded investment guarantees.

JEL-Classification: D81, G11, G13, G22

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1. Introduction

In recent years, there has been an increasing demand for investment products with financial guarantees. For example, sales of unit-linked life insurance products have seen substantial growth.[1] These contracts are typically mutual funds with investment guarantees that additionally offer term insurance. Thus, the maturity payout depends on the performance of the underlying fund. From the investors' perspective, these mutual fund products are generally attractive due to the possibility of participating in positive market developments combined with a guaranteed minimum payoff at maturity.

Brennan and Schwartz (1976) and Boyle and Schwartz (1977) were the first to investigate asset guarantees in unit-linked life insurance products. Goldman, Sosin, and Gatto (1979), Conze and Viswanathan (1991), Gerber and Shiu (2003a), and Lin and Tan (2003) derive closed-form solutions for the valuation of different exotic options, including lookback options and dynamic fund protection with both deterministic and stochastic guaranteed levels. Gerber and Shiu (2003b) treat dynamic fund protection in the context of equity-indexed annuities, i.e., for perpetual American options. Lachance and Mitchell (2003) and Kling, Ruß, and Schmeiser (2006) analyze the value of interest rate guarantees in government-subsidized pension products in a Black/Scholes framework.

However, to date, there has been no comparison of interest rate and lookback guarantees for different underlying funds and different investment strategies with respect to pricing and performance, even though this information is an important prerequisite for decision making. The present analysis intends to fill this gap by providing this information to investors with different risk-return preferences.

In this paper, we compare pricing and performance of two mutual funds with different investment guarantees:[2] The first contract provides an interest rate guarantee on the

[1] In the European life insurance market, the share of unit-linked products in total premium volume has increased from 21.8% in 2003 to 24.2% in 2005 (CEA, 2007, p. 11). For instance, in France, the second largest life insurance market in Europe, the growth rate of 12.4% in premium income in 2005 was mainly driven by an increase in sales of unit-linked products (CEA, 2007, p.13). Investment products in general have been enjoying a growth surge. For example, German investment funds currently manage 126 guarantee funds (up from 81 at the end of 2005) with a fund asset value of 15.11 billion Euros (up from 10.01 billion Euros at the end of 2005) (www.bvi.de).

[2] In what follows, we focus on two common form of investment guarantees. However, our comparison can generally be expanded if other form of guarantees in mutual funds—for instance Asian type options—are embedded.

premiums paid into the contract. The second product includes a lookback guarantee, under which the payoff is defined by the number of units the client acquired over the contract term multiplied by the highest value of unit price achieved before maturity. The payoff for each product is highly dependent on the underlying fund strategy. In the case of a conventional fund with fixed average rate of return and standard deviation, guarantee costs can be derived. Alternatively, guarantees can be secured using a Constant Proportion Portfolio Insurance (CPPI) strategy via dynamic reallocation of the investment in risky and riskless assets.

Initial guarantee costs are determined using option pricing theory in a Black/Scholes framework. This pricing approach assumes the replicability of cash flows, which is a realistic assumption for product providers, but not usually feasible for investors. Thus, a comparative assessment of two investment alternatives (i.e., a mutual fund with either a lookback guarantee or an interest rate guarantee) will typically depend on risk-return preferences and can be based on performance measures. Further, if investors pay the same premium for either type of contract, only the risk-return profile of the maturity payout matters in the performance measurement.

To account for these issues and to obtain a comprehensive picture of the characteristics of mutual funds with investment guarantees, we employ the following procedure. We first calibrate guarantee costs to be the same for both guarantee products. Next, we investigate the characteristics of the maturity payoffs of these products by calculating descriptive statistics and by using three performance measures (Sharpe ratio, Omega, and Sortino ratio) for the two fund strategies (CPPI and average return and standard deviation). We also test for first, second, and third order stochastic dominance. Empirical results are derived for different μ-σ-efficient diversified portfolios based on stock, bond, real estate, and money market indices.

Comparing products with different guarantees is often difficult due to different maturity guarantees, different underlyings, and different payments by the client (caused by different guarantee costs). Hence, in order to ensure comparability, the premium payment is assumed to be the same for all cases under consideration. We first compare the situation where both investment funds provide a minimum interest rate guarantee of 0% (i.e., a money-back guarantee) and both funds' underlying is managed on the basis of a CPPI strategy. Because of the possibility of a (on average) higher stock portion in the case of an investment fund with an interest rate guarantee in a CPPI framework, we find a considerably higher expected payoff and standard deviation of the maturity payoff compared to the situation involving the lookback guarantee. Second, we analyze a case in which both products provide the same conventional underlying fund and the same implied guarantee costs. Even though both funds have quite similar expected payoffs in this case, the mutual fund with a lookback guarantee has roughly a 2.5% probability of resulting in a payoff below the minimum maturity guarantee promised by an interest rate guarantee. Furthermore, we find that neither investment alternative dominates the other by first, second, or third degree. Overall, the results illustrate the strong effect of fund volatility on the lookback guarantee, which can rapidly become very expensive compared to the interest rate guarantee.

The remainder of the paper is organized as follows. In Section 2, the model framework for the two different guarantee types is introduced. In Section 3, two different investment strategies concerning the underlying funds are derived. Section 4 provides the valuation of the implied guarantees and an analysis of the maturity payoff using descriptive statistics and different performance measures. Several numerical examples based on a Monte Carlo simulation are provided in Section 5. Section 6 concludes.

2. Model Framework

We assume that both products under consideration have a term of T years with constant monthly premium payments P at time $t_0 = 0$, $t_1, \ldots, t_{N-1}$ (with $\Delta t = t_j - t_{j-1} = 1/12$). The premiums are invested in a traded mutual fund and yield a stochastic payoff in $t_N = T$. The mutual fund is split into units, where $S(t_i)$ denotes the unit price of the fund at time t_i. Hence, the number of units acquired at time t_i is given by the premium payment divided by the unit price, i.e.,

$$n_{t_i} = \frac{P}{S_{t_i}}, \quad i \in \{0, \ldots, N-1\},$$

and the total number of units at time t_i before paying the $(i+1)^{st}$ premium is

$$N_{t_i} = \sum_{j=0}^{i-1} n_{t_j}, \quad i \in \{1, \ldots, N-1\}.$$

Mutual fund with interest rate guarantee

A fund with an interest rate guarantee provides the investor a minimum interest rate guarantee g on the premiums paid into the contract. Thus, the guaranteed maturity payment results in

$$G_T = P \cdot \sum_{j=0}^{N-1} e^{g(T-t_j)}.$$

For $g = 0$, this implies $G_T = N \cdot P$ and for g > 0, we obtain $G_T = P \cdot e^{gT} \cdot \frac{1-e^{-gT}}{1-e^{-g\Delta t}}$.

The value of the investment in T, F_T, is given by the number of acquired units N_T times the value of a unit, S_T, leading to

$$F_T = N_T \cdot S_T = P \cdot \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}},$$

or, equivalently, at time t

$$F_t = \left(F_{t-1} + P\right) \frac{S_t}{S_{t-1}}.$$

At maturity, the investor receives the terminal payoff L_T^G, which consists of the value of the investment in the underlying fund, which will be at least the guaranteed payment G_T, i.e.,

$$\begin{aligned} L_T^G &= \max\left(F_T, G_T\right) = \max\left(P \cdot \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}}, P \cdot \sum_{j=0}^{N-1} e^{g(T-t_j)} \right) \\ &= P \cdot \max\left(\sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}}, \sum_{j=0}^{N-1} e^{g(T-t_j)} \right) = P \cdot \tilde{L}_T^G. \end{aligned}$$

Thus, the amount of premium payments only serves as a scalar of the actual payoff. The payoff to the investor in T, L_T^G, can be written as the value of the underlying assets plus a put option on this value with strike G_T, such that

$$L_T^G = \max\left(F_T, G_T\right) = F_T + \max\left(G_T - F_T, 0\right). \quad (1)$$

Mutual fund with lookback guarantee

The fund with the lookback feature guarantees a payoff of the highest value (or peak) H_T of the index that has been attained during the policy term, where

$$H_T = \max_{j \in \{0,\ldots,N-1\}} S_{t_j}.$$

Thus, the payoff in T depends on the previous $N-1$ unit prices and can be written as

$$L_T^H = N_T \cdot H_T = P \cdot \sum_{j=0}^{N-1} \frac{\max_{j \in \{0,\ldots,N-1\}} S_{t_j}}{S_{t_j}} = P \cdot \tilde{L}_T^H.$$

The lookback guarantee's maturity payoff benefits from ups and downs in unit price. The worst case for the investor would be if the unit price of the underlying fund does not move at all, but remains constant over the contract term. As before, the exact amount of premium payments only serves as a scaling factor.

3. Investment Strategies of Underlying Funds

In the following, we compare two investment strategies: first, we model the underlying assets of a fund with fixed average rate of return and standard deviation during the policy term (the "conventional fund"). The second case involves an underlying fund that utilizes a Constant Proportion Portfolio Insurance (CPPI) strategy.

Conventional fund

Let $\left(W_t\right)$, $0 \le t \le T$, be a standard Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $(\mathcal{F}_t)$, $0 \le t \le T$, be the filtration generated by the Brownian motion. In the standard Black/Scholes framework, for the conventional fund, the unit price evolves according to a geometric Brownian motion. Hence, it can be described by the stochastic differential equation (under the objective measure $\mathbb{P}$)

$$dS_t = S_t\left(\mu dt + \sigma dW_t\right),$$

with constant drift μ, volatility σ, and a standard $\mathbb{P}$-Brownian motion W, assuming a complete, perfect, and frictionless market. The stochastic differential equation is solved by (see, e.g., Björk, 2004)

$$\begin{aligned} S_{t_j} &= S_{t_{j-1}} \cdot e^{\left(\mu-\sigma^2/2\right)\cdot\left(t_j-t_{j-1}\right)+\sigma\sqrt{\left(t_j-t_{j-1}\right)}\left(W_{t_j}-W_{t_{j-1}}\right)} \\ &= S_{t_{j-1}} \cdot e^{\left(\mu-\sigma^2/2\right)\cdot\left(t_j-t_{j-1}\right)+\sigma\sqrt{\left(t_j-t_{j-1}\right)}Z_{t_j}} \\ &= S_{t_{j-1}} \cdot R_{t_j}, \end{aligned}$$

where Z_{t_j} are independent standard normally distributed random variables. Hence, the continuous one-period return $r_{t_j} = \ln\left(R_{t_j}\right)$ is normally distributed with an expected value of $\mu - \sigma^2/2$ and standard deviation σ.

Constant Proportion Portfolio Insurance (CPPI) managed fund

In case of a conventional fund, guarantees have to be secured using risk management measures like, e.g., hedging, reinsurance, or equity capital. Instead of investing in risk management measures, guarantees can be secured using portfolio insurance strategies, which dynamically reallocate the investment portfolio so as to reach the maturity guarantee and, also, participate in rising markets (see O'Brien, 1988). Portfolio insurance was developed by Leland (1980) and Rubinstein and Leland (1981). In this context, Perold and Sharpe (1988) showed that these payoff strategies have to be convex, i.e., an increasing portion invested in stock when stock prices go up, and vice versa. CPPI was first introduced by Black and Jones (1987). CPPI secures the guarantees via continuous dynamic reallocation of the investment between two asset classes, namely, a risky and a riskless asset.[3]

Under the objective measure $\mathbb{P}$, the risky investment A evolves according to a geometric Brownian motion $dA_t = A_t\left(\tilde{\mu}_A dt + \sigma_A dW_t\right)$, and the riskless investment is a bond process B with a constant riskless rate of return r resulting in $dB_t = B_t r dt$. For a dis-

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[3] However, portfolio insurance programs may fail in case of high transaction costs, due to market liquidity risk, discontinuous price process (including jump components), or unexpected changes in the volatility of the underlying stocks (Rubinstein and Leland, 1981, p. 66). In the stock market crash in 1987 the two important preconditions market liquidity and continuous price processes were violated at the same time (Rubinstein, 1988, p. 39). In this case, an investor may not be able to adjust its stock positions in the asset portfolio to the degree demanded by the underlying trading strategy.

crete (monthly) adjustment of the share α_{t_j} in the risky asset and the share $\left(1-\alpha_{t_j}\right)$ in the riskless asset, the evolution of the underlying fund is given by

$$S_{t_j} = S_{t_{j-1}} \cdot \left(\alpha_{t_{j-1}} \cdot \frac{A_{t_j}}{A_{t_{j-1}}} + \left(1-\alpha_{t_{j-1}}\right) \cdot \frac{B_{t_j}}{B_{t_{j-1}}} \right) = S_{t_{j-1}} \cdot \left(\alpha_{t_{j-1}} \cdot e^{r_{t_j}^A} + \left(1-\alpha_{t_{j-1}}\right) \cdot e^{r\Delta t_j} \right).$$

In this setting, $r_{t_j}^A = \mu_A \cdot \Delta t_j + \sigma_A \cdot \sqrt{\Delta t_j} \cdot Z_{t_j}$ denotes the continuous one-period return of the risky investment with yearly expected value $\mu_A = \tilde{\mu}_A - \sigma_A{}^2/2$ and yearly standard deviation σ_A. The value of the accumulated investment in the mutual fund *S* in t_i before paying the *i*-th premium is given by

$$F_{t_i} = \left(F_{t_{i-1}} + P\right) \cdot \frac{S_{t_i}}{S_{t_{i-1}}} = \left(F_{t_{i-1}} + P\right) \cdot \left(\alpha_{t_{i-1}} \cdot e^{r_{t_i}^A} + \left(1-\alpha_{t_{i-1}}\right) \cdot e^{r\Delta t_i} \right),$$

$$F_0 = 0.$$

The guarantee to be secured in the case of the lookback guarantee is

$$G_{t_i}^H = P \cdot \sum_{j=0}^{i} \frac{\max_{0 \le k \le i} S_{t_k}}{S_{t_j}},$$

and in the case of the interest rate guarantee by

$$G_{t_i}^G = P \sum_{j=0}^{i} e^{g(T-t_j)}.$$

The cushion *C* for the risky investment results from the difference between the current fund value (including the current premium payment) and the present value of the guarantee *G*, giving

$$C_{t_i} = \left(F_{t_i} + P\right) - e^{-r(T-t_i)} \cdot G_{t_i}.$$

The stock exposure α_{t_i} in period $[t_i, t_{i+1})$ is limited by the factor α_0 and can be calculated as the product of the multiplier (or leverage) *m* and the cushion *C*, i.e.,

$$\alpha_{t_i} = \min \left\{ \max \left(\frac{m \cdot C_{t_i}}{F_{t_i}}, 0 \right), \alpha_0 \right\}.$$

The multiplier *m* corresponds to the investor's risk aversion. A high multiplier implies heavy participation in positive market developments through a high exposure in the risky investment. A low multiplier reduces the shortfall probability of the CPPI strategy.

4. Valuation of the Investment Guarantees and Performance Measurement

Net present value calculations are based on the replicability of the contract's cash flows with assets traded on the capital market. An individual that is able to replicate a cash flow will decide in favor of a contract if its net present value is positive. Hence, this decision does not depend on, e.g., the individual's degree of risk aversion. Even though the replicability of the contract's cash flow can be regarded as practicable for product providers, we think it is usually not feasible for buyers of mutual funds as, e.g., short selling of assets is in general required. Thus, a comparative assessment of two investment alternatives (here: a mutual fund with a lookback or an interest rate guarantee) from the viewpoint of a product buyer will typically depend on risk preferences.

One common way to proceed in this situation is to compare the expected discounted value of the contract's cash flows given a time separable utility function. In our case, the same sequence of premiums is paid into both contracts and hence, a preference dependent valuation of the maturity payoff is sufficient for comparison. Instead of adopting specific utility functions, risk return models can be used, which form the basis for performance measures. These models have the advantage of being easier to handle and only require explicit measures of risk and return as well as a functional relationship between risk and return. In what follows, we focus on three performance measurers, namely the Sharpe ratio, the Omega, and the Sortino ratio. The form of utility functions that makes a decision based on the Sharpe ratio, Omega or the Sortino ratio consistent with the concept of expected utility maximization is shown in, e.g., Fishburn (1977), Sarin and Weber (1993), Farinelli and Tibiletti (2008).

Valuation of the investment guarantee

In the case of a "conventional fund" (i.e., with given average rate of return and standard derivation for the contract term), prices for investment guarantees at time $t = 0$ will be obtained using risk-neutral valuation technique. Under the unique equivalent martingale measure $\mathbb{Q}$ (see Harrison and Kreps, 1979), the drift of the unit price process changes to the riskless rate of return *r*, leading to

$$dS_t = S_t\left(rdt + \sigma dW_t^{\mathbb{Q}}\right),$$

where $W^{\mathbb{Q}}$ is a standard $\mathbb{Q}$-Brownian motion. The net present value of the investment guarantee Π_0 at time $t = 0$ is given as the difference between the expected present value of the contract's payoff under the risk-neutral measure $\mathbb{Q}$ and the present value of the premiums paid, discounted with the riskless interest rate *r*:

$$\Pi_0 = E^{\mathbb{Q}}\left(e^{-rT}L_T\right) - P\cdot\sum_{j=0}^{N-1}e^{-rt_j} = P\cdot\left(E^{\mathbb{Q}}\left(e^{-rT}\tilde{L}_T\right) - \sum_{j=0}^{N-1}e^{-rt_j}\right).$$

The guarantee costs must be paid by the investor at time $t = 0$ in addition to the ongoing premium payments and the provider must invest them in risk management measures such as hedging strategies, equity capital, or reinsurance. In the case of a mutual fund with an interest rate guarantee, Π_0 can also be written as (see Equation (1))

$$\Pi_0^G = e^{-rT}\cdot E^{\mathbb{Q}}\left(\max\left(G_T - F_T, 0\right)\right),$$

which is the price of a European put option on the fund value at maturity with strike G_T.

Analysis of the maturity payoff

To analyze the maturity payoff L_T, we calculate its expected value $E\left(L_T\right) = P\cdot E\left(\tilde{L}_T\right)$ and standard deviation $\sigma\left(L_T\right) = P\cdot\sigma\left(\tilde{L}_T\right)$ under the objective measure $\mathbb{P}$. Furthermore, these figures can be used for performance measurement by way of the Sharpe ratio (see Sharpe, 1966). As a performance measure, the Sharpe ratio (*SR*) takes risk and return into account. For our case, we define the Sharpe ratio as the difference between the contract's expected payoff $E\left(L_T\right)$ and the value of the premium payments compounded to maturity $Y_T\left(= P\cdot\sum_{j=0}^{N-1}e^{r(T-t_j)}\right)$, divided by the standard deviation of the maturity payoff $\sigma\left(L_T\right)$:

$$Sharpe\ ratio\left(L_T\right) = \frac{E\left(L_T\right) - Y_T}{\sigma\left(L_T\right)}.$$

In addition to the Sharpe ratio, two other common performance measures—the Omega and the Sortino ratio—are employed, which use lower partial moments as the relevant risk measure. Lower partial moments belong to the class of downside-risk measures that describe the lower part of a density function; hence only negative deviations are taken into account (see, for example, Fishburn (1977), Sortino and van der Meer (1991)). The lower partial moment of order k is given as

$$LPM_k\left(L_T, Y_T\right) = E\left(\max\left(Y_T - L_T, 0\right)^k\right).$$

For decision making, the degree of risk aversion can be controlled by varying the power k. For $k = 0$, only the number of shortfall occurrences is counted; for $k = 1$, all deviations are weighted equally. Hence, the Omega (see Shadwick and Keating, 2002) and the Sortino measures (see Sortino and van der Meer, 1991) can be obtained by

$$Omega\left(L_T\right) = \frac{E\left(\max\left(L_T - Y_T, 0\right)\right)}{LPM_1\left(L_T, Y_T\right)},$$

$$Sortino\,ratio\left(L_T\right) = \frac{E\left(\max\left(L_T - Y_T, 0\right)\right)}{\sqrt{LPM_2\left(L_T, Y_T\right)}}.$$

The probabilities ψ that the fund value at maturity does not cover the promised guarantees are given by

$$\Psi^G = P\left(G_T > F_T\right),$$

and

$$\Psi^H = P\left(L_T^H > F_T\right)$$

for the funds with interest rate guarantee and lookback guarantee, respectively. The performance measures, as well as the probabilities ψ, do not depend on the amount of premiums paid into the contract.

Stochastic dominance

The two alternative investments are further tested for stochastic dominance. A discussion of the relation between stochastic dominance criteria and utility theory can be found in, e.g., Bawa (1975) and Levy (1992). Let F_1 denote the cumulative distribution function of L_T^G and F_2 denote the cumulative distribution function of L_T^H on the interval $[a, b]$. Then

- L_T^G dominates L_T^H by the first degree (FSD) if and only if $F_1(x) \leq F_2(x)\ all\ x \in [a,b]$;

- L_T^G dominates L_T^H by the second degree (SSD) if and only if $\int_{-\infty}^{x} F_1(t)\,dt \leq \int_{-\infty}^{x} F_2(t)\,dt\ all\ x \in [a,b]$;

- L_T^G dominates L_T^H by the third degree (TSD) if and only if $\int_{-\infty}^{x}\int_{-\infty}^{v} F_1(t)\,dt\,dv \leq \int_{-\infty}^{x}\int_{-\infty}^{v} F_2(t)\,dt\,dv\ all\ x \in [a,b]$ and $\int_{-\infty}^{b} F_1(t)\,dt \leq \int_{-\infty}^{b} F_2(t)\,dt$ (e.g., Aboudi and Thon, 1994).

All three cases require strict inequality for at least one x.

5. Simulation Analyses

Input parameters

Providers of investment products with guarantees typically hold a worldwide diversified portfolio of stocks, bonds, real estate, and money market instruments. In our analysis, we include one market index for each asset class: For stocks, we use the Equity Market Proxy used in Fama and French (1993) and Carhart (1997), which is a value-weighted portfolio of all NYSE, Amex, and Nasdaq stocks. Bonds, real estate and money market indices are given by JPM Global Government Bond, GPR General PSI Global, and the JPM US Cash 3 Month (as a proxy for the risk-free rate of return), respectively. We extracted monthly returns between January 1994 and December 2005 from the Datastream database to obtain mean and standard deviation of the annualized returns (Table 1), and to calculate their correlations (Table 2).

Table 1: Expected value μ_A and standard deviation σ_A of annualized return for selected indices

Asset class	Index	Abbreviation	μ_A	σ_A
Stocks	Equity Market Proxy	(S)	10.27%	16.16%
Bonds	JPM Global Goverment Bond	(B)	5.26%	6.49%
Real estate	GPR General PSI Global	(R)	9.47%	11.63%
Money Market	JPM US Cash 3 Month	(M)	3.57%	-

Notes: JPM: JPMorgan Chase & Co., GPR: Global Property Research, PSI: Property Share Index

Table 2: Correlation matrix for selected indices in Table 1

Index	(S)	(B)	(R)
(S)	1.00	-0.06	0.55
(B)	-0.06	1.00	0.09
(R)	0.55	0.09	1.00

The selected market indices can usually be acquired over index funds with low transaction costs. In addition, they are broadly diversified so that they are generally well suited for performance measurement (for criteria to select representative benchmark indices, see, e.g., Sharpe, 1992). Based on the indices' returns and their correlation, we calculate μ-σ-efficient portfolios under shortselling restrictions. Table 3 sets out efficient portfolios that will be used as the basis for the underlying funds in the simulation analysis.

Table 3: Efficient portfolios based on indices in Table 1

Portfolio	μ_A	σ_A	Share in (S)	Share in (B)	Share in (R)	Share in (M)
CPPI						
CP 1	6.00%	5.80%	7.65%	83.78%	8.58%	-
CP 2	7.50%	6.75%	16.34%	49.82%	33.85%	-
r	3.57%	-	-	-	-	100%
Conventional						
CV 1	6.00%	4.16%	10.30%	26.60%	21.91%	41.20%
CV 2	7.50%	6.74%	16.65%	43.02%	35.43%	4.90%

The money market index JPM US Cash 3 Month (M) represents the riskless investment for the CPPI managed fund (first part in Table 3). To obtain input data for the risky investment, efficient portfolios CP 1 and CP 2 were calculated based on stock

(S), bond (B), and real estate (R) indices, without the money market index. For the conventional fund (second part in Table 3), all four indices were included in the calculation of efficient portfolios. Thus, portfolios CV 1 and CV 2 will serve as underlying for the conventional fund.

The time to maturity of both products is given with T = 10 years with monthly premium payments P = 100. Concerning the CPPI strategy, the multiplier is fixed at m = 2 and ensures, in the numerical examples provided in the following section, the adherence of the investment guarantee. For the product with underlying CPPI strategy, the stock exposure α is initially limited to α_0 = 50%.

Because of path dependence and periodic premium payments, there is generally no closed-form solution for the payoff structure of the contracts.[4] Hence, numerical valuation is conducted using Monte Carlo simulation (see Glasserman, 2004) on the basis of 100,000 simulation runs with monthly reallocation of the portfolio when using CPPI strategy. To ensure that the simulation results for the two products are comparable, we used the same sequence of random numbers for all simulations.[5] To test for first, second, and third order stochastic dominance the method proposed in Porter, Wart, and Ferguson (1973) is implemented. The algorithm makes further use of two properties of the ordering rule that simplify the test procedure. First, a larger mean is a necessary condition for dominance; second, F_1 (L_T^G) cannot dominate F_2 (L_T^H) if the smallest value of L_T^H is larger than the smallest value of L_T^G. Hence, if these conditions are violated neither alternative dominates the other (that is if, e.g., $E\left(L_T^G\right) > E\left(L_T^H\right)$ but the smallest value of L_T^G is lower than the smallest value of L_T^H).

Two different cases are analyzed. First, we compare the two investment guarantees when both products are managed with CPPI and have the same minimum rate of return g of 0%. Second, both products are assumed to have the same underlying conventional fund with fixed parameters over the contract term. To increase comparability and be able to analyze the pure impact of the investment guarantees, we calibrate the guarantee such that the products have identical guarantee costs. We further analyze results for different efficient portfolios with different expected value and standard deviation of the return. In both cases under consideration, the sum of premiums paid into the con-

4 Under the Black-Scholes framework, closed-form solutions for path-dependent options such as the lookback guarantee exist in the case of an up-front premium payment, see, e.g., Conze and Viswanathan (1991), Goldman, Sosin, and Gatto (1979), Kat and Heynen (1995).

5 In the following calculations, the standard error for $E\left(L_T\right)$ is between 0.026% and 0.048%.

tract is 12,000. We have also compared other scenarios in order to test the robustness of results and have found that the general tendencies remain stable when varying the input parameters.

Investment guarantees with CPPI managed underlying fund

First, we study the case where the funds of both investment products are managed with CPPI and have a minimum rate of return of g = 0%. Descriptive statistics, performance, and cumulative distribution functions for interest rate and lookback guarantee are displayed in Figure 1. In Part a), the fund is given by portfolio CP 1; Part b) contains outcomes based on portfolio CP 2 with higher volatility (see Table 3).

Due to using CPPI, guarantee costs are implicitly contained in the contract's payoff. This also implies that the CPPI strategy leads to a zero probability ψ. Figure 1 illustrates that the fund with an interest rate guarantee has a higher expected value and a much higher standard deviation than the fund with the lookback guarantee, despite a minimum rate of return of 0%. When changing the underlying fund to portfolio CP 2 (Part b)), the maturity payoff's standard deviation in case of the interest rate guarantee is increased from 868 to 1,103, while the change in the standard deviation of the lookback guarantee is more moderate.

These results are due to the possibility of a (on average) higher stock portion α when using the CPPI strategy in the mutual funds with an interest rate guarantee compared to the case of a lookback guarantee. Further analysis showed that when raising the maximum share in the risky investment from α_0 = 50% to α_0 = 100%, mainly the standard deviation of the maturity payoff of the fund with interest rate guarantee is concerned. The standard deviation increases substantially due to even higher shares in the risky investment. In contrast, the distribution of the lookback guarantee shows almost no changes.

For these reasons, the fund with the interest rate guarantee has a considerably higher percentage of maturity payoffs that are above 14,500, and also several above 18,000, whereas the lookback guarantee payoffs peak out at 16,500. These results are evidence that a larger standard deviation combined with a minimum rate of return allows participation in positive market developments, thus leading to a higher probability of receiving a high payoff at maturity and, at the same time, be assured of receiving at least the guaranteed payoff.

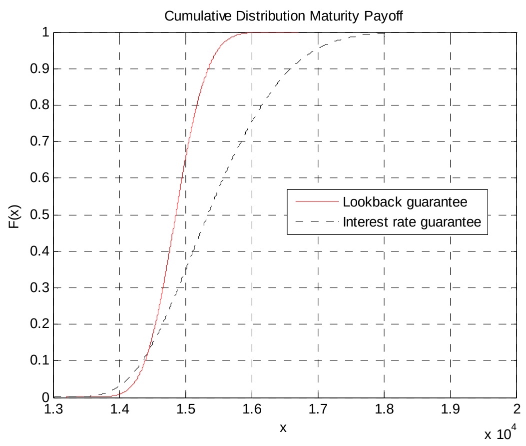
Figure 1: Results for interest rate and lookback guarantees for CPPI managed underlying fund

a) Portfolio CP 1: $\mu_A = 6.00\%$, $\sigma_A = 5.80\%$

Descriptive statistics and performance

	Interest	Lookback
Guarantee	12,000 ($g = 0\%$)	12,000 ($g = 0\%$)
ψ	0%	0%
$E(L_T)$	15,421	14,866
$\sigma(L_T)$	868	372
Sharpe ratio	1.13	1.14
Omega	27.75	20.69
Sortino ratio	7.70	5.61
FSD, SSD, TSD	none	none

Cumulative distribution functions

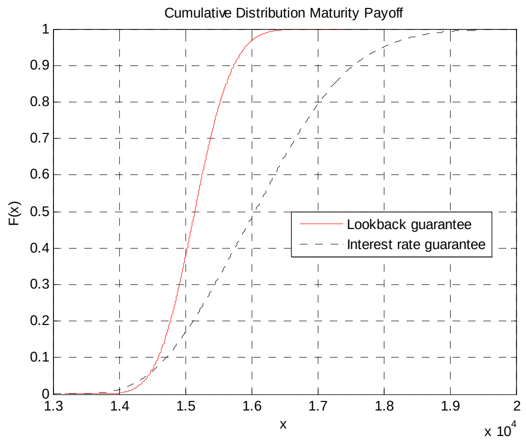


b) Portfolio CP 2: $\mu_A = 7.50\%$, $\sigma_A = 6.75\%$

Descriptive statistics and performance

	Interest	Lookback
Guarantee	12,000 ($g = 0\%$)	12,000 ($g = 0\%$)
ψ	0%	0%
$E(L_T)$	16,109	15,157
$\sigma(L_T)$	1,103	449
Sharpe ratio	1.51	1.59
Omega	112.11	86.23
Sortino ratio	19.93	14.44
FSD, SSD, TSD	none	none

Cumulative distribution functions



Notes: g = minimum rate of return; ψ = probability that value of fund at maturity is lower than guaranteed payoff; L_T = contract's payoff at maturity T; FSD, SSD, TSD = First, Second, Third Order Stochastic Dominance.

Furthermore, the performance of the two products depends on the type of measure chosen. Specifically, for portfolio CP 1 and CP 2 in Figure 1, the Sharpe ratio is slightly higher for the lookback guarantee; the Omega and the Sortino ratio are higher for the interest rate guarantee. This observation illustrates that it makes a difference how deviations from the expected value are taken into account. The Sharpe ratio uses the standard deviation as a measure of risk, which also includes upside deviations and thus chances. This leads to better results for the mutual fund with lookback guarantee. In contrast, the Omega and Sortino ratio solely evaluate downside risk using lower partial moments. Under these risk preferences, an investor would choose the fund with interest rate guarantee in this example.

Investment guarantees with conventional underlying fund

Second, results for the two guarantee products for a conventional underlying fund are compared. Figure 2 sets out results for interest rate guarantee and lookback guarantee. Part a) shows results for portfolio CV 1; Part b) is based on portfolio CV 2 (see Table 3 for details on the portfolios).

To ensure the guaranteed payments at maturity, both products require guarantee costs Π_0 that are due at the contract's inception, in addition to the monthly premium payments (see Figure 2, first row in list of descriptive statistics). Other financial guarantees are not embedded in the contracts under consideration and hence do not influence the guarantee costs. In particular, we assume that the investment guarantees expire if the investor stops paying premiums (paid-up case) or if the investor cancels the contract before maturity (surrender case).[6]

The lookback guarantee has a minimum rate of 0%, which would occur only if the unit price does not change at all and remains constant over the whole contract term. To make the contracts comparable, we calibrate the minimum rate of return g for the fund with an interest rate guarantee such that the initial guarantee costs are the same as for the lookback guarantee. This is achieved by setting the guaranteed interest rate to g = 2.80% in the case of portfolio CV 1 (Part a) in Figure 2), leading to guarantee costs of around 149.

The guarantee costs Π_0 need to be paid only in the case of a conventional underlying fund, if securitization is not achieved with CPPI. This implies that the guarantees have a positive probability ψ that the fund value is below the promised guaranteed maturity payment. For the interest rate guarantee, this probability is 1.57%; for the lookback

[6] In practice, instead of requiring an up-front payment, guarantee costs can also be embedded in the management expense ratio to be deducted each month from, e.g., the return or the deposited premiums. This means that the up-front guarantee costs are distributed over the whole contract term (but still remain the same in terms of the present value). Since we do not include other options to early terminate the contract in our analysis, and the additional premium imposed for covering the guarantee is immediately deducted and thus not invested in the mutual fund, distributing guarantee costs payments over the contract term does not influence the results.

guarantee, the probability of $\{L_T^H > F_T\}$ is much higher with $\psi = 54.16\%$. These shortfall events are secured through the initial up-front payment Π_0.

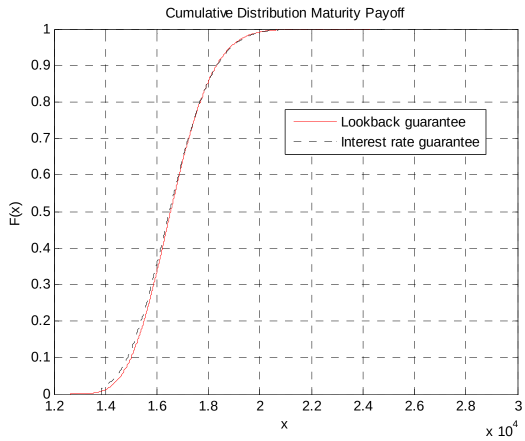
Figure 2: Results for interest rate and lookback guarantees for conventional underlying fund

a) Portfolio CV 1: $\mu_A = 6.00\%$, $\sigma_A = 4.16\%$

Descriptive statistics and performance

	Interest	Lookback
Π_0	149	149
Guarantee	13,865 ($g = 2.80\%$)	12,000 ($g = 0\%$)
ψ	1.57%	54.16%
$E(L_T)$	16,562	16,612
$\sigma(L_T)$	1,350	1,298
Sharpe ratio	1.57	1.67
Omega	23.43	23.67
Sortino ratio	120.27	163.12
FSD, SSD, TSD	none	none

Cumulative distribution functions

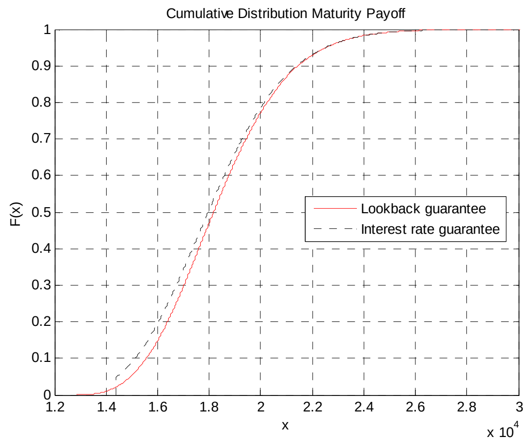


b) Portfolio CV 2: $\mu_A = 7.50\%$, $\sigma_A = 6.74\%$

Descriptive statistics and performance

	Interest	Lookback
Π_0	492	492
Guarantee	14,374 ($g = 3.48\%$)	12,000 ($g = 0\%$)
ψ	4.63%	62.57%
$E(L_T)$	18,181	18,384
$\sigma(L_T)$	2,410	2,323
Sharpe ratio	1.55	1.70
Omega	247.96	47.04
Sortino ratio	1,121.48	390.93
FSD, SSD, TSD	none	none

Cumulative distribution functions



Notes: Π_0 = value of the guarantee in $t = 0$; g = minimum rate of return; ψ = probability that value of fund at maturity is lower than guaranteed payoff; L_T = contract's payoff at maturity T; FSD, SSD, TSD = First, Second, Third Order Stochastic Dominance.

In contrast to the case with the CPPI managed underlying fund, the descriptive statistics show that the interest rate guarantee has a slightly lower expected maturity payoff but a higher standard deviation than the lookback guarantee. The standard deviation of the interest rate guarantee is mainly an upturn deviation, since the minimum maturity payoff is set at 13,865.

These characteristics of the payoff distribution affect the three performance measures (the Sharpe ratio, the Omega, and the Sortino ratio), which are displayed in the lower part of the table. The performance measures typically react very sensitively with respect to the risk of the payoff distribution. Hence, all three performance measures lead to lower results for the fund with an interest rate guarantee compared to the results for the fund with a lookback guarantee. Overall, the results are quite similar for both products, which are also confirmed by the cumulative distribution functions of the maturity payoffs.

Stronger differences can be observed in the case of the underlying portfolio CV 2 with a volatility of σ_A = 6.74% (Part b) in Figure 2). Here, the costs for the lookback guarantee increase to 491. This corresponds to 3.40% of the sum of premium payments (compared to 1.03% for portfolio CV 1). To reach this value, the guaranteed interest rate g needs to be raised considerably to 3.48%. At the same time, the probability ψ increases to 4.63% compared to 1.57% for portfolio CV 1 (where g was 2.80%) for the fund with interest rate guarantee, and to 62.57% for the fund with lookback guarantee.

Compared to Part a), especially the standard deviation of the maturity payoffs is much higher, which is also illustrated by the cumulative distribution functions. According to these curves, there is one difference in the range of maturity payoffs below 14,374 where the lookback guarantee exhibits nearly 2.50% of such realizations. In contrast, the payoff of the interest rate guarantee does not fall below 14,374 due to the minimum guarantee of g = 3.48%. At the same time, the lookback guarantee has a slightly higher probability for larger payoffs.

These outcomes illustrate that in the case of the lookback guarantee, the characteristics of the underlying fund play a central role. In particular, the underlying fund's volatility should be moderate, since otherwise, the lookback guarantee becomes very expensive. For portfolio CV 1 with μ_A = 6.00% and a low volatility of σ_A = 4.16%, for instance, the lookback guarantee corresponds (in terms of guarantee costs) to a guaranteed interest rate of 2.80%. This is considerable, given that the riskfree rate r is 3.57%. For portfolio CV 2, the guaranteed rate must even be raised to 3.48%.

6. Summary

This paper compares the pricing and performance of mutual funds having interest rate guarantees with those having lookback guarantees. The impact of the underlying fund with respect to the embedded guarantees was analyzed by comparing a conventional fund with constant parameters over the contract term and a CPPI managed fund. Results were derived in a simulation analysis for different diversified portfolios based on stock, bond, real estate, and money market indices.

Private investors typically make a decision on which of two products to purchase based on risk preferences. Hence, to compare the mutual fund with interest rate guarantee to the mutual fund with lookback guarantee, we examined the characteristics of these investment guarantees for differently managed underlying funds by calculating descriptive statistics of the maturity payoffs (expected value and standard deviation) and three performance measures that reflect different risk preferences (Sharpe ratio, Omega, Sortino ratio). We further determined the probability that the maturity fund value is below the guaranteed payment, and tested for first, second, and third order stochastic dominance. Premium payments were assumed to be the same for all cases under consideration.

In our numerical analysis, we first analyzed the case where both products have a minimum interest rate guarantee of 0% (money-back guarantee) and an underlying fund that is CPPI managed. In contrast to a conventional underlying, securitization was achieved through CPPI and thus its costs are implicitly contained in the payoff, without the investor needing to make an additional payment at inception of the contract. In this example, the fund with an interest rate guarantee had a higher expected maturity payoff, a much higher standard deviation, and a higher probability of large maturity payoffs. These results were even stronger when the underlying portfolio was changed to one with a higher standard deviation. These outcomes are due to the possibility of an on average higher stock exposure in the case of the mutual fund with an interest rate guarantee for the CPPI managed underlying. Due to the high volatility of the interest rate guarantee product, the Sharpe ratio was slightly higher for the fund with the lookback guarantee. The Omega and Sortino ratio were higher for the fund with the interest rate guarantee due to using lower partial moments and thus downside risk measures.

Second, we considered the case where both investment guarantees have a conventional underlying fund and the same guarantee costs. The results were very similar for both products. One difference was that the lookback guarantee had an approximately 2.5% probability of having a maturity payoff below the minimum guaranteed payment of the product with an interest rate guarantee, but a higher probability for larger maturity payouts. Due to the higher standard deviation of the interest rate guarantee payoff, all three performance measures were higher for the lookback guarantee, a result that changes when the underlying fund's volatility increases. In this scenario, the Omega and Sortino ratio led to higher values for the interest guarantee due to the limitation of downside risk. For all cases under consideration, neither investment alternative dominated the other by first, second, or third order.

Our results show that the maturity payout for funds with a lookback guarantee is very sensitive to the underlying fund's volatility. Unless the volatility is kept low, the lookback guarantee becomes very expensive. This is also confirmed in the case of the underlying CPPI strategy, where volatility is substantially reduced due to a higher share in the riskless investment. This is different from the interest rate guarantee product, which allows for higher volatility when managed via CPPI and thus a higher upside potential for investors.

In our examples, investors with a risk-return profile reflected in the Omega and Sortino ratio performance measures (both based on lower partial moments) would prefer the mutual fund with an interest rate guarantee in the case of a CPPI managed underlying or in the case of a conventional underlying fund with higher volatility. If an investor's decisions are based on the Sharpe ratio, risk is taken into account using the payoff's standard deviation, which includes upside deviations as well. Given this criterion, for this investor, the lookback guarantee would be preferable over the interest rate guarantee. Overall, the cases examined in this paper provide insight into two forms of investment guarantees and their corresponding risk and return profiles regarding the payoff distribution at maturity, information of great relevance to potential investors having different risk-return preferences.

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