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QUANTIFYING CREDIT AND MARKET RISK UNDER SOLVENCY II: STANDARD APPROACH VERSUS INTERNAL MODEL

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ABSTRACT

Even though insurers predominantly invest in bonds, credit risk associated with government and corporate bonds has long not been a focus in their risk management. After the crisis of several European countries, however, credit risk has recently been paid greater attention. Nevertheless, the latest version of the Solvency II standard model (QIS 5), provided by regulators for deriving solvency capital requirements, still does not require capital for credit risk inherent in, e.g., EEA issued government bonds from Greece or Spain. This paper aims to provide an alternative approach and compares the standard model with a partial internal risk model using a rating-based credit risk model that accounts for credit, equity, and interest rate risk inherent in a portfolio of stocks and bonds. The findings demonstrate that solvency capital requirements strongly depend on the quality and composition of an insurer's asset portfolio and that model risk in regard to model choice and calibration plays an important role in the quantification.

Keywords: Solvency II; internal model; rating-based credit risk model; market risk; credit risk
JEL Classification: G22, G32, G38

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1. INTRODUCTION

Credit risk has long not been in the focus of many insurance companies, even though a major part of their capital investments are held in government and corporate bonds generally exposed to credit risk. However, the recent crises in Greece, Ireland, Spain or Portugal have demonstrated the strong need for adequate credit risk models for insurers, since it cannot generally be taken for granted that highly indebted countries obtain the needed financial support. For this reason, credit risk modeling has received increased attention in insurers' risk management. In this paper, we compare the latest proposed standard model of 2010/2011 to be used in the European supervisory system Solvency II (planned to be in force from 2013 on) to quantify market and credit risk for a non-life insurance company with a partial internal model that assesses the market risk situation of an insurer. Special focus is paid to the effect of credit risk while further examining the impact of dependencies between the relevant processes with respect to diversification benefits. Furthermore, we analyze model risk for the internal approach regarding the model choice as well as the model calibration.

Since the Basel II reform of European banking supervision in 2006, insurance supervision has also fundamentally been reformed. In particular, the European Union (EU) Solvency II regulation will impose risk-based capital requirements for insurance companies and is planned to be implemented after 2013, thereby explicitly accounting for market and credit risks. To calculate the solvency capital requirements (SCR), insurers have the option to choose between five different methods. Besides the standard formula provided by the regulator, the SCR can be calculated by using the standard model with a partial internal model, with undertaking-specific parameters, with simplifications, or by modeling the insurers' risks with a full internal model approved by the supervisors (see European Parliament and of the Council, 2009, Article 112, No. 1 to 7).[1] However, the latest proposed standard model of 2010/2011 does not require capital concerning credit spread risk for investments in government bonds issued by countries of the European Economic Area (EEA) or borrowings guaranteed by one of these states, including, e.g., Greece or Ireland.

While credit risk has been extensively researched in the context of the valuation of defaultable bonds[2], applications to insurance companies have hardly been addressed so far. With respect

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1 See Gatzert and Wesker (2012) for a detailed overview of the different methods to derive the SCR according to Solvency II along with a comparison to Basel II/III, and Eling, Schmeiser, and Schmit (2007) for an overview of the Solvency II framework in general.

2 See Black and Scholes, 1973, Merton, 1973, 1974, Black and Cox, 1976, Leland 1994, and Longstaff and Schwarz, 1995, for structural models, and Jarrow and Turnbull, 1995, Das and Tufano, 1996, Jarrow, Lando,

to Solvency II, a study by Fitch Ratings (Piozot et al., 2011) discusses the implications of the new regulatory regime in regard to the insurers' asset allocation and the attractiveness of different asset classes. One main finding is that insurers will likely have to increase their investments in higher-rated corporate and government bonds. At the same time, Solvency II implies investments in shorter-term bonds instead of long-term debt due to *ceteris paribus* higher capital requirements for long durations (while the asset-liability mismatch is simultaneously reduced) as well as a low level of equity holdings.

In addition, as a consequence of the special treatment in terms of the capital requirements for government bonds from members of the EEA under Solvency II, the authors also assume that these bond exposures will gain more importance for the insurers' asset allocation. However, in this context, it should also be taken into account that even if the standard model is used instead of an internal model, according to Solvency II's Pillar 2, the insurer's own risk and solvency assessment (ORSA) requires an adequate assessment of the company's risk situation, which also includes the consideration of credit risk. Thus, even if the standard model currently does not require adequate capital for credit risk of EEA states, this risk will certainly have to be taken into account within the ORSA process.

A different aspect is analyzed in a study by Mittnik (2011), which focuses on the applied calibration procedure for the standard model and particularly points out flaws regarding the use of the rolling-window annualization procedure. For instance, wrongly implied correlations between asset classes, even if returns are independent, do not adequately reflect diversification benefits. Christiansen, Denuit, and Lazar (2012) examine the calibration of the aggregation formula ("square-root formula") used to derive the life underwriting risk in the Solvency II standard model. Applying a stochastic model for an internal approach, they identify the correlation matrix in the life module of the Solvency II standard model as not appropriate, leading to an overestimation for the underlying German data set. Further critical discussions about the aggregation formula in the Solvency II standard model can be found in Sandström (2007) and Pfeifer and Strassburger (2008). Sandström (2007) shows how the standard formula needs to be recalibrated if the probability distributions of the underlying risk factors are skewed (instead of being symmetric and normally distributed, even if risks are independent) in order to ensure consistency. Pfeifer and Strassburger (2008) point out how the overall SCR is misspecified even if the aggregate probability distribution of the risks is symmetric and if the underlying risks are uncorrelated but dependent.

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and Turnbull, 1997, Duffie and Singleton, 1997, Lando, 1998, and Duffie and Singleton, 1999, regarding reduced-form models.

Thus, the question arises to what extent an internal model leads to different capital requirements as compared to the Solvency II standard model in the current calibration if market risks including equity, interest rate and credit spread risk are adequately quantified. Therefore, the aim of this paper is to compare an alternative internal approach with the standard model in the case of a non-life insurance company, thereby focusing on the asset side and the induced market risk. We specifically investigate the importance of an adequate quantification of credit spread risk with regard to the capital investment of insurers, and thus focus on the insurer's asset side, looking at stocks and bonds as the relevant asset classes. In a first step, it is demonstrated how the solvency capital has to be determined for a given portfolio of stocks, government bonds, and corporate bonds by using the latest proposed Solvency II standard approach as laid out in the QIS 5 quantitative impact study. Second, the market and credit risk for stocks and bonds is modeled based on an internal approach. Based upon these results, we analyze the SCR of the two approaches to compare their effectiveness in identifying major market risk sources. In addition, model risk associated with the two approaches that may arise from a misestimation of input parameters in the calibration process is studied.

In the internal market and credit risk model, Monte Carlo simulation is used to derive the necessary solvency capital based on the Value at Risk at a 99.5% confidence level as required under Solvency II. To quantify the market risk of stocks, the development of equity prices is described by a geometric Brownian motion. Concerning the market and credit risk of fixed income government and corporate bonds, two main types of risks are taken into account. Interest rate risk is quantified based on the model by Cox, Ingersoll, and Ross (1985) (CIR) to derive the risk-free term structure of interest rates. Credit risk is integrated to calculate the market value of a bond portfolio at the end of a given period using the rating-based credit risk model of Jarrow, Lando, and Turnbull (1997) (JLT), which includes risk factors with respect to credit default and credit spread. The rating transition process is determined by a time-homogenous Markov chain based on empirical transition rates published by rating agencies. The partial internal model for the market and credit risk also enables examination of the impact of different dependencies between stock price and interest rate. To account for model risk regarding the choice of the underlying processes, the Heston (1993) model is integrated for stocks, the CIR model is replaced by the Vasicek (1977) approach and the reduced-form model by Duffie and Singleton (1999) is implemented as an alternative for credit risk modeling.[3]

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3 Regarding credit risk modeling, we focus on reduced-form credit risk models since the lack of firm-specific data limits the application of structural models, in particular for portfolios with a large number of credit risk sensitive assets.

Our results show the considerable discrepancy between the SCR for market risk calculated by the Solvency II standard formula and the internal approach. In particular, depending on the portfolio composition, capital requirements can be reduced through an internal approach using more distinguished assumptions and fully accounting for diversification benefits. Conversely, they can also be increased, which is especially the case for, e.g., low-rated bonds that appear to be underestimated in the standard model. Moreover, the findings emphasize the importance of considering the credit risk of government bonds issued by members of the EEA and AAA- or AA-rated non-EEA government bonds. Furthermore, model risk regarding processes and calibration plays an important role and should be taken into account when quantifying credit and market risk.

The remainder of this paper is structured as follows. Section 2 provides a general overview and introduction to Solvency II and the standard model with focus on the market risk module. Section 3 presents the quantitative framework of the Solvency II standard model and the alternative internal model approach. The results of the numerical analysis are discussed in Section 4, and Section 5 concludes.

2. OVERVIEW: MARKET AND CREDIT RISK UNDER SOLVENCY II

According to the Directive 2009/138/EC (Solvency II), Article 101, the insurance company is treated as a going concern for a period of twelve months. Hence in quantifying its SCR, the insurance company has to take the existing business as well as the new business in this time period into consideration based on expected values.[4] With the objective to cover unexpected losses, the Value at Risk is chosen as the relevant risk measure by the EU. Thus, the SCR is defined as “the Value-at-Risk of the basic own funds (…) subject to a confidence level of 99.5% over a one-year period” (see European Parliament and of the Council, 2009, Article 101, No. 3).

QIS 5 is said to constitute the final official test before implementing Solvency II after 2013. The standard model is designed as a bottom-up approach, divided into six different risk modules for determining the basic SCR (BSCR) as exhibited in Figure 1, including life, non-life, health, market, and default risk as well as intangibles. Additionally, operational risk and adjustments for loss absorbency of technical provisions and for loss absorbency of deferred taxes have to be taken into account to obtain the total SCR.

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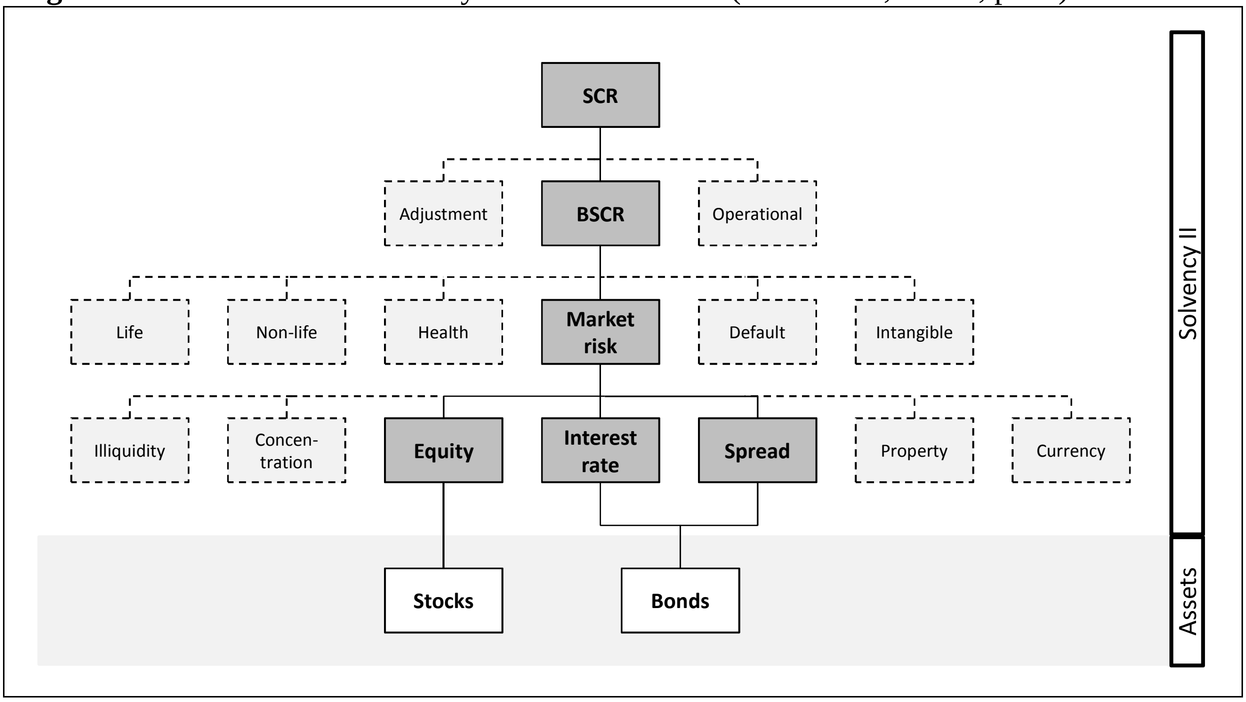
[4] With respect to new business, the expected loss has to be taken into consideration in addition to the unexpected loss.

The latest report on the Solvency II standard approach, published by the Federal Financial Supervisory Authority in Germany (BaFin), identified market risk as the largest risk driver for life and health insurers and the second largest in the property-casualty sector of the German insurance industry (see BaFin, 2011, pp. 16, 18, 21).[5] As shown in Figure 1, the market risk module of the Solvency II framework is divided into seven sub-modules. With respect to credit risk, three modules are of relevance, including spread risk and market risk concentrations, as a part of the market risk module as well as counterparty default risk. The latter default risk module is an extension of the spread risk sub-module, containing counterparty default risks that are not defined as market risk. This includes, for example, other risk mitigating contracts, cash at banks or receivables from intermediaries (see EIOPA, 2010a, pp. 134-135). Concentration default risk "(…) is restricted to the risk regarding the accumulation of exposures with the same counterparty" (see EIOPA, 2010a, p. 127) considered in the equity risk, property risk and spread risk sub-module. Excluded are concentration risks to geographical areas or industry sectors, governments issued by members of the EEA or OECD and assets covered by the counterparty default risk module (see EIOPA, 2010a, pp. 127, 131).

Figure 1: Structure of the Solvency II SCR calculation (see EIOPA, 2010a, p. 90)



According to the QIS 5 report from the BaFin for the German insurance industry, it is particularly spread risk that requires the second largest solvency capital in the market risk sub-module after interest rate risk in the case of life and health insurers (see BaFin, 2011, pp. 16,

5 Excluding diversification effects.

18). For German property-casualty insurers, only equity risk requires more capital in the market risk module (see BaFin, 2011, p. 21).[6] The spread risk module thereby contains all risks that result from changes in the credit spread (over the risk-free interest rate term structure). Hence, it combines default risk, rating transition risk, and all other risks responsible for variations in the market value for all assets and liabilities sensitive to changes in the credit spread in one sub-module. For the selected asset portfolio of stocks and bonds in the following analysis, we consider the market risk sub-modules equity risk, interest rate risk and spread risk, which represent the main important risk drivers and are highlighted in Figure 1.

Based on predefined scenarios, the capital requirements for the market risk module cover the variation of the market value of financial instruments for a time horizon of one year through a mark to market approach (see EIOPA, 2010a, p. 92; European Parliament and of the Council, 2009, Article 101, No. 3). The calculation is based on basic own funds, which are defined as the difference between the market value of assets and the best estimates of the liabilities. The Solvency II bottom-up approach also takes diversification effects into account through correlations in the aggregation process of risk modules. In the standard model, the aggregated solvency capital requirements (SCR_{ag}) for the BSCR and each risk module market, life, health and non-life, SCR_r, are defined through the so-called “square-root formula” given by

$$SCR_{ag} = \sqrt{\sum_{r,c} CorrSCR_{r,c} \cdot SCR_r \cdot SCR_c}, \tag{1}$$

where $CorrSCR_{r,c}$ denotes the pairwise correlation coefficients of module r and c, given by a predefined correlation matrix (see EIOPA, 2010a, pp. 95-96, 107-108, 148-149, 166, 196-197).[7]

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6 Excluding diversification effects.

7 The square-root formula in Equation (1) produces an exact result for the Value at Risk only for jointly normally distributed risks, since it only results in a coherent risk measure. Artzner et al. (1999) show that the Value at Risk satisfies the characteristic of subadditivity (and therefore is a coherent risk measure) for normally distributed risks for a significance level with $\alpha \leq 0.5$. More generally, Embrechts, McNeil, and Straumann (2002) expand the Value at Risk as a coherent risk measure for elliptically contoured distributions with $\alpha \leq 0.5$.

3. MODEL FRAMEWORK

3.1 Standard approach under Solvency II

The basis for the calculation of the SCR under the Solvency II standard model for market risk is the net asset value (*NAV*). In line with the definition of basic own funds, the *NAV* is defined as the difference between assets *A* and liabilities *L*, excluding subordinated liabilities, which are sensitive to the considered risk of the particular sub-module (see EIOPA, 2010a, pp. 91-92). Changes in the *NAV* as a result of a shock scenario are denoted by ΔNAV for a considered (sub-) module. Hence, a positive ΔNAV implies a loss as a consequence of a given scenario; in the case of a negative ΔNAV, it is set to zero (see EIOPA, 2010a, p. 92). In the following analysis, we focus on the asset side and thus assume that the liabilities of the non-life insurer are not affected by changes in credit and market risk, i.e. $L = L | shock$. Hence, ΔNAV is defined as

$$\begin{aligned}\Delta NAV &= \max\left(NAV - \left(NAV \mid shock\right), 0\right) = \max\left(\left(A-L\right) - \left(\left(A-L\right) \mid shock\right), 0\right) \\ &= \max\left(A - \left(A \mid shock\right), 0\right).\end{aligned} \tag{2}$$

Based on this definition, the ΔNAV for the equity, interest rate and spread risk (sub-) module represents the basis for the SCR calculations, which is referred to as Mkt_{eq}, Mkt_{int} and Mkt_{sp}.

SCR in the equity risk sub-module

The SCR for market risk resulting from fluctuations in equity prices of all equity price sensitive assets is calculated based upon the market value $MV_{eq,i}(0)$ for the investment exposure *i* in the equity risk sub-module. The shock scenario differentiates between two investment classes to determine the SCR in this sub-module. First, the risk class “Global” includes all exposures transacted in countries that are members of the EEA or the Organisation for Economic Co-operation and Development (OECD) (see EIOPA, 2010a, p. 113). In this case, the scenario approach assumes a decrease in equity by 30%. Based on the market value $MV_{eq,i}(0)$ at time t = 0, the SCR for the risk class “Global”, $Mkt_{eq,Global}$, results from

$$Mkt_{eq,Global} = \max\left(0.3 \cdot MV_{eq,Global}(0), 0\right) = \max\left(0.3 \cdot \sum_{i \in Global} MV_{eq,i}(0), 0\right).$$[8]

Second, "Other" is defined as the class of higher risks, which contains all other equity price sensitive assets such as hedge funds, alternative investments, non-listed equities as well as exposures in emerging markets. Here, the shock scenario is given by a drop of 40%, implying

$$Mkt_{eq,Other} = \max\left(0.4 \cdot MV_{eq,Other}(0), 0\right) = \max\left(0.4 \cdot \sum_{i \in Other} MV_{eq,i}(0), 0\right).$$

In the case of strategic participations, a stress factor of 22% is assumed for both risk categories in the equity risk sub-module. Participations that are not strategically oriented are stressed with the general factors of 30% and 40%. An exceptional position is given to participations that are excluded from the scope of group supervision according to the Directive 2009/138/EC, Article 214, with a stress of 100% (see EIOPA, 2010a, p. 282). Furthermore, participations in financial or credit institutions do not have to be stressed, but are excluded from own funds, which implicitly produces a stress of 100%.

The standard capital stress (SCS) in the equity risk sub-module is set to 39% ("Global") and 49% ("Other"). To mitigate potential pro-cyclical effects of adverse capital market developments, a symmetric adjustment of -9 percentage points is applied in QIS 5 for both risk classes (see EIOPA, 2010a, p. 114; European Parliament and of the Council, 2009, Article 106). Thus, the following analysis generally uses 30% and 40%, respectively, until stated otherwise. In general, the symmetric adjustment mechanism is calibrated based on the MSCI World Developed price index $I(t)$ at time t by an adjustment term $adj(t)$ and a beta factor $\beta(t)$, limited by an upper and lower bound of +/-10%:

$$adj(t) \cdot \beta(t) = \begin{cases} -0.1 & \text{, if } adj(t) \cdot \beta(t) < -0.1 \\ adj(t) \cdot \beta(t) & \text{, if } adj(t) \cdot \beta(t) \in [-0.1, 0.1] \\ 0.1 & \text{, if } adj(t) \cdot \beta(t) > 0.1 \end{cases}, \text{ with } adj(t) = \frac{I(t) - \frac{1}{780} \cdot \sum_{s=t-1}^{t-780} I(s)}{\frac{1}{780} \cdot \sum_{s=t-1}^{t-780} I(s)}.$$

The adjustment at time t is thus defined as a function of the MSCI index and a weighted average of the MSCI index to a period of three years (780 trading days).[9] Furthermore, through a

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[8] Following the ΔNAV approach in Equation (2), the SCR for the risk class "Global" in the equity price sub-module is given by $\Delta NAV = MV_{eq,Global}(0) - (1-0.3) \cdot MV_{eq,Global}(0) = 0.3 \cdot MV_{eq,Global}(0)$, thus assuming a decrease in equity prices by 30% (to 70%).

regression of the MSCI index on its weighted average (based on 780 trading days), the beta function $\beta(t)$ is determined, such that the adjusted capital stress (ACS) at time t is given by $ACS(t) = SCS + adj(t)\cdot\beta(t)$.[10] The shock parameters for the Solvency II standard approach are summarized in Table 1.

Table 1: Equity shock scenarios in the Solvency II standard model given the adjustment against pro-cyclical effects *ACS* according to QIS 5 (EIOPA, 2010a, p. 114)

	Equity	Strategic Participation	Non-Strategic Participation	Financial Participation*	Excluded Participation
Global	0.30	0.22	0.30	0.00	1.00
Other	0.40	0.22	0.40	0.00	1.00

** Participations in financial or credit institutions are directly excluded from own funds.*

Table 2: Correlations in the equity risk sub-module of the Solvency II standard model (see EIOPA, 2010a, p. 115)

	Global	Other
Global	1.00	0.75
Other	0.75	1.00

Correlations between the two risk classes are taken into account in a last step through a predefined correlations matrix presented in Table 2.[11] The correlation parameters induce diversification effects between the risk classes by calculating the SCR for the equity risk sub-module Mkt_{eq} according to the square-root formula (see also Equation (1)) given by

$$Mkt_{eq} = \max\left(\sqrt{\sum_{r_{eq},c_{eq}} CorrIndex_{r_{eq},c_{eq}} \cdot Mkt_{eq,r_{eq}} \cdot Mkt_{eq,c_{eq}}}, 0\right), \; r_{eq}, c_{eq} \in \{Global, Other\}. \qquad (3)$$

Concerning the SCR calculation in the equity risk sub-module, the market value $MV_{eq,i}(0)$ of stock exposure i is determined by the invested capital $A_{S,i}(0)$ at the starting time $t = 0$ with

$$MV_{eq,i}(0) = A_{S,i}(0).$$

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9 In QIS5, the symmetric adjustment $adj(t)$ is calculated based on an weighted average of three years. However, CEIOPS suggests a time period of one year (see EIOPA, 2011, pp. 15-16).

10 See EIOPA, 2010c, pp. 41-50. For simplification, $\beta(t)$ can be set to one (see EIOPA, 2010c, p. 45).

11 A critical analysis of the derivation of the correlation structure of assets within the class “Other” (set to 1) is presented in Mittnik (2011), where flaws regarding the annualization procedure of asset returns are pointed out that induce a misleading assessment of correlations and thus neglect diversification benefits.

SCR in the interest rate risk sub-module

The influence of a change in the term structure of the interest rate is determined by the interest rate risk sub-module. The present value of all interest rate sensitive exposures PV_{int} is given by discounting the respective cash flows using the risk-free interest rate $r_f(t)$ at time t and given by the European Commission, such that

$$PV_{int} = \sum_{t=1}^{T} \frac{CF(t)}{\left(1+r_f(t)\right)^t},\ T = \max\left(t \mid CF(t) \neq 0\right),\ CF(t) = \sum_j CF_j(t), \tag{4}$$

where $CF_j(t)$ is the single cash flow of exposure j at time t. The interest rate risk sub-module distinguishes between two stress scenarios, namely an increase and a decrease of the interest term structure. Thus, the stressed present value is calculated twice, adding an upward $s^{up}(t)$ and a downward movement $s^{down}(t)$ to the risk-free interest term structure that depends on time t. These two stressed present values are denoted by PV_{int}^{up} and PV_{int}^{down} with

$$PV_{int}^{k} = \sum_{t=1}^{T} \frac{CF(t)}{\left(1+r_f(t)\cdot\left(1+s^k(t)\right)\right)^t},\ T = \max\left(t \mid CF(t) \neq 0\right),\ k \in \{up, down\}$$

Table 3 exhibits the predefined stress parameters in the Solvency II standard model for selected point of times. Maturities less than one year will be stressed with the one-year stress parameters. For durations larger than 25 years, the shock is determined by the relative change of 0.25 for the upward and -0.30 for the downward scenario.

Table 3: Interest rate shock in the Solvency II standard model (see EIOPA, 2010a, p. 111)

Maturity t (years)	Relative change $s^{up}(t)$	Relative change $s^{down}(t)$
1	0.70	-0.75
2	0.70	-0.65
3	0.64	-0.56
…	…	…
25	0.26	-0.30
>25	0.25	-0.30

Finally, to obtain the SCR for the interest rate risk sub-module with the standard approach Mkt_{int}, the differences of present value without stress and the present values under stress Mkt_{int}^{up} and Mkt_{int}^{down} have to be calculated, i.e.

$$Mkt_{int} = \max\left(Mkt_{int}^{up}, Mkt_{int}^{down}\right), \text{ where } Mkt_{int}^{k} = PV_{int} - PV_{int}^{k},\ \ k \in \{up, down\}.$$

The cash flows $CF_j(t)$ of a bond exposure j with maturity $T_j = \max\left\{t \mid CF_j(t) \neq 0\right\}$ are determined by the annual coupon payments $C_j(t)$, the face value FV_j, and the number of bonds n_j,

$$CF_j(t) = \begin{cases} C_j(t) \cdot FV_j \cdot n_j & , \ t < T_j \\ \left(1 + C_j(t)\right) \cdot FV_j \cdot n_j & , \ t = T_j. \end{cases} \tag{5}$$

The face value is set to one for the different bonds, $FV_j = 1$. Thus, the number of bonds in Equation (5) is determined by

$$n_j = \frac{A_{B,j}(0)}{B_j^{FV=1}(0)},$$

where $A_{B,j}(0)$ denotes the invested capital in bond exposure j and $B_j^{FV=1}(0)$ the price of the bond with $FV_j = 1$ at time $t = 0$.

SCR in the spread risk sub-module

The impact of changes of the credit spread (over the risk-free interest rate term structure) on exposures is considered in the rating-based spread risk sub-module of the Solvency II standard approach. The SCR for spread risk Mkt_{sp} is calculated based on three uncorrelated groups of exposures, including the SCR for bonds Mkt_{sp}^{bonds}, the SCR for structured credit products Mkt_{sp}^{struct}, and the credit derivatives SCR Mkt_{sp}^{cd}, yielding

$$Mkt_{sp} = Mkt_{sp}^{bonds} + Mkt_{sp}^{struct} + Mkt_{sp}^{cd}.$$

In the following, we only focus on the SCR of bond exposures. Analogously to the equity sub-module, the SCR calculation for the spread risk of bonds is based on the market value $MV_{sp,j}(0)$ of asset j. The extent of the asset-individual stress in this rating-based approach depends on the modified duration and a rating-specific stress parameter. The modified duration of exposure j, denoted by $duration_j$, is the weighted average time to maturity divided by the yield to maturity. As the technical specifications of QIS 5 do not specify this point, we use the Macaulay (1938) duration modified with the discounted yield to maturity r_{YtM},[12] given by

$$duration_j = \frac{\sum_{t=1}^{T_{\max}} t \cdot CF_j(t) \cdot \left(1+r_f(t)\right)^{-t}}{\sum_{t=1}^{T_{\max}} CF_j(t) \cdot \left(1+r_f(t)\right)^{-t}} \cdot \frac{1}{1+r_{YtM}},\ T_{\max} = \max\left(t \mid CF(t) \neq 0\right),$$

with the risk-free interest term structure r_f provided by the European Commission. The duration is limited by a lower limit (floor) and an upper limit (cap). The shock parameter in regard to the credit quality, $F^{up}(rating_j)$, depends on the current credit rating and the type of bond and is exhibited in Table 4 for corporates and non-EEA governments.

Table 4: Spread shock for corporates and non-EEA governments in the Solvency II standard model (see EIOPA, 2010a, pp. 122-123)

	Spread shock corporates			Spread shock non-EEA governments		
Rating	F^{up}	Duration Floor	Duration Cap	F^{up}	Duration Floor	Duration Cap
AAA	0.9%	1	36	-	-	-
AA	1.1%	1	29	-	-	-
A	1.4%	1	23	1.1%	1	29
BBB	2.5%	1	13	1.4%	1	23
BB	4.5%	1	10	2.5%	1	13
B or lower	7.5%	1	8	4.5%	1	10
Unrated	3.0%	1	12	3.0%	1	12

If several different ratings exist for one exposure, the second best rating has to be applied (see EIOPA, 2010a, p. 121). A special treatment in this sub-module is dedicated to (mortgage and public sector) covered bonds with an AAA credit rating. If a bond meets the requirements of "undertakings for collective investment in transferable securities" from the European Parliament and of the Council (see European Parliament and of the Council, 2005, Article 22, No. 4), the stress parameter of this best quality bond is set to 0.6% instead of 0.9% with a duration upper limit of 53 years. Furthermore, an exceptional position is given to exposures of government bonds of EEA states issued in their domestic currency or a currency of an EEA country (see CEIOPS, 2010). According to the Solvency II standard approach, no solvency capital has to be allocated for such investments. Besides government bonds, this treatment also in-

[12] The yield to maturity r_{YtM} of bond exposure j is given by solving the equation $PV_{int,j} = \sum_{t=1}^{T_j} CF_j(t) \cdot \left(1+r_{YtM}\right)^{-t}$, where $PV_{int,j}$ denotes the present value using the risk-free interest rate $r_f(t)$ at time t, given by the European Commission as exhibited in Equation (4).

cludes borrowings issued by multilateral development banks, international organizations or the European Central Bank, irrespective of the assets' currency. The asset class of "non-EEA governments" includes all in the domestic currency denominated and funded government bonds from non-EEA states or central banks.

In contrast to the equity risk sub-module, no correlation effects between the assets are assumed at this point. Hence, the SCR for the spread risk for bonds *j* is given by

$$Mkt_{sp}^{bonds} = \max\left(\sum_j MV_{sp,j}(0) \cdot duration_j \cdot F^{up}\left(rating_j\right), 0\right).$$

The market value $MV_{sp,j}(0)$ of bond exposure *j* at time $t = 0$ for the spread risk sub-module in Solvency II is, analogously to the equity risk sub-module, given by the invested capital $A_{B,j}(0)$ with

$$MV_{sp,j}(0) = A_{B,j}(0).$$

Aggregation to the market risk module

The SCR of the market risk module SCR_{mkt}^{SII} is calculated by aggregating the SCRs of the isolated sub-modules using the square-root formula and taking into account dependencies between the single risk categories,

$$SCR_{mkt}^{SII} = \sqrt{\sum_{r,c} CorrMkt_{r,c} \cdot Mkt_r \cdot Mkt_c}, \ where\ r, c \in \{eq, int, sp\}. \tag{6}$$

The correlation parameters $CorrMkt_{r,c}$ for the sub-modules are given in Table 5, whereby the correlation parameter *C* for the interest rate risk varies in the standard model depending on the adopted stress scenario in the interest rate risk sub-module:

$$C = CorrMkt_{eq,int} = CorrMkt_{sp,int} = \begin{cases} 0 & , if\ Mkt_{int} = Mkt_{int}^{up} \\ 0.5 & , if\ Mkt_{int} = Mkt_{int}^{down}. \end{cases}$$

In the case where market risk is only relevant for the asset side, $Mkt_{int} = Mkt_{int}^{up}$.

Table 5: Correlations in the market risk module in the Solvency II standard model (see EIOPA, 2010a, pp. 108-109)

	Interest	Equity	Spread
Interest	1.00	C	C
Equity	C	1.00	0.75
Spread	C	0.75	1.00

3.2 Partial internal model

Modeling stocks

In the partial internal model, we assume that stocks follow a geometric Brownian motion,

$$dS(t) = \mu_S \cdot S(t)\,dt + \sigma_S \cdot S(t)\,dW_S^{\mathbb{P}}(t) \tag{7}$$

with constant drift μ_S and volatility σ_S, and $W_S^{\mathbb{P}}$ being a standard $\mathbb{P}$-Brownian motion on the probability space $(\Omega_S, \mathcal{F}_S, \mathbb{P})$ with filtration $\mathcal{F}_S$ and real world probability measure $\mathbb{P}$. For an initial value $S(0)$, the solution of the stochastic differential equation in (7) is given by

$$S(t) = S(0) \cdot e^{\left(\mu_S - \frac{\sigma_S^2}{2}\right) \cdot t + \sigma_S \cdot \sqrt{t} \cdot Z_S(t)},$$

where $Z_S(t)$ denotes independent standard normally distributed random variables (see Björk, 2009). Hence, the market value of a portfolio of N_S stocks at time t = 1, $MV_S(1)$, is given by

$$MV_S(1) = \sum_{i=1}^{N_S} S_i(1),$$

where $S_i(1)$ is the market value of stock i at time 1.

Modeling and valuation of bonds

To determine the risks arising from the stochasticity of the interest term structure, we first study the market value of a non-defaultable zero coupon bond $p(t,h)$ at time t that pays out one monetary unit at time h, $t \leq h$ and $p(h,h) = 1$. The zero bond price is defined through the short rate $r(t)$ on the probability space $(\Omega_r, \mathcal{F}_r, \mathbb{Q})$, where $\mathcal{F}_r$ is the filtration generated by the Brownian motion under the risk-neutral probability measure $\mathbb{Q}$ and is given by (see Björk, 2009)

$$p(t,h) = E_t^{\mathbb{Q}}\left(e^{-\int_t^h r(s)ds}\right),$$

where $r(t)$ is given by the Cox, Ingersoll, and Ross (1985) model,

$$dr(t) = \kappa_r \cdot (\theta_r - r(t))dt + \sigma_r \cdot \sqrt{r(t)}dW_r^{\mathbb{Q}}(t). \tag{8}$$

Here, κ_r controls the speed of the mean reversion to the long-term mean θ_r, σ_r is the volatility, and $W_r^{\mathbb{Q}}$ is a standard $\mathbb{Q}$-Brownian motion on the probability space $(\Omega_r, \mathcal{F}_r, \mathbb{Q})$. The CIR process is characterized through a mean reverting drift, for which the condition $2 \cdot \kappa \cdot \theta \geq \sigma_r^2$ provides a strictly positive short rate for all t. In this setting, the zero bond price can be specified in a closed affine form, given by

$p(t,h) = e^{A(t,h) - B(t,h) \cdot r(t)}$, where

$$A(t,h) = \frac{2 \cdot \kappa \cdot \theta}{\sigma_r^2} \cdot ln\left(\frac{2 \cdot a \cdot e^{(\kappa + a) \cdot \frac{h-t}{2}}}{(\kappa + a) \cdot \left(e^{a \cdot (h-t)} - 1\right) + 2 \cdot a}\right),\ B(t,h) = \frac{2 \cdot \left(e^{a \cdot (h-t)} - 1\right)}{(\kappa + a) \cdot \left(e^{a \cdot (h-t)} - 1\right) + 2 \cdot a},$$
$$a = \sqrt{\kappa^2 + 2 \cdot \sigma_r^2}.$$

Under the real world probability measure $\mathbb{P}$, Equation (8) changes to

$$\begin{aligned} dr(t) &= \left(\kappa_r \cdot \theta_r - (\kappa_r - \gamma_0 \cdot \sigma_r) \cdot r(t)\right)dt + \sigma_r \cdot \sqrt{r(t)}dW_r^{\mathbb{P}}(t) \\ &= \hat{\kappa}_r \cdot \left(\hat{\theta}_r - r(t)\right)dt + \sigma_r \cdot \sqrt{r(t)}dW_r^{\mathbb{P}}(t), \end{aligned}$$

where the market price of risk $\gamma(t, r(t))$ is derived from $\gamma(t, r(t)) = \gamma_0 \cdot \sqrt{r(t)}$ (see Brigo and Mercurio, 2007). Furthermore, we assume that stocks and interest rates are correlated with $dW_r^{\mathbb{P}} dW_S^{\mathbb{P}} = \rho_{r,S} dt$.

With respect to credit risk, we use the reduced-form model by Jarrow, Lando, and Turnbull (JLT) (1997) in order to harmonize the procedure of the SCR calculation with the Solvency II standard model that is also based on credit transition. In the work of Jarrow and Turnbull (1995), the state of a defaultable bond is only described by the default or non-default state. The framework of Jarrow, Lando, and Turnbull (1997) extends this model by quantifying the credit risk through credit ratings and the probability of a change in the credit rating. Transition rates for credit ratings allow taking the consequences of up- and downgrading the credit quality into account. Furthermore, independence between interest rate and the default process and deterministic credit spreads are assumed. The spreads are equal for a given rating and vary by migration. In particular, the credit spreads that affect the bond value are taken into account, but are treated as deterministic variables. Das and Tufano (1996) extend the work of Jarrow, Lando, and Turnbull (1997) by incorporating stochastic recovery rates that are correlated with the interest rate process. A framework that further takes into consideration correlations between the interest rate process and the default intensity (credit spread) is introduced by Lando (1998) and Duffie and Singleton (1999).[13] Following the JLT model, the credit transition is assumed to follow a Markov process X,

$$X = \left(x(t), t \in \mathbb{N}_0\right) \tag{9}$$

on the probability space $\left(\Omega_x, \mathcal{F}_x, \mathbb{Q}\right)$ and distribution $\left(\Lambda_x\right)_{x\in E}$, which is given by

$$\Lambda(t,h) = \begin{pmatrix} \lambda_{1,1}(t,h) & \cdots & \cdots & \lambda_{1,k}(t,h) \\ \vdots & & & \vdots \\ \lambda_{k-1,1}(t,h) & \cdots & \cdots & \lambda_{k-1,k}(t,h) \\ 0 & 0 & \cdots & 1 \end{pmatrix} \tag{10}$$

with transition probabilities $\left(\lambda_{i,j}(t,h)\right)_{i,j\in E}$ for a state space E = {1,…k}. The transition distribution represents the probabilities of attaining state j at time h when starting at state i at time t, satisfying the conditions $\lambda_{i,j}(t,h) \geq 0$, $i \neq j$, and $\lambda_{i,i}(t,h) = 1 - \sum_{\substack{j=1 \\ i\neq j}}^{k} \lambda_{i,j}(t,h)$. By setting a discrete state space E with dimension k, the stochastic transition process in Equation (9) corresponds to a Markov chain in discrete time ($t \in \mathbb{N}_0$). The Markov chain is adapted to the filtration $\mathcal{F}_\tau = \left(\mathcal{F}_{\tau,t}\right)_{t\in\mathbb{N}_0}$ with the stopping time τ described by

$$\tau = \inf\left\{t \in \mathbb{N} : x(t) = k\right\},$$

[13] In addition, Jarrow and Turnbull (2000) discuss the problem of quantifying credit risk in terms of the intersection of market and credit risk. While economic theory and empirical evidence confirm the intrinsic relation between market and credit risk, regulators as well as practitioners generally calculate both risks separately due to the complex determination of the correlation between market risk and credit risk. Reduced-form credit risk models as from, e.g., Jarrow and Turnbull (1995) and Jarrow, Lando, and Turnbull (1997) consider market and credit risk jointly with the assumption of independency. However, the reduced-form approaches published by, e.g., Lando (1998) and Duffie and Singleton (1999) allow for implicit correlations between risk factors, which, however, are very difficult to calibrate (see Jarrow and Turnbull, 2000), which in turn increases the risk of misestimation.

where the state k is absorbent, also shown in the last row of the matrix in Equation (10). The transition matrix in Equation (10) assumes a complete and arbitrage-free market. Furthermore, apart from non-defaultable bonds, the JLT model assumes the existence of defaultable zero coupon bonds for all maturities. In the case of default, only a deterministic and exogenously given fraction of a non-defaultable zero coupon bond, the recovery rate (of treasury) δ, will be paid out at maturity h.[14] According to the transition process in Equation (9), the probability of default depends on state $x(t)$ at time t. Assuming independence between the interest rate and the transition process, the price of a defaultable zero coupon bond $\hat{p}_{x(t)=i}(t,h)$ with rating $x(t) = i$ is given by

$$\begin{aligned}\hat{p}(t,h)_{x(t)=i} &= E_t^{\mathbb{Q}}\left(\mathbb{I}_{\{\tau>h\}}\cdot e^{-\int_t^h r(s)ds}+\mathbb{I}_{\{\tau\le h\}}\cdot\delta\cdot e^{-\int_t^h r(s)ds}\right)=E_t^{\mathbb{Q}}\left(e^{-\int_t^h r(s)ds}\cdot\left(\mathbb{I}_{\{\tau>h\}}+\mathbb{I}_{\{\tau\le h\}}\cdot\delta\right)\right)\\ &= p(t,h)\cdot\left(\delta+(1-\delta)\cdot\left(1-\lambda_{i,k}(t,h)\right)\right),\end{aligned}$$

where $\mathbb{I}_{\{\tau\le h\}}$ represents the indicator function, which is equal to one if a default occurs until time h and zero otherwise, and $1-\lambda_{i,k}(t,h)$ denotes the probability of non-default from time t to h. The risk-neutral transition probabilities $\left(\lambda_{i,j}(t)\right)_{i,j\in E}$ can be interpreted as risk-adjusted transitions and are received by an adjustment of the real world distribution $\left(\tilde{\Lambda}_x\right)_{x\in E}$. In particular, to obtain risk-neutral transition probabilities that ensure an arbitrage-free market, the real world transition probabilities have to be adjusted in the JLT model by a risk premium $\pi_{x(t)=i}(t)$ that depends on time t and rating $x(t) = i$, setting

$$\lambda_{i,j}(t,t+1)=\begin{cases}\pi_{x(t)=i}(t)\cdot\tilde{\lambda}_{i,j} & ,\ i\neq j\\ 1-\pi_{x(t)=i}(t)\cdot\left(1-\tilde{\lambda}_{i,j}\right) & ,\ i=j,\end{cases} \tag{11}$$

where the second constraint ($i = j$) in Equation (11) ensures a row sum of one in the risk-neutral distribution in Equation (10).[15] In matrix form, Equation (11) can be written as

$$\Lambda(t,h)=\Pi(t)\cdot\left(\tilde{\Lambda}(t,h)-\mathrm{I}\right)+\mathrm{I}$$

with a $k\times k$ matrix $\Pi(t)=\mathrm{diag}\left(\pi_{x(t)=1}(t),\ldots,\pi_{x(t)=k-1}(t),1\right)$ and a $k\times k$ identity matrix I. Furthermore, the Markov chain of the real world distribution is assumed to be a time-

[14] The JLT model offers a recovery rate δ that is paid out at maturity (recovery of treasury value assumption) and depends on the issuers' seniority. In contrast, we assume a constant recovery rate δ for all considered bond exposures.

[15] The risk premium $\pi_{x(t)=i}(t)$ is assumed to be a non-stochastic function independent of migration state j.

homogenous process (see McNeil, Frey, and Embrechts, 2005), achieved through the assumption of constant distributions of $\left(\tilde{\Lambda}_x\right)_{x\in E}$, exhibited by

$$\tilde{\Lambda}(t,t+1)=\left(\tilde{\lambda}_{i,j}(t,t+1)\right)_{i,j\in E}=\left(\tilde{\lambda}_{i,j}\right)_{i,j\in E}=\tilde{\Lambda}.$$

Based on the results of the zero coupon bond valuation, the price for a defaultable fixed income bond exposure j at time t, $B_j(t)$ is calculated as the sum of future cash flows $CF_j(h)$ (see also Equation (5)), multiplied with the defaultable zero coupon bond prices (see Björk, 2009) that are given by the JLT credit risk model, which includes interest rate risk, spread risk, and credit risk for bonds. Therefore, the bond price at time t with maturity T_j is calculated by

$$B_j(t)=\sum_{h=t+1}^{T_j} CF_j(h)\cdot \hat{p}_{x(t)=i}(t,h). \tag{12}$$

The stochastic market value $MV_B(1)$ at time t = 1 of a portfolio with N_B bonds (without reinvestment) is given by

$$MV_B(1)=\sum_{j=1}^{N_B}\left(\mathbb{I}_{\{\tau_j>1\}}\cdot\left(B_j(1)+CF_j(1)\right)+\mathbb{I}_{\{\tau_j\le 1\}}\cdot\delta\cdot FV_j\cdot n_j\right),$$

where n_j is the number of bonds of type j (see Equation (5)).

SCR in case of the internal model

In accordance with Solvency II, the SCR of the internal model for market risk is defined as the capital needed to cover the change in the net asset value over one year, which in the case of market risk corresponds to the change in the market value of assets during a one-year period. Thus, the Value at Risk of the change in basic own funds is calculated for a confidence level of 99.5% (see EIOPA, 2010a, p. 92), yielding to an SCR of[16]

[16] Under the simplified assumption of unchanged liabilities (in particular not affected by market risk in case of a non-life insurer), $L(0)=e^{-\int_0^1 r(t)dt}\cdot L(1)$ as well as the properties of translation invariance and positive homogeneity (see McNeil, Frey, and Embrechts, 2005), the Value at Risk of basic own funds is given by $VaR_\alpha\left(\left(A(0)-L(0)\right)-e^{-\int_0^1 r(t)dt}\cdot\left(A(1)-L(1)\right)\right)=VaR_\alpha\left(A(0)-e^{-\int_0^1 r(t)dt}\cdot A(1)\right)=A(0)-VaR_{1-\alpha}\left(e^{-\int_0^1 r(t)dt}\cdot A(1)\right)$, where the expression $\int_0^1 r(t)dt$ is approximated using the composite trapezoidal rule (Newton-Cotes formula) for numerical integration (see Press et al., 2007).

$$SCR_{mkt}^{IM} = MV_{S+B}(0) - VaR_{0.005}\left(e^{-\int_0^1 r(t)dt} \cdot MV_{S+B}(1)\right), \tag{13}$$

where $MV_{S+B}(t)$ denotes the market value of an asset portfolio consisting of fixed income bonds and stocks at time t, given by $MV_{S+B}(t) = MV_S(t) + MV_B(t)$. Furthermore, the market value of the portfolio at time $t = 1$ is discounted with the risk-free interest rate $r(t)$, given by the CIR short rate model.[17]

3.3 Model risk

To assess model risk associated with the use of an internal model, in addition to varying the input parameters, we further replace the relevant key processes for stocks, bonds, and credit risk. Toward this end, first the Heston (1993) approach is adopted as an alternative to the geometric Brownian motion for modeling stock prices, which is characterized by a stochastic variance process,

$$dS(t) = \mu_S \cdot S(t)dt + \sqrt{V(t)}dW_S^{\mathbb{P}}(t),$$
$$dV(t) = \hat{\kappa}_V \cdot \left(\hat{\theta}_V - V(t)\right)dt + \sigma_V \cdot \sqrt{V(t)}dW_V^{\mathbb{P}}(t).$$

Here, μ_S denotes the drift of the price process $S(t)$ and the variance process $V(t)$ reverts to the long-term variance $\hat{\theta}_V$ with a speed of mean reversion $\hat{\kappa}_V$ and a standard deviation σ_V. $W_S^{\mathbb{P}}$ and $W_V^{\mathbb{P}}$ are standard $\mathbb{P}$-Brownian motions on the probability space $\left(\Omega_r, \mathcal{F}_r, \mathbb{P}\right)$ with $dW_S^{\mathbb{P}} dW_V^{\mathbb{P}} = \rho_{S,V} dt$, where $\rho_{S,V}$ is the coefficient of correlation.

Second, in regard to the interest rate process, the short rate model of Vasicek (1977) is used instead of the CIR model, which is given by

$$dr(t) = \kappa_r \cdot \left(\theta_r - r(t)\right)dt + \sigma_r dW_r^{\mathbb{Q}}(t).$$

with speed of the mean reversion κ_r, long-term mean θ_r, volatility σ_r and standard $\mathbb{Q}$-Brownian motion $W_r^{\mathbb{Q}}$.[18]

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[17] A discussion of different SCR definitions and the discounting factor in Solvency II can be found in Christiansen and Niemeyer (2012).

[18] The relationship between the empirical and the risk-neutral parameter is given by $\kappa = \hat{\kappa}$ and $\theta = \hat{\theta} - (\gamma_0 \cdot \sigma)/\kappa$ with market price of risk γ_0 (see Vasicek, 1977).

Third, concerning the impact of the model choice with respect to credit risk, the reduced-form credit risk model of Duffie and Singleton (1999) is applied, where the default event is modeled as a function of the hazard rate, i.e. through a Cox process. The price of a defaultable zero coupon bond is thus expressed by

$$\hat{p}(t,h)_{x(t)=i} = E_t^{\mathbb{Q}}\left(e^{-\int_t^h r(s)+h_i(t,s)\cdot(1-\delta)ds}\right)$$

with short rate $r(t)$, hazard rate $h_i(t,s)$ for rating $x(t) = i$, and recovery rate δ at time t.[19] Here, the expression $sp(t,s) = h_i(t,s)\cdot(1-\delta)$ represents a time-dependent credit spread from time t to s. In contrast to the JLT model, the Duffie and Singleton (1999) approach thus implies a recovery of market value at the time of default and, furthermore, explicitly allows for correlations between interest rate and default risk.[20]

3.4 Portfolio building and diversification effects

When calculating the SCR of a portfolio of stocks or bonds or a portfolio composed of both asset classes, diversification benefits imply a reduction in the aggregated SCR, both in case of the standard model and the internal model. Considering stocks or bonds, the diversification effect for the first level diversification d_1 is defined as

$$d_1(K) = \frac{SCR_K}{\sum_{k\in K} SCR_k} - 1,\ K \in \{S, B\} \tag{14}$$

with SCR_k denoting the SCR for individual assets k (from the asset classes of stocks (S) or bonds (B)), and SCR_K denoting the SCR for a portfolio of stocks or a portfolio of bonds. To quantify the diversification benefits for a portfolio of both stocks and bonds, the second level diversification d_2 is defined as

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[19] This assumes constant recovery rates δ for all exposures analogously to the JLT model.

[20] The model by Duffie and Singleton (1999) allows for integrating correlations between interest rate $r(t)$ and credit spread dynamic $sp(t,s)$ depending on a state variable $Y(t) = (Y_1(t),\ \ldots, Y_n(t))$ at time t:
$r(t) = a_0 + a_1 \cdot Y_1(t) + \ldots + a_n \cdot Y_n(t)$,
$sp(t) = b_0 + b_1 \cdot Y_1(t) + \ldots + b_n \cdot Y_n(t)$.
The state variable $Y(t)$ can be modeled by an affine process, e.g. an CIR process, with independent Brownian motions and leads to correlated interest rate and credit spread dynamics (see, e.g. Duffie and Singleton, 1997, 1999).

$$d_2\left(S+B\right)=\frac{SCR_{(S+B)}}{SCR_S+SCR_B}-1 \tag{15}$$

with SCR_S denoting the SCR for the stock portfolio, SCR_B the SCR for the bond portfolio and SCR_{S+B} the SCR for the portfolio including stocks and bonds. In this way, the isolated diversification benefit of combining a stock portfolio and a bond portfolio can be assessed, as diversification benefits within each asset class are already accounted for when calculating SCR_B and SCR_S.

4. Numerical Results

In the following numerical analysis, the SCR calculation for market risk with respect to stocks and fixed income bonds for the standard approach of Solvency II and the internal model is illustrated. In a first step, the SCR of the asset class of stocks is analyzed for a company investing $S(0)$ = €100 million at $t = 0$, i.e. $MV_S(0) = S(0)$, (for business in force). Second, we consider the SCR calculation for corporate and government bonds only, thereby distinguishing between government bonds of EEA states and non-EEA members, also investing $MV_B(0) = B(0)$ = €100 million in the bond market. The effect of investing €100 million in an asset portfolio that consists of stocks and bonds (50% each) on the SCR, accounting for diversification effects, is then examined in a third step. Finally, the impact of varying the stock portion α on the SCR is studied for the scenario-based Solvency II standard model and the simulation-based internal approach. Numerical results for the market risk according to the internal model are derived through Monte Carlo simulation with 100,000 paths.[21] Moreover, for all portfolios considered in the numerical analysis, we assume that capital is equally distributed between the different types of stocks and bonds.

4.1 Input parameters

In the following, we assume a stock portfolio consisting of three “Global” stocks and one riskier stock investment from the risk class “Other” (defined by EIOPA, 2010a, p. 113). Table 6 shows the corresponding annualized expected return $m_S = \mu_S - 0.5 \cdot \sigma_S^2$ with standard deviation σ_S.[22] The considered fixed income corporates and governments are given in Table 7 and

[21] For robustness, all numerical results have been calculated using different sets of sample paths to ensure stability.

[22] Regarding the Heston (1993) model, the mean reverting variance process of the DAX 30 price index (S_1) is calibrated to the volatility index VDAX that specifies the implicit volatility of the DAX 30 index (see Grünbichler and Longstaff, 1996). Maximum likelihood estimation techniques are used based on monthly da-

differ by time to maturity, the coupon payment, various credit ratings, as well as exposures issued by members of the EEA or not.

Table 6: Stock portfolio (with annualized parameters)

S_i	Index	Type	m_S	σ_S
1	DAX 30	Global	0.0637	0.2164
2	FTSE 100	Global	0.0436	0.1658
3	Dow Jones Industrial	Global	0.0755	0.1784
4	India BSE 100	Other	0.1043	0.3309
5	MSCI World	Global	0.0509	0.1574

Notes: The parameters are estimated based on monthly data from 01/1988 to 07/2011 with S_1: DAX 30 (price index), S_2: FTSE 100 (price index), S_3: Dow Jones Industrials (price index), S_4: India BSE 100 (price index) and S_5: MSCI World (price index).

Table 7: Bond portfolio

B_i	Type	Rating	EEA	Maturity (years)	Coupon p.a. (%)
1	Corporate	AA	-	3	1.25
2	Corporate	A	-	5	3.15
3	Corporate	B	-	5	9.25
4	Government	AAA	Yes	5	2.00
5	Government	A	Yes	3	2.75
6	Government	B	Yes	5	6.10
7	Government	AAA	No	5	2.75
8	Government	BBB	No	7	7.85
9	Government	B	No	7	8.95

Notes: The data are taken from a database of straight fixed-income bonds, issued in the period range 2/2010 to 5/2011, with B_1: Colgate-Palmolive Company, B_2: Woolworth Ltd. Company, B_3: Air Canada, B_4: Germany (Government of), B_5: Czech Republic (Government), B_6: Greece (Republic of), B_7: Canada (Government), B_8:Russian Federation (Government) and B_9: Belarus (Republic of).

The distribution of the time-homogenous Markov process in Equation (9), describing the credit transition, is based on a report from Standard & Poor's (see Vazza, Aurora, and Kraemer, 2010). This presents the average one-year transition rates for global corporate bonds measured based on bond data from 1981 to 2009 (see Appendix, Table A.1). The last column of Table A.1 refers to corporates no longer rated (NR) by Standard & Poor's. The average one-year transition rates for foreign currency ratings of governments, based on data from 1975 to 2010 (see Appendix, Table A.2), are also published by Standard & Poor's (see Chambers, Ontko, and Beers, 2011). To deal with transition rates identified as "NR", we follow the pro-

ta from 12/2005 to 07/2011, resulting in $\hat{\theta}_V$ = 0.0555, $\hat{\kappa}_V$ = 3.3497, and σ_V = 0.3593 (see, e.g. Brigo et al., 2009). The initial value is set to $V(0)$ = 0.0555 and the market price of risk is assumed to be zero (γ_0 = 0). Besides the price index and interest rate correlation, given by $\rho_{S,r}$ = -0.19 (see Table 8), the correlation between the price and the prices' instantaneous variance is estimated with $\rho_{S,V}$ = -0.4533.

cedure of Bangia et al. (2002) and disregard this information by distributing the rate proportionally to all seven rating classes and the default state. The transition rates after eliminating the “NR” column with the procedure mentioned above are given in Tables A.3 and A.4 in the Appendix.[23]

To determine the recovery level in case of default of fixed income bonds for the internal model, we follow the specification of the internal ratings based (IRB) approach to credit risk of Basel II, the international standards for banking regulations, where the supervisory value for the recovery rate of senior claims on corporates and sovereigns not secured by recognized collateral is set to a value of 55% (see BIS, 2006, p. 67).[24] In the numerical simulation of the internal model, correlations between stock prices and interest rate are taken into consideration analogously to the diversification effects in the Solvency II standard approach. Here, the Gauss copula of a multivariate normal distribution is used (see McNeil, Frey, and Embrechts, 2005).[25] The parameters for the linear correlation matrix P are given in Table 8, where r represents the interest rate term structure given by the CIR model and S_i is the equity price for stock i.

Regarding calibration of the CIR model, the three month “Euro Interbank Offered Rate” (EURIBOR) is used with monthly data from 01/1999 to 12/2011 based on maximum likelihood estimates.[26] The long-term mean level and the speed of mean reversion are thus estimated

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[23] The hazard rate $h_i(t,h)$ with rating $x(t) = i$ from time t to h of the Duffie und Singleton (1999) model is defined by $h_i(t,h) = \left(\lambda_{i,k}(t,h) - \lambda_{i,k}(t-1,h)\right) / \left(1 - \lambda_{i,k}(t-1,h)\right)$ (see McNeil, Frey, and Embrechts, 2005). In the numerical analysis, we refrain from applying a state variable $Y(t)$ for taking correlations into account, as is similarly done by practitioners and regulators, which generally measure credit and market risk separately (see Jarrow and Turnbull, 2000). Brigo and Pallavicini (2007) apply a generalization of the Duffie and Singleton (1999) model to integrate correlations between default and interest rate risk. Their empirical results show that the consideration of these dependencies can be relevant for the valuation of default and interest rate sensitive assets, in particular for small default probabilities.

[24] In comparison to Basel II, Standard & Poor’s (see Vazza, Aurora, and Kraemer, 2010) identifies a recovery for senior unsecured corporate bonds with mean of 43%, median of 39.2% and a standard deviation of 32.8%, based on global data in the time horizon from 1987 to 2009. Considering sovereign defaults from 1983 to 2007, Moody’s publishes a recovery mean (based on average trading price) of 31% (see Cantor et al., 2008). For all types of bonds, the deterministic value of the IRB approach of Basel II was chosen for the recovery rate and conducted sensitivity analysis.

[25] Instead of linear dependence (Gauss copula), an asymmetric (non-linear) dependence (e.g. t copula) could be integrated within the asset class of stocks and between stocks and interest rates. While Garcia and Tsafack (2011) find a strong non-linear dependence for international assets within the stock and bond market, the non-linear dependence is weak between stock and bond markets between countries as well as within the same country.

[26] More details regarding the procedure to estimate the parameters of mean reverting processes using maximum likelihood methods are given in, e.g., Brigo et al. (2009).

to $\hat{\theta}_r$ = 0.0161 and $\hat{\kappa}_r$ = 0.1036 with a standard deviation of σ_r = 0.039. The initial value is set to r(0) = 0.01 and the market price of (interest rate) risk in the CIR model is assumed to be zero (γ_0 = 0). The risk premium in the JLT credit risk model is set to $\pi_{x(t)=i}(t) = 1.4,\ \forall t \in \mathbb{N}_0^+,\ \forall i \in \{1,\ldots,k-1\}$ and is constant for corporates and governments relating to all rating classes and times.[27] The parameters of the Vasicek (1977) short term interest rate model are calibrated based on the same dataset and also using maximum likelihood methods, resulting in $\hat{\theta}_r$ = 0.0149, $\hat{\kappa}_r$ = 0.095, and σ_r = 0.0069.

Table 8: Correlations for the internal model

	S_1	S_2	S_3	S_4	S_5	r
S_1	1.00					
S_2	0.68	1.00				
S_3	0.65	0.76	1.00			
S_4	0.31	0.31	0.30	1.00		
S_5	0.73	0.72	0.57	0.26	1.00	
r	-0.19	-0.27	-0.14	-0.21	-0.26	1.00

Notes: The correlations in row S_1, S_2, S_3, S_4, S_5 and r are estimated by monthly data from 01/1988 to 07/2011 with S_1: DAX 30 (price index), S_2: FTSE 100 (price index), S_3: Dow Jones Industrials (price index), S_4: India BSE 100 (price index), S_5: MSCI World (price index) and r: EURIBOR (1 month offered rate, data from 01/1999 to 07/2011).

4.2 SCR for stocks

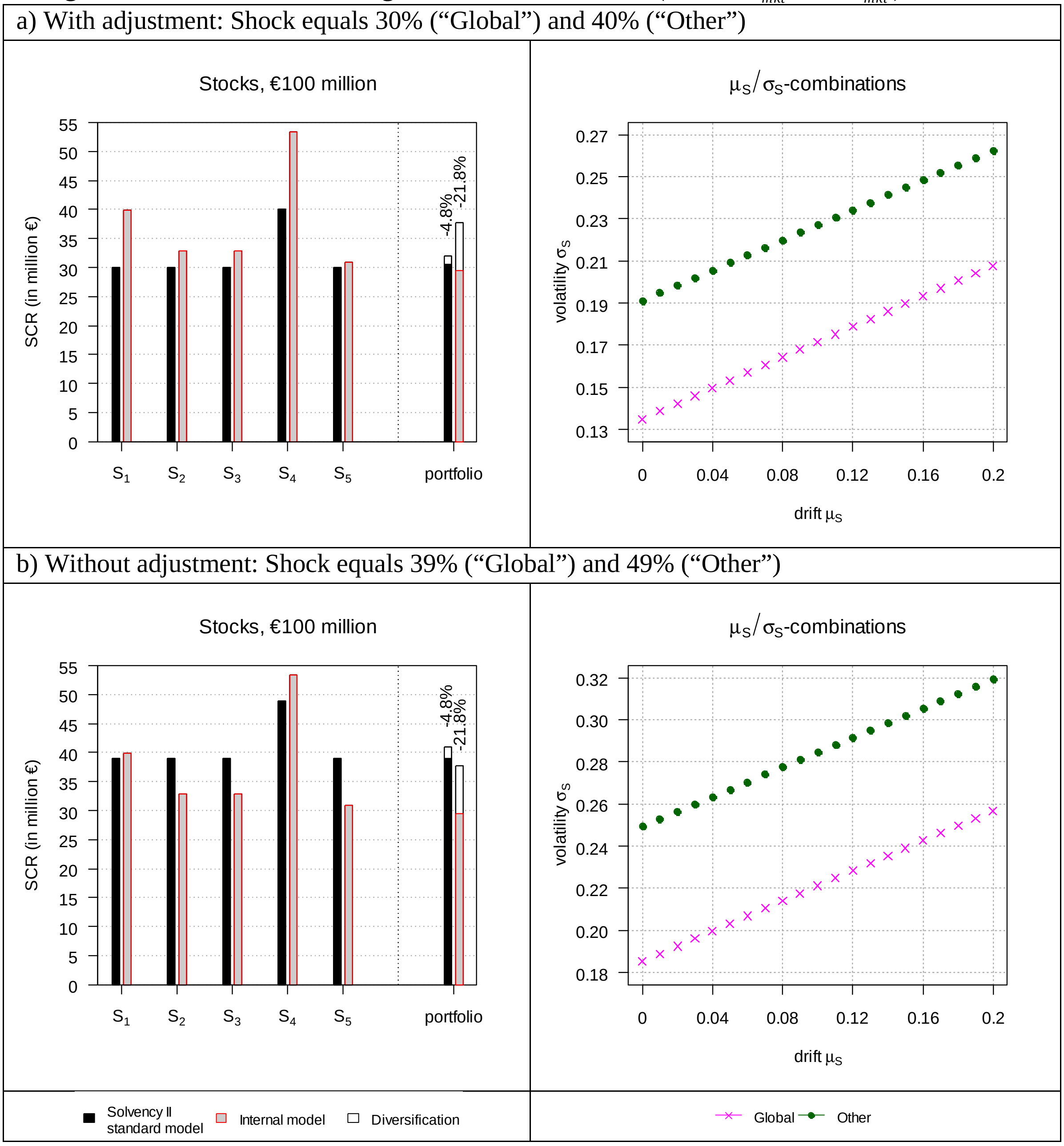
First, we focus on the SCR for a stock investment of €100 million, which is individually invested in every single stock given in Table 6. As a next step, €100 million are invested in a portfolio of these five stocks in equal portions, i.e. investing €20 million in each stock, and then the diversification benefit d_1 is calculated (see Equation (14)). Results regarding the capital requirements are displayed in the left graphs in Figure 2 a) and b), where Figure 2 a) represents the case where an adjustment in the shock scenario of -9 percentage points is made to counteract possible pro-cyclical effects in adverse market environments as defined in QIS 5 and Figure 2 b) represents the standard case.

In particular Figure 2 a) (left graph) shows considerably higher solvency capital requirements induced by the internal model calculated according to Equation (13) as compared to the standard approach of Solvency II (determined according to Equation (6)), especially for stocks with higher risk as indicated by the standard deviation. If no adjustment is made in the standard case (see the left graph in Figure 2 b), the internal model tends to a lower SCR for the

[27] Regarding the parameter of the risk premium $\pi_{x(t)=i}(t)$, we follow the assumption of Cairns (2004).

single stocks compared to the standard approach for the four global stocks, but to a higher SCR for the India BSE 100 price index S_4, which belongs to the class of "Other".

Figure 2: Solvency capital requirement for stocks (standalone and portfolio with equal proportions) and μ_S / σ_S-combinations for the geometric Brownian motion (without discounting) leading to the same SCR according to the standard model (i.e. $SCR_{mkt}^{IM} \overset{!}{=} SCR_{mkt}^{SII}$)



Notes: S_1: DAX 30 (price index), S_2: FTSE 100 (price index), S_3: Dow Jones Industrials (price index), S_4: India BSE 100 (price index) and S_5: MSCI World (price index) (see Table 6).

When looking at the portfolio of the five stocks in Figure 2 a), the SCR derived based on the internal model is still lower than the SCR of the standard model even though the individual SCRs are all higher in case of the internal model. This is a result of the considerably higher diversification benefit that is fully accounted for when using the internal model, in contrast to the Solvency II standard model provided by the regulators (-23.0% versus -4.8% in Figure 2 a). In the standard model, only diversification effects between the risk classes "Global" and "Other" are taken into account (see Table 2) in the square-root formula in Equation (3); the internal approach for stocks accounts for all individual correlations exhibited in Table 8.

In the right graphs of Figure 2, μ_S / σ_S-combinations of the geometric Brownian motion are displayed that imply the same capital requirements as the equity risk sub-module in the standard model of Solvency II, e.g. "Global" (30%) and "Other" (40%) in Figure 2 a), where for illustration purposes, $r(t)$ is set to zero to avoid discounting effects. The graph shows that the volatility σ_S is an increasing function of the drift term μ_S and that the curve for "Other" lies above the one for "Global". All parameter combinations that lie on or below the curves would imply lower capital requirements than 30% or 40%, respectively. Combinations above the curve imply higher SCR values calculated according to the internal model. Hence, none of the μ_S / σ_S-combinations of the stocks displayed in Table 6 satisfies the Solvency II standard model requirements (with adjustment for equity risk). The following formula shows the exact solution of $SCR_{mkt}^{IM} \overset{!}{=} SCR_{mkt}^{SII}$ (see Equations (6) and (13)) for $r(t)$ = 0, when considering one stock according to the internal model with σ_S and μ_S as an example:

$$
\begin{aligned}
SCR_{mkt}^{IM} &= S(0) - VaR_{\alpha}\left(S(1)\right) \\
&= S(0) - S(0) \cdot e^{\left(\mu_S - 0.5 \cdot \sigma_S^2\right) + N_{\alpha} \cdot \sigma_S} \overset{!}{=} SCR_{mkt}^{SII}, \text{ with } SCR_{mkt}^{SII} = S(0) \cdot x,\ x \in \{0.3, 0.39, 0.4, 0.49\} \\
&\Leftrightarrow e^{\left(\mu_S - 0.5\sigma_S^2\right) + N_{\alpha}\sigma_S} = (1 - x) \\
&\Leftrightarrow \mu_S - 0.5\sigma_S^2 + N_{\alpha}\sigma_S = \ln(1 - x) \\
&\Leftrightarrow \left(\sigma_S - N_{\alpha}\right)^2 = N_{\alpha}^2 - 2\ln(1 - x) + 2\mu_S \\
&\Leftrightarrow \sigma_S = \sqrt{N_{\alpha}^2 - 2\ln(1 - x) + 2\mu_S} + N_{\alpha},
\end{aligned}
$$

where N_{α} is the α-quantile of the standard normal distribution, which is negative for α = 0.5%.

From the formula above, the effect of an increase in μ_S on σ_S can be immediately seen due to the third term in the equation that leads to an increase of σ_S. Furthermore, the equation clearly shows that the standard formula implies that an increase in x (the capital charge in the standard model) *ceteris paribus* allows a higher volatility for a given drift term in the standard model. Thus, the fixed scenario factor x in the standard formula has several implications, as the SCR amounts to 30% or 40% of the current market value of the asset, independent of the volatility or drift of the asset. The fact that the μ_S / σ_S-curve for "Other" lies considerably above the curve for "Global" shows that stocks in the category of "Other" are generally assumed to have a higher risk. Thus, the internal model will imply lower capital requirements for stocks with μ_S / σ_S-combinations that are below the upper or lower curve in the graph, depending on the classification, which suggests a stock picking depending on the classification and volatility. Overall, a partial internal model for equity risk is particularly beneficial for insurers in case of diversification effects.

Table 9 shows the safety levels of the five stock indices under consideration based on a partial internal model using a geometric Brownian motion that correspond to a risk capital of 30% / 39% and 40% / 49%. The safety levels are obtained by solving the equation above for α. Thus, safety levels $1-\alpha$ above 99.5% mean that the actual safety level that is achieved with a risk capital of x% exceeds the required one. In general, taking into account the adjustment in the shock scenario of -9 percentage points to mitigate possible pro-cyclical effects, the safety level is principally lower than 99.5% except for the MSCI World price index (S_5), which implies 99.52%. Without the adjustment and thus higher risk capital requirements, the associated safety levels exceed 99.5% with the exception of the India BSE 100 price index (S_4) of the risk class "Other", which does not satisfy the specified safety level in both cases.

Table 9: Safety level of stock indices for given solvency capital requirements of the standard model without discounting

	S_1	S_2	S_3	S_4	S_5
With adjustment (30% / 40%)	97.40%	99.21%	99.23%	96.85%	99.52%
Without adjustment (39% / 49%)	99.50%	99.94%	99.93%	99.06%	99.97%

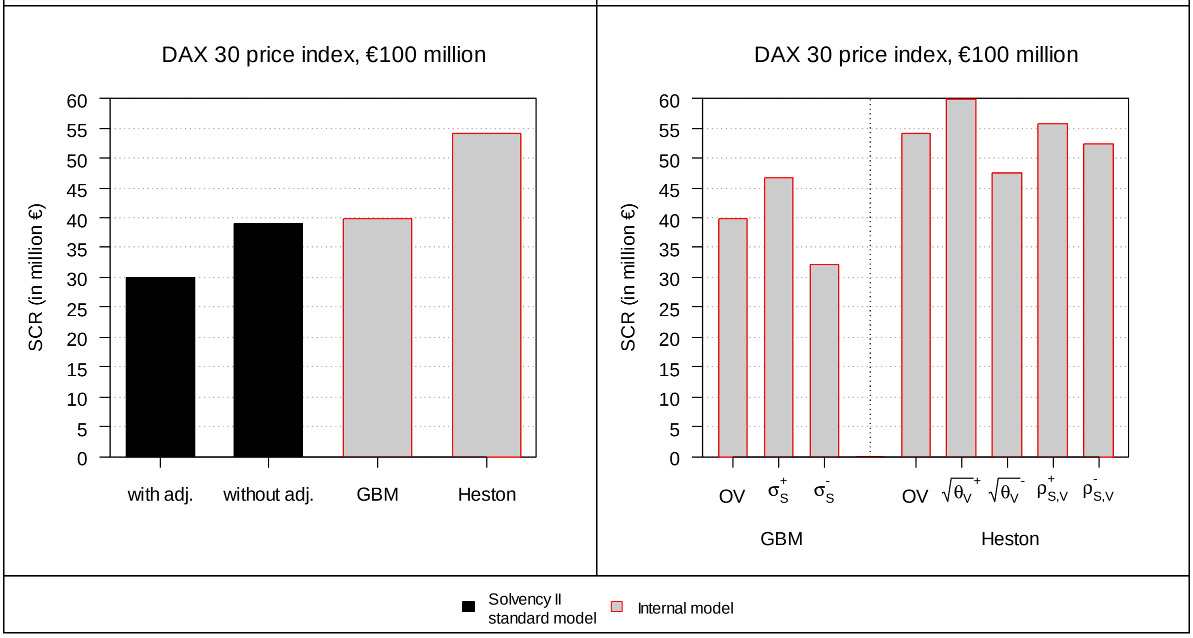
Notes: S_1: DAX 30 (price index), S_2: FTSE 100 (price index), S_3: Dow Jones Industrials (price index), S_4: India BSE 100 (price index) and S_5: MSCI World (price index) (see Table 6).

The impact of model risk regarding the choice of processes when calculating the SCR for stocks is illustrated in Figure 3. It displays the SCR for the DAX 30 price index using the standard model of Solvency II (with and without adjustment) as well as the two internal approaches, using the geometric Brownian motion and, alternatively, the stochastic volatility model of Heston (1993). Figure 3 shows that for the calibrated parameters, substantially higher capital requirements result when using the Heston (1993) model with its stochastic variance as compared to the geometric Brownian motion. In addition, with capital requirements of almost 54%, the Heston (1993) model considerably exceeds even the SCR of the standard model without adjustment (39%). Furthermore, estimations errors regarding the input parameters, particularly the long-term mean in the volatility process $\sqrt{\theta_V}$ (including the initial value

$\sqrt{V(0)}$) and the correlation between the stock price and its instantaneous variance $\rho_{S,V}$, can yield a completely different picture of an insurer's risk situation. Figure 3 b) emphasizes the relevance of model risk in regard to the processes used in the internal models as well as the input parameters by varying the relevant parameters for the geometric Brownian motion and the Heston (1993) model separately by a factor of +/-20%.

Figure 3: Solvency capital requirement for the DAX 30 price index using the Solvency II standard model with adjustment (30%) and without adjustment (39%) ("adj."), the geometric Brownian motion (GBM), and the Heston (1993) model

a) Solvency capital requirement	b) Model risk for internal models



Notes: The model risk in the long-term mean in the volatility process implies also a shock to the initial value, since we assume $\sqrt{\theta} = \sqrt{V(0)}$ *; Model risk: +/-20% of original value (OV).*

4.3 SCR for bonds

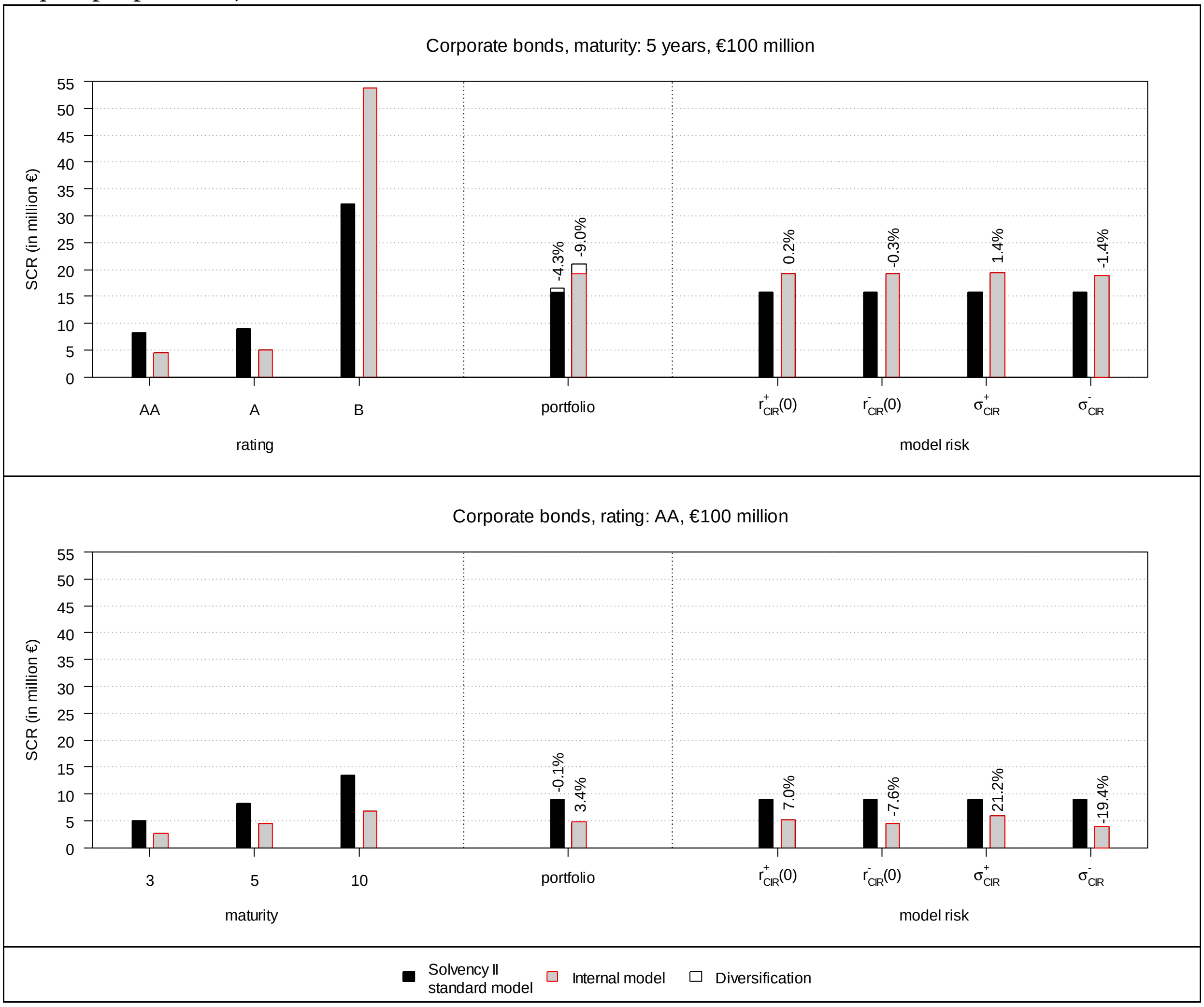
Next, we study the SCR for corporate and government bonds for different maturities and credit quality, given by the individual rating. Again the focus is first on single bonds separately and then on the consequence of building a portfolio of bonds as well as the effect of model risk with respect to the parameters of the CIR model in the portfolio context. To quantify the model risk, the starting value $r(0)$ and the standard deviation σ_r of the CIR process are separately shocked by a factor of 20% and -20%. In Solvency II, diversification effects arise due to imperfect correlations between the interest rate risk and the spread risk sub-modules.

Starting with corporate bonds, Figure 4 displays the SCR when investing €100 million in one single corporate bond with a maturity of five years with varying credit qualities (upper graph) and for single corporates with different maturities and a given credit rating of AA (lower graph). In addition, the SCR of a portfolio of the three bonds is calculated, assuming an investment of equal shares (€33.33 million in each bond). The results show that the internal model, in contrast to the case of stocks (equity risk, see the previous subsection), leads to a considerably lower SCR for the considered investment grade credit quality bonds as compared to the Solvency II standard model. However, for the B-rated corporate bond in the upper graph of Figure 4, the internal model more strongly accounts for a possible default of the issuer as compared to the standard model and thus implies a considerably higher SCR. Thus, particularly low rated bonds may be severely underestimated in the standard model.

When looking at the portfolio in the upper part of Figure 4 (5-year bonds), similarly to the observations in case of equity risk SCR, diversification benefits are twice as high in the case of the internal model as compared to the standard model (-9.0% versus -4.3%). However, the lower graph of Figure 4 shows a diversification effect $d_1(B)$ of 3.4%, indicating an increasing SCR when building the portfolio of bonds. This observation is due to the lack of subadditivity for the Value at Risk when calculating the SCR and does not occur when using the Tail Value at Risk.

With respect to model or estimation risk regarding the input parameters, Figure 4 indicates that it is especially the standard deviation of the interest rate process σ_r that generates a significantly higher or lower SCR for a portfolio of high quality bonds with different maturities, e.g., when increasing or decreasing σ_r to 120% or 80% of the original value, respectively.

Figure 4: Solvency capital requirement for corporate bonds (standalone and portfolio with equal proportions)



Notes: Straight fixed-income bonds issued in the period range 2/2010 to 5/2011: Colgate-Palmolive Company (rating (r): AA, maturity (m): 5, coupon (c): 1.375%), Woolworth Ltd. Company (r: A, m: 5, c: 2.55%), Air Canada (r: B, m: 5, c: 9.25%), Colgate-Palmolive Company (r: AA, m: 3, c: 1.25%), Colgate-Palmolive Company (r: AA, m: 10, c: 2.95%); model risk: +/-20% of original value.

We next consider the SCR for government bonds issued by members of the EEA as shown in Figure 5. Besides the internal model that fully quantifies credit and spread risk, we further consider the SCR for a reduced internal model that, analogously to the Solvency II standard model, excludes spread risk for EEA governments.[28] First, an investment of €100 million in an EEA government bond with a varying credit quality and a maturity of five years is considered in the upper graph of Figure 5. Second, different maturities for a given rating are analyzed in the lower graph. Furthermore as before, portfolios consisting of equal shares (€33.33

[28] For the calculation of the internal model without spread risk, Equation (12) is set to $B_j(t) = \sum_{h=t+1}^{T_j} CF_j(h) \cdot p_{x(t)=i}(t,h)$ with the non-defaultable zero coupon bond price $p(t,h)$.

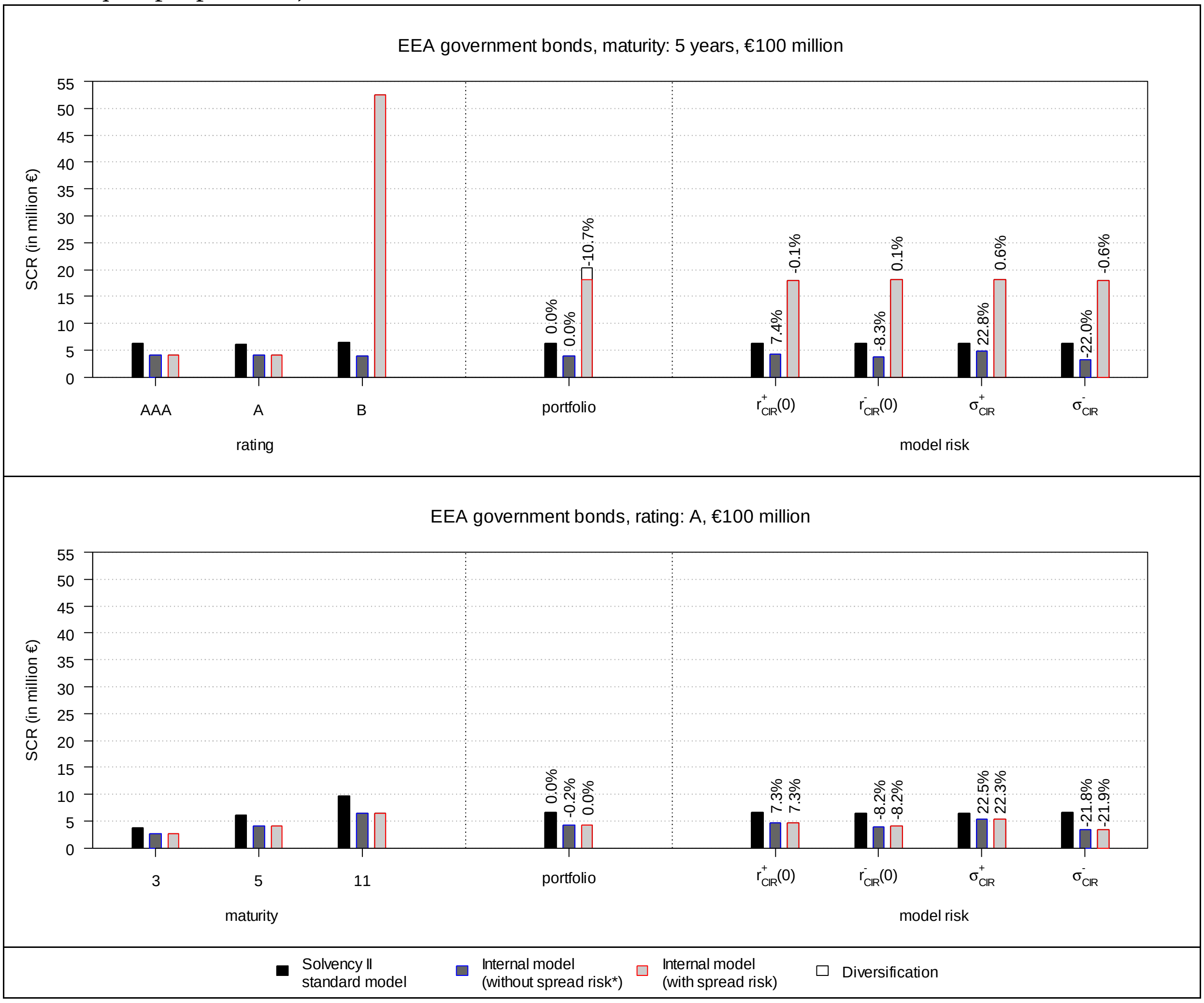
million for every bond) are analyzed together with a model risk assessment for relevant input parameters.

The upper graph in Figure 5 shows that the SCR increases even stronger for lower credit qualities as compared to the case of corporate bonds. In the case of the B-rated government bond with lower credit quality (speculative grade), the internal model's SCR with spread risk exceeds the SCR of the other two models by a multiple, while for higher rated bonds, the standard model implies higher SCRs. The internal model without spread risk for EEA governments in contrast exhibits a very low SCR, which further emphasizes the importance of taking the actual spread risk into consideration in order to obtain an adequate picture of the risk of a bond investment. In particular, this observation clearly illustrates the effect of the special regulations for EEA governments that do not account for spread risk, which implies a severe underestimation of risk especially for bonds with lower credit quality.

As before, diversification benefits in the Solvency II standard model do not exist, since the market value of bonds is summed up without accounting for correlations and spread risk is not applicable. Besides the rating, another important factor for the SCR of bonds is their maturity, which is exhibited in the lower graph in Figure 5 by means of A-rated EEA government bonds. In this case, the SCR quantified by the internal model with spread risk lies below the standard model for all maturities due to the comparably low spread risk. The SCR is increasing for longer maturities due to interest rate risk and credit risk, which is increasing over time.

Furthermore, the analysis of the model risk illustrates that the initial value $r(0)$ and particularly the standard deviation σ_r of the CIR process have a strong impact on the SCR, which is highly prone to errors in the input parameters. The SCR even varies around +/-19% for the internal models both in the upper and lower graph when the actual standard deviation σ_r is 20% higher or lower than the one originally assumed. Thus, a sensitivity analysis regarding the standard deviation is vital for insurers to assess the impact of potential model risk.

Figure 5: Solvency capital requirement for EEA government bonds (standalone and portfolio with equal proportions)

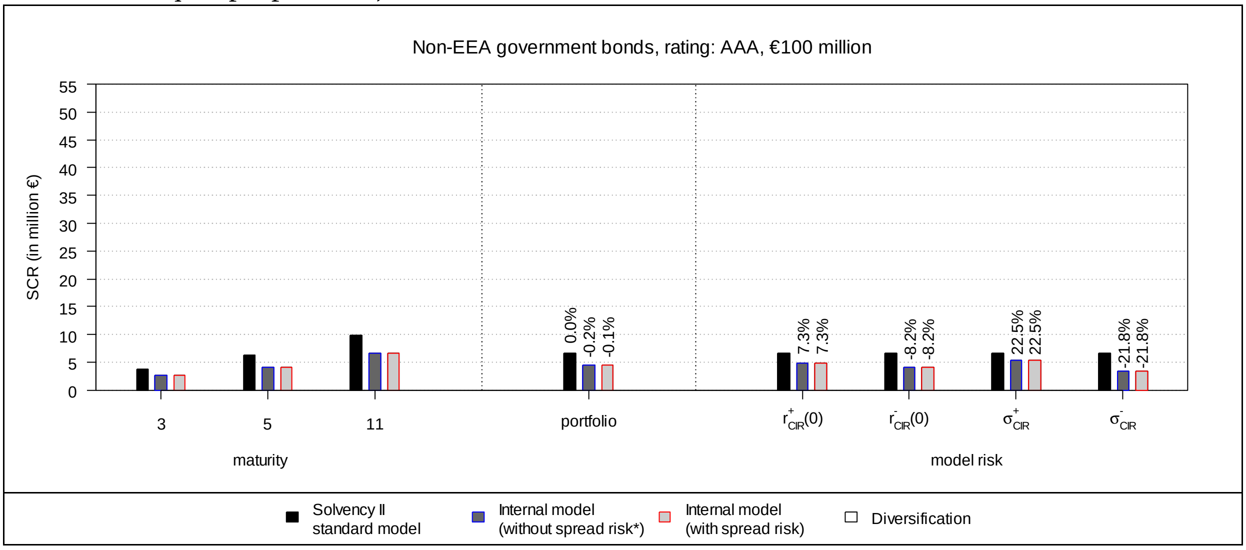


** Spread risk is excluded for EEA government bonds.*

Notes: Straight fixed-income bonds, issued in the period range 2/2010 to 5/2011: Germany (Government of, r: AAA, m: 5, c: 2.00%), Czech Republic (Government, r: A, m: 5, c: 3.40%), Greece (Republic of, r: B, m: 5, 6.10%), Czech Republic (Government, r: A, m: 3, c: 2.75%), Czech Republic (Government, r: A, m: 11, c: 3.85%); model risk: +/-20% of original value.

The special regulations with respect to spread risk do not only comprise EEA government bonds, but also AAA- and AA-rated government bonds issued by non-EEA states. The SCR calculations of an AAA-rated non-EEA government bond are given in Figure 6, again distinguishing between an internal model with and without accounting for spread risk. Here, one can observe again that the SCR increases with longer maturities and that the internal model implies lower SCR values as compared to the standard model, also with respect to the portfolio view. In addition, the difference between the two internal models is similarly minor for the high rated non-EEA government bonds as in Figure 5. Regarding estimation risk, too, similar observations can be made as in the case of EEA government bonds, specifically showing high deviations in case of the interest rate volatility.

Figure 6: Solvency capital requirement for non-EEA government bonds (standalone and portfolio with equal proportions)



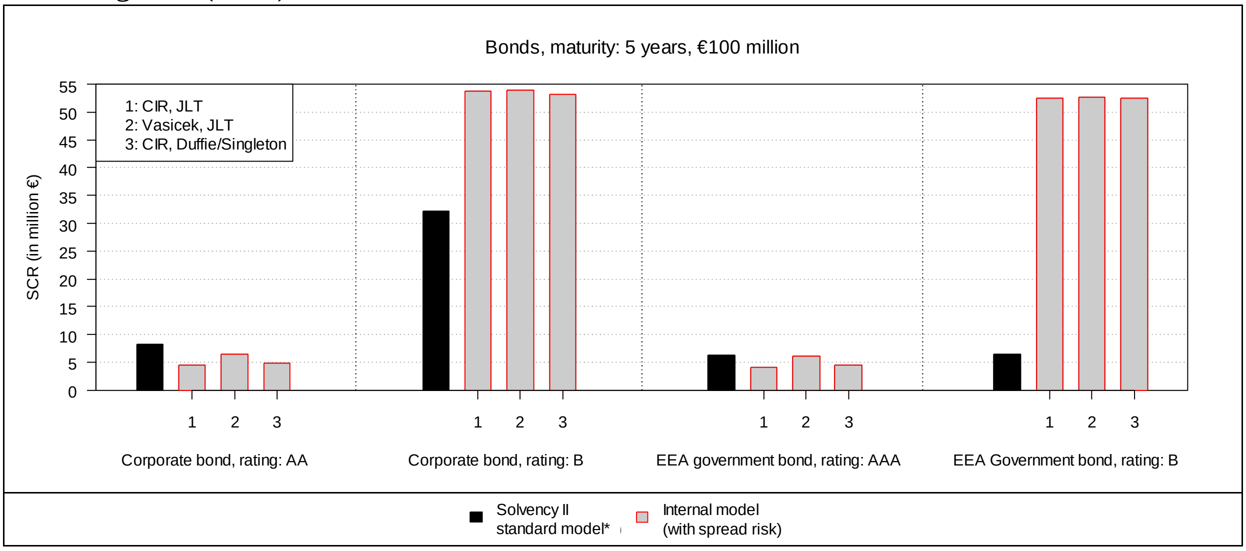
** Spread risk is excluded for AAA- and AA-rated non-EEA government bonds.*
Notes: Straight fixed-income bonds, issued in the period range 2/2010 to 5/2011: Canada (Government, r: AAA, m: 3, c: 2.25%), Canada (Government, r: AAA, m: 5, c: 2.75%), Canada (Government, r: AAA, m: 11, c: 3.25%); model risk: +/-20% of original value.

The impact of model risk in regard to the choice of risk processes for deriving the SCR of bond investments is laid out in Figure 7, thereby distinguishing between the underlying process for the interest rates and the credit spread. Thus, Figure 7 displays three different internal models, starting 1) with the CIR model and the JLT credit risk model; 2) substituting the CIR model with the Vasicek (1977) model while keeping the JLT credit risk model; and 3) using the CIR model for interest rates but integrating the Duffie and Singleton (1999) credit risk model. The graph displays the SCR for corporate and government bonds with different ratings. For high-rated corporate and government bonds, the results show higher SCRs when using the Vasicek (1997) instead of the CIR model (compare cases 1 and 2 in Figure 3). The SCR using internal model 2 even approximates the SCR of the Solvency II standard model, especially for AAA government bonds. For the B-rated bonds, the differences between the two internal models almost vanish as credit risk is the major risk driver for this type of bond exposures.

When comparing the internal models 1 and 3, where the credit risk process is defined differently using the model from Duffie and Singleton (1999), the SCR results are quite similar as in the case with the JLT model. However, when using the Duffie and Singleton (1999) model instead of the JLT model, the SCR tends to a slightly higher level for the high-rated bonds and to a slightly lower level for low-rated bond investments. The differences in the SCR for the credit risk models are mainly due to the underlying assumption in terms of the recovery rate.

The JLT model supposes a recovery of treasury where the recovery is paid out at maturity as a fraction of a corresponding non-defaultable bond. In the model of Duffie and Singleton (1999), the recovery is received at the time of default as a fraction of the bonds' market value (recovery of market value). While this appears minor at first glance, it can still have a considerably impact when taking into account the large volumes of bonds. Thus, model risk in regard to interest and credit risk should also be assessed in order to obtain a more detailed picture of the firm's risk situation.[29]

Figure 7: Solvency capital requirements for single bonds using the Solvency II standard model, the CIR and Vasicek (1977) model for interest rate risk as well as the JLT and Duffie and Singleton (1999) model for credit risk



** The face value and, hence, the nominal annual coupon payment in the Solvency II standard model is calibrated with the CIR and JLT model. Notes: Straight fixed-income bonds issued in the period range 2/2010 to 5/2011: Colgate-Palmolive Company (rating (r): AA, maturity (m): 5, coupon (c): 1.375%), Air Canada (r: B, m: 5, c: 9.25%), Germany (Government of, r: AAA, m: 5, c: 2.00%), Greece (Republic of, r: B, m: 5, c: 6.10%).*

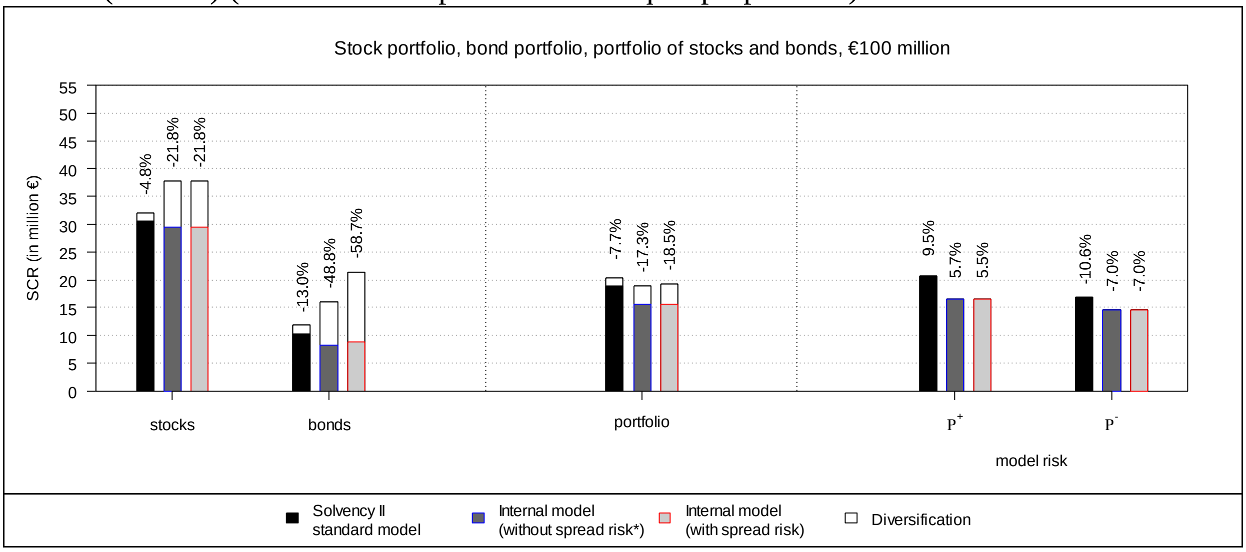
4.4 SCR for the stock and bond portfolio

We next study the implications of the SCR when investing €100 million in a stock portfolio and a bond portfolio as given in Tables 6 and 7 to assess diversification effects. In a second step, €100 million are put into an asset portfolio consisting half of the stock portfolio and half of the bond portfolio, i.e. the stock portion is set to 50%. Model risk for the internal approach is analyzed with respect to the risk of misestimated correlations included in the model. Thus, the correlation matrix P with the parameters given in Table 5 is changed by +/-20%. When

[29] The interest rate and credit risk models in Figure 7 are calibrated based on the same data set in order to ensure comparability and to isolate the effect of model risk. The general results thereby remain stable when changing, e.g., the length of the interest rate data set.

building a portfolio of stocks and bonds, correlation effects arise when using the square-root formula in Equation (3) for the equity risk sub-module and in Equation (6) for the market risk module of Solvency II with the predefined correlations. For the internal approach, the correlation matrix *P* (with the parameters given in Table 8) is used.

Figure 8: Solvency capital requirement for a portfolio of stocks (Table 6) and a portfolio of bonds (Table 7) (standalone and portfolio with equal proportions)

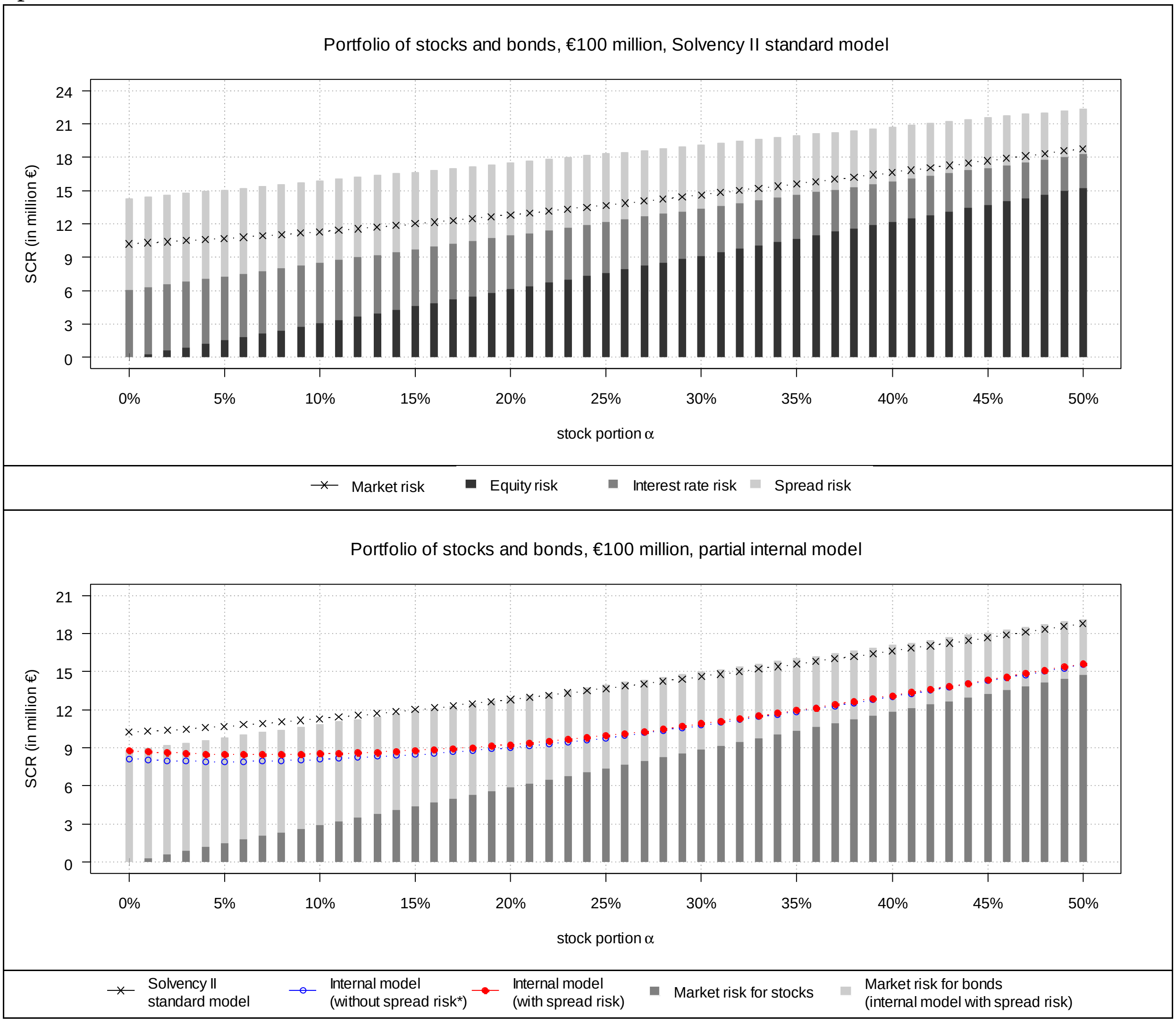


** Spread risk is excluded for EEA governments and AAA- and AA-rated non-EEA governments.*
Notes: S_1: DAX 30 (price index), S_2: FTSE 100 (price index), S_3: Dow Jones Industrials (price index), S_4: India BSE 100 (price index), S_5: MSCI World (price index), B_1: Colgate-Palmolive Company, B_2: Woolworth Ltd. Company, B_3: Air Canada, B_4: Germany (Government of), B_5: Czech Republic (Government), B_6: Greece (Republic of), B_7: Canada (Government), B_8:Russian Federation (Government), B_9: Belarus (Republic of) (see Tables 6 and 7); model risk: +/-20% of original value.

From Figure 8, it can be seen that the SCR obtained by the Solvency II standard model for a portfolio of stocks is slightly higher than the SCR of the internal models, even though the SCR for individual stocks is lower in case of a 30% / 40% capital charge (see the upper graph in Figure 2). This is due to the considerably higher diversification effects that are fully accounted for in case of the internal model (-21.8%) as opposed to the standard model (-4.8%), which was already apparent in the analysis of the SCR of equity risk only. The diversification effects are even stronger in case of the bond portfolio, implying a considerably lower SCR in case of the full internal model (-58.7% diversification benefit), while the standard model implies an SCR that is almost twice as high, despite diversification effects of -13.0%. Regarding the portfolio of stocks and bonds in Figure 8, the diversification effects of the internal models are higher (17.3% and 18.5%, respectively) than the one of the standard model (7.7%), leading to similar amounts of SCR in all three models.

Concerning model risk, the SCR from the internal models vary until +/-7% when changing the correlation values by +/-20%. The model risk in case of the standard model induces a change in the SCR of around +/-10% and thus demonstrates a considerably larger impact of parameter uncertainty as compared to the internal model in the considered example.

Figure 9: Solvency capital requirement for a stock and bond portfolio as a function of stock portion α



** Spread risk is excluded for EEA governments and AAA- and AA-rated non-EEA governments.*
Notes: S_1: DAX 30 (price index), S_2: FTSE 100 (price index), S_3: Dow Jones Industrials (price index), S_4: India BSE 100 (price index), S_5: MSCI World (price index), B_1: Colgate-Palmolive Company, B_2: Woolworth Ltd. Company, B_3: Air Canada, B_4: Germany (Government of), B_5: Czech Republic (Government), B_6: Greece (Republic of), B_7: Canada (Government), B_8: Russian Federation (Government), B_9: Belarus (Republic of) (see Tables 6 and 7).

To analyze the impact of the different SCR components for different stock portions α, Figure 9 displays the composition of the Solvency II sub-modules in the market risk module for the considered asset portfolio and an overall increasing SCR for a higher portion of stocks. The total SCR for market risk as shown by the line with stars accounts for diversification benefits. The lower graph in Figure 9 shows that the SCR of the internal approach lies always below the scenario-based Solvency II approach. Moreover, as α increases, the gap between the internal models with and without spread risk becomes smaller, which is due to the decreasing impact of spread risk in the asset portfolio for an increasing stock portion.

Finally, we consider a representative asset portfolio consisting of 20% stocks (in equal proportions: S_1, S_2, S_3, S_4, S_5; see Table 6), 60% investment grade corporate and government bonds (in equal proportions: B_1, B_2, B_4, B_5, B_6, B_7; see Table 7) and 20% of corporate and government bonds with speculative grade credit ratings (in equal proportions: B_3, B_6, B_9; see Table 7). Based on this asset portfolio, the partial internal model leads to a considerable lower SCR compared to the Solvency II standard approach. Investing €100 million, the internal model (with spread risk) results in an SCR of €8.27 million, while the standard model implies a capital requirement of €11.85 million, which is almost twice as high.

5. Conclusions

This paper examines the differences of calculating the SCR for market and credit risk using the current standard model of Solvency II and an internal approach. In doing so, we concentrate on the asset classes of stocks and bonds and on the most important sub-modules for market risk in Solvency II: equity risk, interest rate risk, and spread risk. To obtain comparability between the standard model and the internal approach, the latter approach includes the same risks considered in the respective sub-modules of the Solvency II framework. Considering the asset class of bonds, we distinguish between corporate bonds and government bonds and quantify the risk of changes in the term structure of interest rates and changes of the credit spread (over the risk-free interest rate term structure). The risk of fluctuations in prices of equity investments is quantified in the Solvency II equity risk-sub-module.

The procedure of quantifying the SCR in the spread risk sub-module is based on the current credit rating of the corporate or government bonds, which determine the spread risk factor in the sub-module. Therefore, to obtain comparability with the standard model of Solvency II, we also use a rating-based credit risk model for the internal approach along with the Cox-Ingersoll-Ross (1985) model for interest rate risk. The development of equity prices is described by a geometric Brownian motion. In the rating-based internal approach for bonds, the credit risk model by Jarrow, Lando, and Turnbull (1997) is used, which quantifies the credit risk through credit ratings and the probability of a change in the credit rating. Transition rates for credit ratings allow the consideration of consequences of up- and downgrading the credit quality.

One major result is that even though the standard approach is easier to use, the insurance company's risk situation is generally not sufficiently reflected by the predefined scenarios, both over- or underestimating the risk associated with investments, depending on the actual underlying asset risk. The internal model allows for adjustments and differentiated assumptions to better reflect the insurer's individual and actual credit and market risk situation. The dimension of underestimating the credit and market risk in Solvency II by ignoring spread risk of EEA governments and AAA- and AA-rated non-EEA governments specifically depends on the credit quality of the bonds. A comparative analysis indicated that compared to the partial internal model considered in this paper, the standard model appears to particularly underestimate the risk of low-rated bonds, while it overestimates the risk of high-rated bonds.

In contrast, the solvency capital of the empirically estimated stock indices studied was generally underestimated when using the standard model and an adjustment factor as applied in QIS 5 that implies a capital charge of 30% and 40% for stocks in the category "Global" and "Other", respectively. However, this result depends on the adjustment factor. In case no adjustment is made (capital charge of 39% and 49%), only the India BSE 100 stock index from the category "Other" implied a higher internal model SCR than the one induced by the standard model, while for the other considered stock indices from the category "Global", the internal model led to lower SCRs than in the case of the standard model.

In general, diversification effects played an important role in the total SCR for market risk. In particular, even though the SCRs of individual stock indices derived according the internal model were above the ones of the standard model, diversification effects implied a considerable reduction in solvency capital requirements, thus generally leading to lower SCR values in case of the internal model. In particular, correlation effects between different stocks within the asset class "Global" (e.g. EEA and OECD) and between "Global" and "Other" were not sufficiently accounted for in the standard model (approximately -5% in case of the standard model versus -22% in case of the internal model in the cases considered), thus not adequately reflecting diversification benefits.

Thus, insurers should use a partial internal model with respect to equity risk and credit spread risk instead of or in addition to the standard model when calculating the necessary solvency capital to achieve a predefined safety level. In case of stocks, this generally allows fully benefiting from diversification benefits and in case of bonds, the actual credit risk can be more detailed and adequately assessed. Additionally, model risk should be taken into consideration by means of an adequate internal model choice, and moreover by conducting sensitivity analyses with respect to the parameter calibration to obtain a more comprehensive picture of an insurer's risk situation. Further analysis should critically study the calibration of the standard model and its adequacy regarding a firm's individual risk situation.

Overall, in addition to the SCR quantification, an internal model further offers the opportunity to integrate the model in the internal control process of the insurer, which is also of high relevance in the context of the insurer's own risk and solvency assessment (ORSA) as required in Solvency II's Pillar 2. Further research should also look at the implications and incentives generated by the standard model and internal models regarding the capital allocation behavior of insurance companies as one of the largest investors in Europe, as systematic and procyclical behavior during adverse capital market developments might severely impact the financial markets.

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APPENDIX

Table A.1: Global corporates average one-year transition rates (%) (see Vazza, Aurora and Kraemer, 2010, p. 27)

	AAA	AA	A	BBB	BB	B	C	D	NR
AAA	88.21	7.73	0.52	0.06	0.08	0.03	0.06	0.00	3.31
AA	0.56	86.60	8.10	0.55	0.06	0.09	0.02	0.02	4.00
A	0.04	1.95	87.05	5.47	0.40	0.16	0.02	0.08	4.83
BBB	0.01	0.14	3.76	84.16	4.13	0.70	0.16	0.26	6.68
BB	0.02	0.05	0.18	5.17	75.52	7.48	0.79	0.97	9.82
B	0.00	0.04	0.15	0.24	5.43	72.73	4.65	4.93	11.83
C	0.00	0.00	0.21	0.31	0.88	11.28	44.98	27.98	14.36

Table A.2: Governments average one-year transition rates (%) (see Chambers, Ontko and Beers, 2011, p. 41)[30]

	AAA	AA	A	BBB	BB	B	C	D	NR
AAA	97.78	2.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	3.37	93.64	2.25	0.00	0.37	0.37	0.00	0.00	0.00
A	0.00	3.60	92.80	3.60	0.00	0.00	0.00	0.00	0.00
BBB	0.00	0.00	6.75	89.03	3.38	0.84	0.00	0.00	0.00
BB	0.00	0.00	0.00	6.14	88.06	4.10	1.02	0.68	0.00
B	0.00	0.00	0.00	0.00	7.20	86.36	3.41	1.89	1.14
C	0.00	0.00	0.00	0.00	0.00	31.82	31.82	36.36	0.00

Table A.3: Global corporates average one-year transition rates derived from Table A.1 accounting for non-rated corporates (NR) (%, rounded values)

	AAA	AA	A	BBB	BB	B	C	D
AAA	91.23	7.99	0.54	0.06	0.08	0.03	0.06	0.00
AA	0.58	90.21	8.44	0.57	0.06	0.09	0.02	0.02
A	0.04	2.05	91.47	5.75	0.42	0.17	0.02	0.08
BBB	0.01	0.15	4.03	90.18	4.43	0.75	0.17	0.28
BB	0.02	0.06	0.20	5.73	83.74	8.29	0.88	1.08
B	0.00	0.05	0.17	0.27	6.16	82.49	5.27	5.59
C	0.00	0.00	0.25	0.36	1.03	13.17	52.52	32.67

[30] To obtain row sums equal to one and thus a cumulative distribution function for each row, we adjust individual values in Table A.2.

Table A.4: Governments average one-year transition rates derived from Table A.2 accounting for non-rated governments (NR) (%, rounded values)

	AAA	AA	A	BBB	BB	B	C	D
AAA	97.78	2.22	0.00	0.00	0.00	0.00	0.00	0.00
AA	3.37	93.64	2.25	0.00	0.37	0.37	0.00	0.00
A	0.00	3.60	92.80	3.60	0.00	0.00	0.00	0.00
BBB	0.00	0.00	6.75	89.03	3.38	0.84	0.00	0.00
BB	0.00	0.00	0.00	6.14	88.06	4.10	1.02	0.68
B	0.00	0.00	0.00	0.00	7.28	87.36	3.45	1.91
C	0.00	0.00	0.00	0.00	0.00	31.82	31.82	36.36