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CREATING CUSTOMER VALUE IN PARTICIPATING LIFE INSURANCE

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ABSTRACT

The value of a life insurance contract may differ depending on whether it is looked at from the customer's point of view or that of the insurance company. We assume that the insurer is able to replicate the life insurance contract's cash flows via assets traded on the capital market and can hence apply risk-neutral valuation techniques. The policyholder, on the other hand, will take risk preferences and diversification opportunities into account when placing a value on that same contract. Customer value is represented by policyholder willingness to pay and depends on the contract parameters, i.e., the guaranteed interest rate and the annual and terminal surplus participation rate. The aim of this paper is to analyze and compare these two perspectives. In particular, we identify contract parameter combinations that—while keeping the contract value fixed for the insurer—maximize customer value. In addition, we derive explicit expressions for a selection of specific cases. Our results suggest that a customer segmentation in this sense, i.e., based on the different ways customers evaluate life insurance contracts and embedded investment guarantees while ensuring fair values, is worthwhile for insurance companies as doing so can result in substantial increases in policyholder willingness to pay.

Keywords: Participating life insurance, risk-neutral valuation, customer value, mean-variance preferences

JEL classification: D46; G13; G22; G28

1. INTRODUCTION

Participating life insurance contracts generally feature a minimum interest rate guarantee, guaranteed participation in the annual return of the insurer's asset portfolio, and a terminal bonus payment. Appropriate pricing of these features is crucial to an insurance company's financial stability. Risk-neutral valuation and other premium principles based on the duplication of cash flow serve well to evaluate contracts from the insurer's perspective. However, these techniques are only relevant, if insurance policies priced according to them actually

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meet customer demand. Since policyholders may not be able to duplicate their claims via capital market instruments for valuation purposes, they will often judge its value based on individual preferences. Thus, their willingness to pay—referred to here as "customer value" of the contract—may be quite different from the fair premium calculated by the insurance company. The aim of this paper is to combine the insurer's perspective with that of the policyholders, which is done by identifying those fair contract parameters (guaranteed interest rate and annual and terminal surplus participation rate) that, while keeping the fair value fixed for the insurer, maximize customer value.

We extend previous literature by combining these two approaches; however, there is a fair amount of previous research on each individual perspective. From the insurer perspective, the relevant area is option pricing theory and its application to participating life insurance contracts. Among this literature, we find in particular Briys and de Varenne (1997), Grosen and Jørgensen (2002), Bacinello (2003), Ballotta, Haberman, and Wang (2006), and Gatzert (2008). All these papers use option pricing models to determine the price of life insurance policies, but their objectives are various. Briys and de Varenne (1997), for example, use a contingent claims approach to derive prices for life insurance liabilities and to compare the durations of equity and liabilities in the insurance and banking industries, respectively. In contrast, Gatzert (2008) analyzes the influence of asset management and surplus distribution strategies on the fair value of participating life insurance contracts.

From the policyholder perspective, the literature on utility theory and, in particular, on the demand for insurance, is relevant. In our paper, the demand for insurance is derived by assuming that the policyholders follow mean-variance preferences, a common assumption in the literature. For example, Berketi (1999) assumes mean-variance preferences in an analysis of insurers' risk management activity, finding that although such activity does reduce the risk of insolvency, it also reduces the expected payments to the policyholders when considering participating life insurance contracts. Berketi (1999) applies a mean-variance framework to analyze policyholder preferences with regard to these activities, but does not derive their willingness to pay. Various other research has been conducted to analyze the demand for insurance by corporate entities (see, e.g., Mayers and Smith, 1982; Doherty and Richter, 2002; Doherty and Tinic, 1981). Generally, demand for insurance depends not only on an individual's preferences, but also on the person's economic situation. Accordingly, Mayers and Smith (1983) examine insurance holdings as one of many interrelated portfolio decisions. Inspired by this paper, Showers and Shotick (1994) conduct an empirical analysis and verify the interdependence between individuals' demand for insurance and household characteristics (e.g., income, number of family members, number of working family members). Ehrlich and Becker (1972) combine expected utility theory with consumption theory and analyze substitution effects. In particular, they examine the relationship between insurance, self-insurance (reduction of the loss extent), and self-protection (reduction of the loss probability). To account for the findings of this research, we consider the special case in which the policyholder's wealth develops stochastically and thus there are diversification opportunities between the private wealth and investment in a life insurance contract.

However, our approach can as well be extended and applied based on different preference models such as prospect theory used by Wakker, Thaler, and Tversky (1997), developed by Kahneman and Tversky (1979), to explain experimental data on the demand for probabilistic insurance. Probabilistic insurance is a type of insurance policy that indemnifies the policyholder with a probability only strictly less than one due to the insurer's default risk. Recent experimental research on demand for insurance under default risk includes Zimmer, Schade, and Gründl (2009), who show that awareness of even a very small positive probability of insolvency hugely reduces customer willingness to pay.

In this paper, we combine the insurer and policyholder viewpoints in the context of participating life insurance contracts. The insurer conducts (preference-independent) risk-neutral valuation and arrives at the fair price of the insurance contract. This fair price is the minimum premium the insurance company needs to charge in order for its equityholders—who could also and simultaneously be policyholders—to receive a risk-adequate return on their investment. Policyholders, who generally cannot duplicate cash flows to the same extent as the insurance company, possibly will not base their decision on risk-neutral valuation. Instead, it is likely that their willingness to pay depends on their individual degree of risk aversion and, in our model, is thus based on mean-variance preferences. On this basis, we are able to derive explicit expressions for policyholder willingness to pay and analyze its sensitivity for changes in the payoff structure of the participating life insurance policy.

Our findings show how an insurance company can alter policy characteristics to increase customer value, while, at the same time, keeping the fair premium value fixed. Furthermore, we investigate whether existing regulatory specifications regarding the design of participating life insurance contracts actually fulfill their intended purpose of protecting policyholder interests. If, by disregarding those specifications, the insurance company can increase customer value, this justification comes into doubt. Our findings are relevant for both the insured and insurance companies, who may be able to realize premiums above the fair premium level by increasing policyholder willingness to pay. Taking the lead from Mayers and Smith (1983), Showers and Shotick (1994), and Ehrlich and Becker (1972), we also aim to investigate the effects on insurance demand when policyholder basis wealth is stochastic and the policyholder thus has diversification possibilities.

The remainder of this paper is organized as follows. In Section 2, the basic setting is introduced. In Section 3, we present the valuation procedures employed by the insurer and by the policyholders. Keeping the fair (from the insurer's perspective) premium value fixed, we optimize customer value in Section 4. Section 5 provides numerical examples. Selected policy implications and a summary are found in Section 6.

2. BASIC SETTING

We analyze participating life insurance contracts similar to those offered in many European countries, including Germany, Switzerland, the United Kingdom, and France. The insurance company's initial assets are denoted by A_0. At inception of the contract, policyholders pay a single up-front premium $P_0\left(=\beta\cdot A_0\right)$ and the insurance company equityholders make an initial contribution of $Eq_0\left(=\left(1-\beta\right)\cdot A_0\right)$. Here, $\beta=P_0/A_0$ can be considered as the leverage of the company. The total value of initial payments $A_0=P_0+Eq_0$ is then invested in the capital market, which leads to uncertainty about the value of the insurer's assets $A(t)$ at time *t = 1,2,…,T*, where *T* denotes the fixed maturity of the contract(s). Having the assets follow a geometric Brownian motion captures this uncertainty. Under the real-world measure $\mathbb{P}$, this stochastic process is characterized by drift μ_A and volatility σ_A leading to

$$A(t) = A(t-1)\cdot\exp\left[\mu_A-\sigma_A{}^2/2+\sigma\left(W_A(t)-W_A(t-1)\right)\right], \tag{1}$$

with

$$A(0) = A_0 = P_0+Eq_0, \tag{2}$$

where W_A is a standard $\mathbb{P}$-Brownian motion. In the case the insurer is solvent at time $t = T$, the assets $A(T)$ should exceed the liabilities to the policyholders. The amount of liabilities at maturity T is determined by three parameters. The first is a guaranteed minimum annual interest rate g regarding the policyholder reserves. In several European countries, this minimum interest rate is determined by law and changed periodically—depending on capital market conditions.

The second parameter is the annual surplus distribution rate α. In general, this rate is regulated, too, similar to the minimum annual interest rate (e.g., Germany, Switzerland, and France). Hence, in the case of positive market developments, the policyholders participate in the insurer's investment returns above the guaranteed interest rate. The participation rate is applied to earnings on book values, which can differ considerably from earnings on market values. We therefore introduce a constant parameter γ, as is done in Kling, Richter, and Ruß (2007), to capture the difference between book and market values. In this sense, the factor γ also serves as a smoothing parameter as it allows the insurance company to build up reserves and thus to even out policyholder payments between years of "low" and "high" investment returns. The parameter γ takes values between 0 and 1.

The third contract parameter is the optional terminal surplus bonus δ. This terminal bonus is not guaranteed, but is optionally credited to the policyholder account according to the initial contribution rate $\beta = P_0 / A_0$ at maturity. As we are mainly interested in the financial risk situation, we do not take early surrender and deaths into account. Under the assumption that mortality risk is diversifiable, it can be dealt with using expected values when writing a sufficiently large number of similar contracts. However, we presume that any additional options are priced adequately and paid for separately. Thus, the policyholder account $P(t)$ in our model is as follows:

$$P(t) = P(t-1)\cdot(1+g) + max\left[\alpha\cdot\gamma\cdot\left(A(t)-A(t-1)\right) - g\cdot P(t-1), 0\right], \tag{3}$$

where $P(0) = P_0$ and γ is the relation of book value to market value. The interest rate and the annual participation payment are locked in each year and thus become part of the guarantee (so-called cliquet-style guarantee). The terminal bonus is given by a fraction δ of $B(T)$, where

$$B(T) = max(\beta \cdot A(T) - P(T), 0). \tag{4}$$

The total payoff to the policyholder at maturity $L(T)$ thus consists of the policyholder's guaranteed accumulated account $P(T)$—including guaranteed interest rate payments and annual surplus participation—as well as an optional terminal surplus participation payment $\delta \cdot B(T)$. The policyholder will receive the guaranteed payoff only if the insurance company is solvent at maturity, i.e., if the market value of assets $A(T)$ is sufficient to cover the guaranteed maturity payoff $P(T)$. If the company is insolvent—$P(T) > A(T)$—policyholders receive only the total market value of the insurer's assets. Hence, the expected cost of insolvency is represented by the default put option $D(T)$:

$$D(T) = max(P(T) - A(T), 0). \tag{5}$$

The default put option is deducted from the policyholder claims (see, e.g., Doherty and Garven, 1986), leading to a total policyholder payoff $L(T)$, with

$$L(T) = P(T) + \delta \cdot B(T) - D(T). \tag{6}$$

The insurance company equityholders have limited liability, which means that they either receive the residual difference between the market value of the assets and the policyholder payoff at time t = T or, in the case of insolvency, nothing:

$$Eq(T) = A(T) - L(T) = max(A(T) - P(T), 0) - \delta \cdot B(T). \tag{7}$$

The first term on the right-hand side of Equation (7) represents a call option on the insurer's assets with strike price $P(T)$, which illustrates the equityholders' limited liability.

3. VALUATION FROM THE PERSPECTIVE OF INSURERS AND POLICYHOLDERS

We now turn to the valuation and determination of fair premiums, which will be different, depending on the perspective taken—policyholder or insurer (equityholder). Since we believe that policyholders generally cannot duplicate cash flows to the same extent as can an insurance company, their valuation and thus their willingness to pay for the contract depends on individual preferences. In this paper, policyholder willingness to pay is referred to as the "customer value" of the insurance contract. From the insurance company point of view, we assume that claims are replicable in order to derive fair (or minimum) premiums. Thus, a preference-free valuation approach, for a given combination of the parameters g, α, and δ, can be applied to provide a risk-adequate return for the company's equityholders. If the customer value exceeds the minimum premium derived, we obtain a positive premium agreement range. If this range is negative, it is not likely the contract will be bought by this particular policyholder.

3.1 Insurer perspective

Assuming an arbitrage-free capital market, the insurer evaluates claims under the risk-neutral measure $\mathbb{Q}$. Under $\mathbb{Q}$, the drift of the asset process changes from μ_A to the risk-free interest rate r,

$$A(t) = A(t-1)\cdot \exp\left[r-\sigma^2/2+\sigma\left(W_A^{\mathbb{Q}}(t)-W_A^{\mathbb{Q}}(t-1)\right)\right] \tag{8}$$

where $W_A^{\mathbb{Q}}$ is a $\mathbb{Q}$-Brownian motion. The values of the policyholder $\left(\Pi^*\right)$ and the equityholder claims $\left(\Pi^E\right)$ under the risk-neutral measure are then given by:

$$\begin{aligned}\Pi^* &= e^{-rT}\cdot E^{\mathbb{Q}}\left(L(T)\right) = e^{-rT}\cdot E^{\mathbb{Q}}\left[\left(P(T)+\delta\cdot B(T)\right)\right] - e^{-rT}\cdot E^{\mathbb{Q}}\left(D(T)\right)\\ &= \Pi - \Pi^{DPO}\end{aligned} \tag{9}$$

and

$$\Pi^E = e^{-rT}\cdot E^{\mathbb{Q}}\left(E(T)\right). \tag{10}$$

An up-front premium P_0 is called “fair” if it equals the market value of the contract under the risk-neutral measure at time t = 0. This is expressed as

$$\Pi^* = P_0, \tag{11}$$

which, due to no arbitrage, is equivalent to solving

$$\Pi^E = Eq_0. \tag{12}$$

The value of the policyholder claim is determined by the guaranteed interest rate g, the annual surplus participation α, and the terminal bonus δ. Keeping all else equal, a decrease in any one of the three parameters – e.g., of g – decreases the fair contract value $\Pi^*(g,\alpha,\delta) < P_0$. However, by increasing the remaining parameters – in this example, α, δ, or both – the value of the contract can be kept constant at $\Pi^* = P_0$. Hence, there are in general an infinite number of contract specifications that all have the same fair value but, because of their different payoff structures, will vary in the degree to which policyholders find them attractive, that is, each variant, although of equal value to the insurer, may have a different customer value.

Any fair premium provides a net present value of zero for the insurance company equityholders. The fair premium P_0 thus provides the lower end of the premium agreement range.

3.2 Policyholder perspective

The upper end of the premium agreement range is determined by policyholder willingness to pay, denoted by P_0^Φ. Assuming mean-variance preferences (see, e.g., Berketi, 1999; Mayers and Smith, 1983), the policyholder’s order of preferences under the real-world measure $\mathbb{P}$ is given by the difference between expected wealth and the variance of the wealth multiplied by the policyholder’s individual risk aversion coefficient a (times one-half; see, e.g., Doherty and Richter, 2002):

$$\Phi = E(Z_T) - \frac{a}{2} \cdot \sigma^2(Z_T). \tag{13}$$

Here, Z_T denotes the policyholder's wealth at maturity. The procedure and analyses can analogously be applied based on other preference models.

To determine policyholder willingness to pay, we compare the preference function for the case of no insurance (*NI*) to the one with insurance (*WI*) (see Eisenhauer, 2004). The maximum willingness to pay is exactly the price at which the customer becomes indifferent between the two cases:

$$\Phi^{WI} = \Phi^{NI} \tag{14}$$

with

$$\Phi^{NI} = E\left(Z_T^{NI}\right) - \frac{a}{2} \cdot \sigma^2\left(Z_T^{NI}\right) \tag{15}$$

and

$$\Phi^{WI} = E\left(Z_T^{WI} + L(T)\right) - \frac{a}{2} \cdot \sigma^2\left(Z_T^{WI} + L(T)\right). \tag{16}$$

The policyholder's initial wealth is denoted by Z_0, where $Z_0 > 0$. In the case without insurance, $Z_0^{NI} = Z_0$. Alternatively, $Z_0^{WI} = Z_0 - P_0^{\Phi}$. The remainder of the initial wealth is either compounded with the risk-free interest rate (if the policyholder has no chance to diversify) or is invested in a stochastic portfolio (i.e., the policyholder can diversify). We distinguish between these two cases below.

Part A—Deterministic wealth of policyholder

In the case of deterministic wealth, the policyholder must choose between investing in the risk-free asset or using at least part of the wealth to purchase the life insurance contract. If the policyholder invests all the wealth in the risk-free investment opportunity, his or her future wealth is given by $Z_0 e^{rT}$. If the policyholder decides to purchase life insurance, initial wealth is reduced by the premium he or she is willing to pay, P_0^{Φ}, i.e., $Z_0^{WI} = Z_0 - P_0^{\Phi}$.

Furthermore, a variance term is deducted from the preference function to account for the risk associated with the life insurance policy's payback. For the two cases, the following holds:

$$\Phi^{NI} = Z_0 \cdot e^{rT} \tag{17}$$

and

$$\Phi^{WI} = E\left(\left(Z_0 - P_0^{\Phi}\right)\cdot e^{rT} + L(T)\right) - \frac{a}{2}\cdot\sigma^2\left(\left(Z_0 - P_0^{\Phi}\right)\cdot e^{rT} + L(T)\right). \tag{18}$$

According to Equation (14), the policyholder solves

$$Z_0 \cdot e^{rT} = \left(Z_0 - P_0^{\Phi}\right)\cdot e^{rT} + E\left(L(T)\right) - \frac{a}{2}\cdot\sigma^2\left(L(T)\right). \tag{19}$$

Hence, maximum willingness to pay does not depend on the policyholder's initial wealth:

$$P_0^{\Phi} = e^{-rT}\cdot\left[E\left(L(T)\right) - \frac{a}{2}\cdot\sigma^2\left(L(T)\right)\right]. \tag{20}$$

Part B—Stochastic wealth of policyholder

Following Mayers and Smith (1983), who emphasize the interaction between demand for insurance and other portfolio decisions, we introduce a stochastic investment opportunity. The policyholder may now invest his or her total initial wealth at time $t = 0$ in the stochastic asset process, or use parts of it to purchase life insurance. We assume that the stochastic asset process of the investment opportunity evolves according to a geometric Brownian motion with drift μ_Z and volatility σ_Z. Under the objective measure $\mathbb{P}$, W_Z—in analogue to the assets process of the insurance company—is a standard $\mathbb{P}$-Brownian motion. Development of the investment opportunity is thus given by

$$Z(t) = Z(t-1)\cdot\exp\left[\mu_Z - \sigma_Z{}^2/2 + \sigma_Z\left(W_Z(t) - W_Z(t-1)\right)\right], \tag{21}$$

with $Z(0)=Z_0^{WI}$ (with insurance) or $Z(0)=Z_0^{NI}$ (no insurance). Furthermore, the two Brownian motions of the insurer's asset process $A(t)$ and the private investment opportunity $Z(t)$ are correlated with a constant coefficient of correlation ρ,

$$dW_A dW_Z = \rho dt. \tag{22}$$

As before, if the policyholder chooses not to purchase life insurance, the initial investment sum equals the initial wealth $\left(Z_0^{NI} = Z_0\right)$. If the policyholder decides to take out an insurance contract, his or her investment sum equals the initial wealth reduced by a premium payment, $Z_0^{WI} = Z_0 - P_0^{\Phi}$.

Again, the policyholder's marginal willingness to pay P_0^{Φ} is derived by comparing the policyholder's preference function for the case with and without insurance (see Equations (14)–(16)). The policyholder thus solves

$$E\left(Z_T{}^{NI}\right)-\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{NI}\right)=E\left(Z_T{}^{WI}+L(T)\right)-\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{WI}+L(T)\right). \tag{23}$$

with

$$Z_0^{WI} = Z_0 - P_0^{\Phi} = Z_0^{NI} - P_0^{\Phi}$$

and (24)

$$\tilde{Z}_T = \exp\left[\left(\mu_Z - \sigma_Z{}^2/2\right)\cdot T + \sigma_Z\cdot\left(W_Z(T)-W_Z(0)\right)\right], \tag{25}$$

which can be rewritten as $Z_T{}^{WI} = \left(Z_0{}^{NI} - P_0^{\Phi}\right)\cdot\tilde{Z}_T$, and $Z_T{}^{NI} = Z_0{}^{NI}\cdot\tilde{Z}_T$. Solving Equation (23) leads to an explicit formula for policyholder willingness to pay, hence for the customer value $\left(P_0^{\Phi}\right)$ of the life insurance contract (see Appendix A for the detailed derivation):

$$P_0^{\Phi} = \left[\left[\frac{\frac{1}{a}\cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T^{NI},\tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^2 - \\ - \frac{\sigma^2\left(L(T)\right) + 2\cdot Cov\left(Z_T^{NI}, L(T)\right) - \frac{2}{a}\cdot E\left(L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^{\frac{1}{2}} \\ - \frac{\frac{1}{a}\cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T^{NI},\tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)}. \tag{26}$$

Hence, this premium P_0^{Φ} stands for the upper end of the premium agreement range in the case where the policyholder has, in addition to the life insurance contract, a second stochastic investment opportunity.

4. CREATING CUSTOMER VALUE FOR FAIR CONTRACTS

This section combines the two valuation approaches presented above so as to analyze how customer value can be maximized and, at the same time, ensure fair contract conditions for the insurer.

The participating life insurance policy under investigation here has three features that affect the policyholder payoff. Even if contracts are calibrated to be fair according to Equation (11), the value to the customer (see Equations (20) and (26)) can differ substantially. From the insurer perspective, maximizing customer value P_0^{Φ} (hence the policyholder willingness to pay) is a worthwhile undertaking toward increasing the chances of obtaining a positive net present value on the insurance market. The corresponding optimization problem can be described as follows:

$$\underbrace{P_0^{\Phi} \rightarrow \max_{g,\alpha,\delta}}_{\substack{\textit{Customer value} \\ \textit{under the real-world measure } \mathbb{P}}} \qquad \textit{such that} \quad \underbrace{P_0 = \Pi^*\left(g,\alpha,\delta\right) = e^{-rT}\cdot E^{\mathbb{Q}}\left(L(T)\right).}_{\substack{\textit{Fair contract} \\ \textit{under the risk-neutral measure } \mathbb{Q}}} \tag{27}$$

Hence, for a fixed nominal premium P_0, a fair parameter combination (g, α, δ) is chosen that leads to the highest customer value, while providing, at a minimum, risk-adequate returns for company's equityholders. A higher customer value increases the premium agreement range and thus may enable the company to realize a higher rate of return for its equityholders. However, these optimal contracts may not comply with regulatory restrictions on minimum interest rates or other legal requirements. We will consider this situation for the case of Germany in the numerical examples conducted in Section 5.

We now use some specific model cases to demonstrate the procedure required by Equation (27). We focus on the case of deterministic wealth (see Section 3.2, Part A) and aim to derive explicit expressions for the customer value of fair contracts. The procedure is, in principle, the same for the case of stochastic wealth (Part B); however, derivation of explicit expressions is far more complex.

For participating life insurance contracts with all three features, that is, g, α, and δ, the accumulated policy reserve at maturity, $P(T) = f(g,\alpha)$, is a function of g and α. For a given g^* and α^*, the fairness condition in Equation (11) is satisfied if δ is given by

$$\delta^* = \frac{P_0 - e^{-rT} \cdot E^{\mathbb{Q}}\left(P(T)\right) + Put}{Call} = h\left(g^*, \alpha^*\right), \tag{28}$$

where $Call = e^{-rT} \cdot E^{\mathbb{Q}}\left[\max\left(\beta \cdot A(T) - P(T), 0\right)\right]$ and Put is a put option with value $e^{-rT} \cdot E^{\mathbb{Q}}\left[\max\left(P(T) - A(T), 0\right)\right]$. In Equation (28), δ^* is a function of g^* and α^* (denoted by $h\left(g^*, \alpha^*\right)$). Thus, $\left(g^*, \alpha^*, \delta^*\right)$ represents a fair parameter combination that serves as a starting point for further calculation of customer value using Equations (20) and (26). Our final goal is to find a fair parameter combination that maximizes customer value as expressed by Equation (27).

4.1 The general case

For the case of deterministic wealth, we replace δ^* with the expression in Equation (28) and rewrite the second term in Equation (20)—the variance term—as

$$\sigma^2\left(L(T)\right)=\sigma^2\left(P(T)+\delta^*\cdot\max\left(\beta\cdot A(T)-P(T),0\right)-\max\left(P(T)-A(T),0\right)\right)$$
$$= f\left(g^*,\alpha^*,h\left(g^*,\alpha^*\right)\right). \tag{29}$$

Hence, the variance of the policyholder payoff $L(T)$ depends on the functions *f* and *h*. The customer value P_0^Φ under fair contract conditions is thus given by

$$\begin{aligned} P_0^\Phi &= e^{-rT}\cdot\left[E\left(L(T)\right)-\frac{a}{2}\cdot\sigma^2\left(L(T)\right)\right] \\ &= e^{-rT}\cdot E\left(P(T)\right)+h\left(g^*,\alpha^*\right)\cdot Call^P\left(g^*,\alpha^*\right) \\ &\quad -Put^P\left(g^*,\alpha^*\right)-e^{-rT}\cdot\frac{a}{2}\cdot\left(f\left(g^*,\alpha^*,h\left(g^*,\alpha^*\right)\right)\right), \end{aligned} \tag{30}$$

where

$$Call^P = e^{-rT}\cdot E\left[\max\left(\beta\cdot A(T)-P(T),0\right)\right],$$
$$Put^P = e^{-rT}\cdot E\left[\max\left(P(T)-A(T),0\right)\right]. \tag{31}$$

Equation (30) shows that P_0^Φ is a function of g^* and α^* only, since the fair δ^* is a function of these two parameters. Thus with δ being replaced by the function $h\left(g^*,\alpha^*\right)$, g^* or α^* can be increased and still satisfy the fairness constraint.

Further, with an increasing risk aversion parameter *a*, P_0^Φ is decreasing if

$$f\left(g^*,\alpha^*,h\left(g^*,\alpha^*\right)\right)>0. \tag{32}$$

The optimization problem in Equation (27) can be solved using the Lagrange method. If, for instance, the guaranteed interest rate is fixed by the regulatory authorities, the annual surplus participation parameter α that maximizes P_0^Φ is given by the implicit solution of the equation

$$\frac{\partial P_0^\Phi\left(g^*\right)}{\partial\alpha}=0 \tag{33}$$

if the second derivative is negative. The partial derivatives can also be used to see how customer value will change when increasing or decreasing g or α given fair contracts. However, more general statements regarding the impact of each contract parameter cannot be derived due to the complexity of the expression. For instance, one cannot be sure that an increasing guaranteed interest rate will raise the customer value under fair contract conditions. This is likely to be the case only for certain intervals, which we will illustrate in numerical examples in Section 5.

4.2 Contracts with one option

Let us now consider the special case of contracts that contain only one of the three parameters: either a guaranteed interest rate, or annual surplus participation, or terminal bonus. Our goal is to derive explicit expressions for willingness to pay for all three contract types and to see which of them generates the highest customer value. Furthermore, these simple types of contracts may generally imply a higher customer value than the more complicated contracts that include all three parameters.

For simplicity, we assume that the equity capital is sufficiently high for a default put option value of approximately zero. This allows derivation of explicit expressions for each fair contract parameter and for the customer value (for a detailed derivation, see Appendix B).

Guaranteed interest rate

For a contract that features only a guaranteed interest rate and does not include annual or terminal surplus participation, i.e., $g > 0$, $\alpha = 0$, $\delta = 0$, we proceed as in the general case and first calibrate g to be fair under the risk-neutral measure $\mathbb{Q}$, resulting in

$$\left(1+g^*\right)^T = e^{rT}. \tag{34}$$

Given g*, we obtain the following expression for the customer value:

$$P_0^\Phi = e^{-rT} \cdot P_0 \cdot \left(1+g^*\right)^T = P_0. \tag{35}$$

This outcome is intuitive since this contract carries no risk. Therefore, the guaranteed interest rate must be equal to the risk-free rate in order to ensure no arbitrage possibilities. Hence, a policyholder would be willing to pay only the nominal value P_0 for a contract that guarantees the risk-free rate.

Annual guaranteed surplus participation

Second, we examine a contract with annual guaranteed surplus participation and a money-back guarantee that, at a minimum, returns the premiums paid into the contract, i.e., $g = 0$, $\alpha > 0$, $\delta = 0$. In this case, the fair annual surplus participation rate is given by (see Equations (B5)-(B8), Appendix B)

$$\alpha^* = \frac{P_0 \cdot \left(1-e^{-rT}\right)}{\sum_{i=1}^{T} \gamma \cdot e^{-rT} \cdot E^{\mathbb{Q}}\left(max\left[\left(A(i)-A(i-1)\right),0\right]\right)}. \tag{36}$$

The customer value for this fair α^* results in (see Equation (B9), Appendix B)

$$\begin{aligned} P_0^{\Phi} &= e^{-rT} \cdot P_0 + P_0 \cdot \left(1-e^{-rT}\right) \cdot \frac{\sum_{i=1}^{T} E\left(\max\left[\left(A(i)-A(i-1)\right),0\right]\right)}{\sum_{i=1}^{T} E^{\mathbb{Q}}\left(\max\left[\left(A(i)-A(i-1)\right),0\right]\right)} \\ &\quad -e^{-rT} \cdot \frac{a}{2} \cdot \sigma^2 \left(P_0 \cdot \left(1-e^{-rT}\right) \cdot \frac{\sum_{i=1}^{T} \max\left[\left(A(i)-A(i-1)\right),0\right]}{e^{-rT} \cdot \sum_{i=1}^{T} E^{\mathbb{Q}}\left(\max\left[\left(A(i)-A(i-1)\right),0\right]\right)} \right). \end{aligned} \tag{37}$$

Terminal bonus payment and money-back guarantee

We finally consider a contract with a terminal bonus payment and a money-back guarantee, $g = 0$, $\alpha = 0$, $\delta > 0$. Similar to the previous case, the fair terminal surplus participation rate is (see Equation (B14), Appendix B)

$$\delta^{*}=\frac{P_0\cdot\left(1-e^{-rT}\right)}{e^{-rT}\cdot E^{\mathbb{Q}}\left(max\left(\beta\cdot A(T)-P_0,0\right)\right)}. \tag{38}$$

Inserting this participation rate into the customer value formula yields (see Equation (B15), Appendix B)

$$\begin{aligned}P_0^{\Phi}&=e^{-rT}\cdot P_0+P_0\cdot\left(1-e^{-rT}\right)\cdot\frac{e^{-rT}\cdot E\left(\max\left(\beta\cdot A(T)-P_0,0\right)\right)}{e^{-rT}\cdot E^{\mathbb{Q}}\left(\max\left(\beta\cdot A(T)-P_0,0\right)\right)}\\&\quad-e^{-rT}\cdot\frac{a}{2}\cdot\sigma^2\left[P_0\cdot\left(1-e^{-rT}\right)\cdot\frac{\max\left(\beta\cdot A(T)-P_0,0\right)}{e^{-rT}\cdot E^{\mathbb{Q}}\left(\max\left(\beta\cdot A(T)-P_0,0\right)\right)}\right].\end{aligned} \tag{39}$$

We can reformulate Equation (39) by using the fact that

$$\beta\cdot e^{-rT}\cdot E\left(\max\left(A(T)-A(0),0\right)\right)=\beta\cdot e^{-rT}\cdot E\left[\sum_{i=1}^{T}\max\left(A(i)-A(i-1),0\right)\right]. \tag{40}$$

However, even though

$$E\left[\sum_{i=1}^{T}\max\left(A(i)-A(i-1),0\right)\right]\geq E\left[\max\left(\sum_{i=1}^{T}A(i)-A(i-1),0\right)\right], \tag{41}$$

a general ranking between, e.g., Equations (37) and (39) cannot be derived due to the ratios of expected values under the real-world and risk-neutral measures contained in these equations. For the same reason, they cannot be explicitly compared to Equation (35) for the contract with a guaranteed interest rate only. It is not clear whether the customer values of the fair contracts with annual or terminal surplus participation are below or above the premium P_0 and thus preferable compared to a contract that contains only a guaranteed interest rate. However, we believe the explicit formulas in Equations (35), (37), and (39) to be useful for practical implementation, as numerical inputs will deliver comparable results.

5. NUMERICAL EXAMPLES

This section illustrates application of the explicit formulas derived in the previous section using numerical examples. In particular, we demonstrate how contract parameters in a participating life policy can be adjusted to lead to fair contracts and, at the same time, increase customer value.

Input parameters

Until otherwise stated, we use the following input parameters as the basis for all our numerical analyses. The case considered reflects the condition of the German market; however, the analysis can easily be adjusted to meet conditions prevalent in other countries.

r = 4.5%, μ_A = 7%, σ_A = 6%, P_0 = 100, Eq_0 = 30, γ = 50%, T = 10.

The assets of the insurance company $A(t)$ are invested in a portfolio with mean annual return of 7% $(=\mu_A)$, and a standard deviation of the annual return of 6% $(=\sigma_A)$; the risk-free interest rate r is set to 4.5%. Further, the fair premium and thus the starting value of the policyholder account is set to 100 $(=P_0)$. The contribution of the equityholders is set to Eq_0 = 30. As in Kling, Richter, and Ruß (2007), the relation of book to market values, which at the same time is an (inverse) flexibility parameter for the insurance company to build up hidden reserves, is set to γ = 50%. The input parameters reflect a high safety level for the insurance company. Numerical results are derived using Monte Carlo simulation, where necessary, on the basis of 100,000 simulation runs.

Currently, e.g., German regulations concerning policy reserves require a minimum annual interest rate of 2.25% until maturity $(=g)$ for all German life insurance contracts issued after January 2007. Furthermore, German law generally ensures that at least 90% of the investment earnings on book values are credited to the policyholder account (α). In the base case, we use these preset parameters and calculate the terminal surplus participation rate (δ) such that the fairness condition of Equation (11) is satisfied. Hence, the present value of the policyholder payoff is equal to the initial nominal premium of Π^* = 100. This is achieved by setting δ = 68%.

Table 1 contains numerical results for the cases of deterministic and stochastic wealth. The left part of the table displays parameter combinations that lead to a fair contract value of Π^* = 100 (fair premium from the insurer perspective in order to achieve a risk-adequate return). To provide an indication of the risk associated with the contracts, we list the corresponding default put option value (DPO) and the shortfall probability. The right part of the table contains the corresponding customer value based on the policyholders' mean-variance preferences for the case of deterministic (first column in the right part) and stochastic wealth (second to seventh column in the right part). Customer values are calculated using the expressions in Equations (20) and (26).

Panel A of Table 1 displays the base case, i.e., the contract satisfying regulatory restrictions. For better comparison, we adjust the risk-aversion parameter *a* such that the customer value in this base case is equal to the fair policy price of 100 $\left(= P_0^{\Phi} = \Pi^*\right)$. Thus, we start the analysis with standardized parameters. For the cases of deterministic and stochastic wealth, these values are given by *a* = 0.0685 and *a* = 0.0105, respectively. In all examples, we first calibrate contract parameters to have the same fair value from the insurer perspective using risk-neutral valuation. Second, we calculate the corresponding customer value for these contracts by using the explicit expressions for deterministic and stochastic wealth derived in the previous sections.

Table 1 illustrate the different values of the contracts to a risk-averse customer with mean-variance preferences, even though all contracts in the left column have the same fair value $\left(\Pi^*\right)$ of 100 for the insurer. In particular, the customer value varies substantially, i.e., contracts can be designed such that policyholder willingness to pay considerably exceeds the minimum premium required to achieve a risk-adequate return on equity.

Table 1: Fair contracts and corresponding customer value for deterministic and stochastic wealth.

Fair contract parameters (insurer perspective)						Customer value P_0^{Φ} (policyholder perspective)						
Guaranteed interest rate (g)	Terminal participation rate (δ)	Annual participation rate (α)	Π^*	DPO	*Shortfall probability*	Part A: deterministic (a = 0.0685)	Part B: stochastic (a = 0.0105)	ρ = 0.9	σ_Z = 8%	σ_Z = 4%	Z_0 = 200	a = 0.0685
Panel A: Contract with regulatory restrictions:												
2.25%	68%	90%	100	0.06	0.02%	100.0	100.0	96.6	106.7	95.5	104.2	133.7
Panel B: Simple contracts with one parameter only:												
4.56%	0%	0%	100	1.07	0.69%	100.9	85.4	85.3	91.7	81.3	89.0	133.8
0.00%	99.89%	0%	100	0.06	0.02%	96.5	**104.1**	**100.3**	**110.9**	**99.5**	**108.4**	130.6
0.00%	0%	130%	100	0.14	0.09%	**101.3**	99.9	96.7	106.7	95.5	104.1	**134.7**
Panel C: Maximizing customer value:												
1.00%	0%	123%	100	0.02	0.01%	**102.4**	98.6	95.6	105.3	94.2	102.7	**135.7**
	40%	117%	100	0.02	0.01%	100.8	99.9	96.6	106.6	95.5	104.1	134.3
	80%	101%	100	0.01	0.00%	98.3	**101.9**	**98.3**	**108.7**	**97.4**	**106.1**	132.0
2.00%	0%	113%	100	0.07	0.03%	**103.5**	96.9	94.1	103.5	92.5	101.0	**136.6**
	40%	105%	100	0.06	0.02%	101.8	98.7	95.5	105.4	94.2	102.8	135.1
	80%	85%	100	0.04	0.01%	98.8	**101.3**	**97.7**	**108.0**	**96.8**	**105.5**	132.5
3.00%	0%	99%	100	0.18	0.08%	**104.3**	94.6	92.2	101.2	90.2	98.5	**137.3**
	40%	89%	100	0.15	0.06%	102.9	97.0	94.1	103.6	92.5	101.0	136.1
	80%	58%	100	0.11	0.04%	99.6	**100.2**	**96.8**	**106.9**	**95.7**	**104.4**	133.2
4.00%	0%	77%	100	0.47	0.26%	**104.3**	91.0	89.3	97.4	86.7	94.8	**137.2**
	40%	58%	100	0.42	0.22%	104.2	94.0	91.6	100.5	89.6	97.9	**137.2**
	50%	47%	100	0.41	0.21%	104.0	**94.6**	**92.2**	**101.2**	**90.3**	**98.6**	137.1
4.30%	0%	67%	100	0.62	0.39%	103.7	89.3	88.1	95.7	85.1	93.0	136.9
	10%	62%	100	0.61	0.39%	104.1	90.0	88.6	96.5	85.8	93.8	137.2
	30%	45%	100	0.60	0.37%	**104.6**	**91.3**	89.6	97.8	87.1	95.1	**137.6**
4.40%	0%	62%	100	0.69	0.42%	103.3	87.7	87.5	95.0	84.4	92.3	136.6
	25%	37%	100	0.67	0.38%	104.4	88.3	88.6	96.4	85.7	93.7	137.5
	27%	9%	100	0.67	0.38%	**104.7**	**90.1**	**88.8**	**96.6**	**85.9**	**93.9**	**137.7**
4.50%	0%	56%	100	0.76	0.45%	102.8	87.7	86.9	94.1	83.6	91.4	136.2
	15%	39%	100	0.75	0.42%	103.6	**88.4**	**87.4**	**94.8**	**84.2**	**92.1**	136.8
	17%	26%	100	0.75	0.42%	**103.7**	**-**	**-**	**-**	**-**	**-**	**136.9**

Part A: Numerical results for deterministic wealth of policyholder

We look first at the results for the case of deterministic wealth. As mentioned above, the risk-aversion parameter for this case is set to $a = 0.0685$ so that the customer value P_0^Φ will be equal to the fair premium $P_0 = 100$ in Panel A of Table 1. When considering fair contracts with only one of the three contract parameters (g, α, δ)—as discussed in Section 4—we find that the customer value can be increased above this level (see Panel B of Table 1). In particular, the highest value for deterministic wealth ($P_0^\Phi = 101.3$) among the three simple contracts is achieved when offering a contract with an annual surplus participation rate and a money-back guarantee ($g = 0\%$) only. To ensure fair contract conditions, this fair annual rate even exceeds 100%. A contract with a guaranteed interest rate on the premium paid is also more valuable to a customer with mean-variance preferences than the fair contract that complies with regulatory restrictions (Panel A of Table 1). In particular, this result demonstrates that the premium agreement range can be increased by freely adjusting contract parameters with the aim of maximizing customer value while continuing to keep the contracts fair from the insurer perspective.

To illustrate this process, Panel C in Table 1 contains customer values for different choices of g, α, and δ. As discussed in Section 4, the results show that customer value is a complex function of these three parameters. For lower fixed values of g (1%, 2%, 3%, 4%), customer value is highest if the terminal bonus participation rate is zero. At the same time, customer value is increasing with increasing guaranteed rate. This pattern changes, however, when the guaranteed rate approaches the risk-free rate. Here, policyholders prefer higher terminal bonus with low annual surplus participation. The highest customer value in the examples considered is obtained for $g = 4.4\%$, $\alpha = 5\%$, and $\delta = 27\%$. However, this combination represents maximum customer value regarding fair contracts only for these numerical examples. Since there are in general an infinite number of parameter combinations leading to one specific fair contract value, analyzing a larger set of contracts may lead to a further increase in customer value.

Part B: Numerical results for stochastic wealth of policyholders

Next, the case of stochastic wealth is considered. Here, we assume that the drift and volatility of the investment open to the policyholders are given by $\mu_Z = 7\%$ and $\sigma_Z = 6\%$, which are the same parameters applicable to the policyholder account. For simplicity, we start by assuming that policyholder and insurer investments are uncorrelated $(\rho = 0)$ and then consider the case of positively correlated cash flows $(\rho = 0.9)$. Results are exhibited in Part B of the right-hand side customer value area in Table 1. In contrast to the case of deterministic wealth, we now find the maximum customer value of P_0^{Φ} =104.1 for a simple contract with a terminal bonus participation rate only.

For a positive correlation coefficient of 0.9 between the payoff from the life insurance contract and insurer investments, customer value is reduced compared to the contract with uncorrelated cash flows. This is due to a lower diversification effect achieved when investing in the life policy. A higher volatility of the wealth process *Z* of $\sigma_Z = 8\%$ makes (ceteris paribus) the less volatile life insurance contract $(\sigma_A = 6\%)$ more attractive from the policyholder perspective and, hence, P_0^{Φ} is increasing. The opposite is observed for a lower wealth process volatility of $\sigma_Z = 4\%$. We further find that a higher initial wealth of 200, compared to 150, increases the customer value of the contract. In addition, if the risk-aversion coefficient is the same as in the case of deterministic wealth (*a* = 0.0685), customer value increases substantially. However, the differences in customer value for different fair parameter combinations are quite small for *a* = 0.0685.

Overall, we find that restrictions on contract parameters can—at least in our model setup—seriously depress customer value. The extent of the loss in utility depends on the preference function of the policyholders.

6. Summary and Policy Implications

Most literature on participating life insurance focuses on pricing from the insurer perspective and does not take into consideration how policyholders might value the contract. In this paper, we examine how insurers can generate customer value for participating life insurance contracts by combining their perspective with that of the policyholders. Participating life insurance contracts feature a minimum interest rate guarantee, a guaranteed annual participation in the surplus generated by the asset portfolio of the insurer, and a terminal bonus. In this paper, customer value is defined as policyholder willingness to pay and is calculated based on mean-variance preferences. We compare the cases of policyholders with deterministic wealth and those with stochastic wealth, i.e., with and without diversification opportunities and derive closed-form solutions for selected cases of fair contract combinations and customer value.

For the insurer, we assume that the preference-free approach of risk-neutral valuation is used (hence, cash flows of an insurance contract can be replicated by means of assets traded on the capital market). We combine customer value and the insurer’s valuation by first calibrating contract parameters so that all contracts have the same fair risk-neutral value from the insurer perspective. In the second step, we derive explicit expressions for the customer value of these same contracts.

Our findings show that customer value varies substantially, even though all contracts have the same value from the insurer perspective. The results suggest that customer segmentation (in this sense) is a viable tool for increasing insurer profit and achieving a shareholder return above the risk-adequate rate. If insurers know how particular segments of the customer population value the financial part of the contracts, they can design contracts (by adjusting the guaranteed interest rate and/or annual and terminal surplus participation rate) to specifically increase customer value compared to standard contracts. In particular, preferred contracts may be simple contracts with, e.g., only one of the three parameters, as illustrated by our numerical example for stochastic policyholder wealth. For instance, a change from the regulatory parameter combination to the case with terminal participation rate increases customer value by approximately 4%, given our input assumptions.

Depending on the respective preferences, customer value may be even further increased for higher default put option values (or shortfall probability). Hence, policyholders may prefer a fair product parameter combination that is associated with higher shortfall risk but are simpler by only including one contract parameter, for instance. Future steps in the customer value analysis should take behavioral aspects into consideration. If the safety level is a main decision variable for policyholders, results may differ and default put option values could have a much more negative impact on customer value.

APPENDIX A

Derivation of the customer value given the case of stochastic wealth

In the following, explicit expressions of customer value in the case of stochastic wealth are derived.

$$\mathrm{E}\left(Z_T{}^{NI}\right)-\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{NI}\right)=\mathrm{E}\left(Z_T{}^{WI}\right)+E\left(L(T)\right)-\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{WI}+L(T)\right)$$
$$\Leftrightarrow P_0^{\Phi}\cdot E\left(\tilde{Z}_T\right)-E\left(L(T)\right)-\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{NI}\right)+\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{WI}+L(T)\right)=0, \qquad \text{(A1)}$$

with

$$\tilde{Z}_T=\exp\left[\left(\mu_Z-\sigma_Z{}^2/2\right)\cdot T+\sigma_Z\cdot\left(W_Z(T)-W_Z(0)\right)\right].$$

Calculation of the last variance term in Equation (A1) leads to:

$$\begin{aligned}
&\sigma^2\left(Z_T{}^{WI}+L(T)\right)\\
&=\sigma^2\left(Z_T{}^{NI}-P_0^{\Phi}\cdot\tilde{Z}_T+L(T)\right)\\
&=\sigma^2\left(Z_T{}^{NI}\right)+\sigma^2\left(P_0^{\Phi}\cdot\tilde{Z}_T\right)+\sigma^2\left(L(T)\right)-2\cdot Cov\left(Z_T{}^{NI},P_0^{\Phi}\cdot\tilde{Z}_T\right)\\
&\;+2\cdot Cov\left(Z_T{}^{NI},L(T)\right)-2\cdot Cov\left(P_0^{\Phi}\cdot\tilde{Z}_T,L(T)\right)\\
&=\sigma^2\left(Z_T{}^{NI}\right)+P_0^{\Phi 2}\cdot\sigma^2\left(\tilde{Z}_T\right)+\sigma^2\left(L(T)\right)-2\cdot P_0^{\Phi}\cdot Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)\\
&\;+2\cdot Cov\left(Z_T{}^{NI},L(T)\right)-2\cdot P_0^{\Phi}\cdot Cov\left(\tilde{Z}_T,L(T)\right)\\
&=P_0^{\Phi 2}\cdot\sigma^2\left(\tilde{Z}_T\right)-2\cdot P_0^{\Phi}\cdot Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)-2\cdot P_0^{\Phi}\cdot Cov\left(\tilde{Z}_T,L(T)\right)\\
&\;+\sigma^2\left(Z_T{}^{NI}\right)+\sigma^2\left(L(T)\right)+2\cdot Cov\left(Z_T{}^{NI},L(T)\right).
\end{aligned} \qquad \text{(A2)}$$

Replacing the variance term in Equation (A1) with the result derived in Equation (A2) leads to

$$P_0^{\Phi}\cdot E\left(\tilde{Z}_T\right)-E\left(L\left(T\right)\right)-\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{NI}\right)+\frac{a}{2}\cdot P_0^{\Phi 2}\cdot\sigma^2\left(\tilde{Z}_T\right)-a\cdot P_0^{\Phi}\cdot Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)$$

$$-a\cdot P_0^{\Phi}\cdot Cov\left(\tilde{Z}_T,L\left(T\right)\right)+\frac{a}{2}\cdot\sigma^2\left(Z_T{}^{NI}\right)+\frac{a}{2}\cdot\sigma^2\left(L\left(T\right)\right)+a\cdot Cov\left(Z_T{}^{NI},L\left(T\right)\right)=0$$

$$\Leftrightarrow\frac{a}{2}\cdot P_0^{\Phi 2}\cdot\sigma^2\left(\tilde{Z}_T\right)-a\cdot P_0^{\Phi}\cdot Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)+P_0^{\Phi}\cdot E\left(\tilde{Z}_T\right)-a\cdot P_0^{\Phi}\cdot Cov\left(\tilde{Z}_T,L\left(T\right)\right)-$$

$$-E\left(L\left(T\right)\right)+\frac{a}{2}\cdot\sigma^2\left(L\left(T\right)\right)+a\cdot Cov\left(Z_T{}^{NI},L\left(T\right)\right)=0$$

$$\Leftrightarrow P_0^{\Phi 2}\cdot\left(\frac{a}{2}\cdot\sigma^2\left(\tilde{Z}_T\right)\right)+P_0^{\Phi}\cdot\left[E\left(\tilde{Z}_T\right)-a\cdot Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)-a\cdot Cov\left(\tilde{Z}_T,L\left(T\right)\right)\right]$$

$$+\left[\frac{a}{2}\cdot\sigma^2\left(L\left(T\right)\right)+a\cdot Cov\left(Z_T{}^{NI},L\left(T\right)\right)-E\left(L\left(T\right)\right)\right]=0$$

$$\Leftrightarrow P_0^{\Phi 2}+P_0^{\Phi}\cdot\frac{2}{\left(\sigma^2\left(\tilde{Z}_T\right)\right)}\cdot\left[\frac{1}{a}\cdot E\left(\tilde{Z}_T\right)-Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)-Cov\left(\tilde{Z}_T,L\left(T\right)\right)\right]$$

$$+\frac{1}{\left(\sigma^2\left(\tilde{Z}_T\right)\right)}\cdot\left[\sigma^2\left(L\left(T\right)\right)+2\cdot Cov\left(Z_T{}^{NI},L\left(T\right)\right)-\frac{2}{a}\cdot E\left(L\left(T\right)\right)\right]=0$$

$$\Leftrightarrow P_0^{\Phi 2}+2\cdot P_0^{\Phi}\cdot\frac{\frac{1}{a}\cdot E\left(\tilde{Z}_T\right)-Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)-Cov\left(\tilde{Z}_T,L\left(T\right)\right)}{\sigma^2\left(\tilde{Z}_T\right)}$$

$$+\frac{\sigma^2\left(L\left(T\right)\right)+2\cdot Cov\left(Z_T{}^{NI},L\left(T\right)\right)-\frac{2}{a}\cdot E\left(L\left(T\right)\right)}{\sigma^2\left(\tilde{Z}_T\right)}=0$$

$$\Leftrightarrow P_0^{\Phi 2}+2\cdot P_0^{\Phi}\cdot\frac{\frac{1}{a}\cdot E\left(\tilde{Z}_T\right)-Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)-Cov\left(\tilde{Z}_T,L\left(T\right)\right)}{\sigma^2\left(\tilde{Z}_T\right)}$$

$$+\left[\frac{\frac{1}{a}\cdot E\left(\tilde{Z}_T\right)-Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)-Cov\left(\tilde{Z}_T,L\left(T\right)\right)}{\sigma^2\left(\tilde{Z}_T\right)}\right]^2$$

$$=\left[\frac{\frac{1}{a}\cdot E\left(\tilde{Z}_T\right)-Cov\left(Z_T{}^{NI},\tilde{Z}_T\right)-Cov\left(\tilde{Z}_T,L\left(T\right)\right)}{\sigma^2\left(\tilde{Z}_T\right)}\right]^2-\frac{\sigma^2\left(L\left(T\right)\right)+2\cdot Cov\left(Z_T{}^{NI},L\left(T\right)\right)-\frac{2}{a}\cdot E\left(L\left(T\right)\right)}{\sigma^2\left(\tilde{Z}_T\right)}$$

$$\Leftrightarrow \left[P_0^{\Phi} + \frac{\frac{1}{a} \cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T^{NI}, \tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^2$$

$$= \left[\frac{\frac{1}{a} \cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T^{NI}, \tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^2 - \frac{\sigma^2\left(L(T)\right) + 2 \cdot Cov\left(Z_T^{NI}, L(T)\right) - \frac{2}{a} \cdot E\left(L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)}$$

$$\Leftrightarrow P_0^{\Phi} = \left[\left[\frac{\frac{1}{a} \cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T^{NI}, \tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^2 - \frac{\sigma^2\left(L(T)\right) + 2 \cdot Cov\left(Z_T^{NI}, L(T)\right) - \frac{2}{a} \cdot E\left(L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^{\frac{1}{2}} \tag{A3}$$

$$- \frac{\frac{1}{a} \cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T^{NI}, \tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)}.$$

APPENDIX B

Derivation of formulas in Section 4.2 (Contracts with one option—deterministic wealth)

i) $g > 0$, $\alpha = 0$, $\delta = 0$.

In this case, we have

$$L(T) = P(T) + \delta \cdot B(T) - D(T) = P(T) = P_0 \cdot (1+g)^T, \tag{B1}$$

and that the contract is fair if

$$P_0 = E^{\mathbb{Q}}\left(e^{-rT} \cdot L(T)\right) = e^{-rT} \cdot P_0\left(1+g^*\right)^T. \tag{B2}$$

Hence, from the insurer perspective, the fair guaranteed interest rate satisfies

$$\left(1+g^*\right)^T = e^{rT}. \tag{B3}$$

For the customer value, Equation (20) implies that

$$\begin{aligned} P_0^{\Phi} &= e^{-rT} \cdot E\left(L(T)\right) - e^{-rT} \cdot \frac{a}{2} \cdot \sigma^2\left(L(T)\right) \\ &= e^{-rT} \cdot E\left(P_0 \cdot \left(1+g^*\right)^T\right) - e^{-rT} \cdot \frac{a}{2} \cdot \sigma^2\left(P_0 \cdot \left(1+g^*\right)^T\right) \\ &= e^{-rT} \cdot P_0 \cdot \left(1+g^*\right)^T = P_0 \cdot e^{-rT} \cdot e^{rT} = P_0. \end{aligned} \tag{B4}$$

ii) $g = 0$, $\alpha > 0$, $\delta = 0$.

For the policy reserves, one obtains

$$\begin{aligned} P(t) &= P(t-1)\cdot(1+g) + max\left[\alpha \cdot \gamma \cdot \left(A(t) - A(t-1)\right) - g \cdot P(t-1), 0\right] \\ &= P(t-1) + \alpha \cdot \gamma \cdot max\left[\left(A(t) - A(t-1)\right), 0\right] \\ &= P(t-2) + \alpha \cdot \gamma \cdot max\left[\left(A(t-1) - A(t-2)\right), 0\right] + \alpha \cdot \gamma \cdot max\left[\left(A(t) - A(t-1)\right), 0\right] \\ &= P_0 + \alpha \cdot \sum_{i=1}^{t} \gamma \cdot max\left[\left(A(i) - A(i-1)\right), 0\right]. \end{aligned} \tag{B5}$$

For the payoff to the policyholder, the money-back guarantee is added, leading to

$$L(T) = P(T) + \delta B(T) - D(T) = P_0 + \alpha \cdot \sum_{i=1}^{T} \gamma \cdot max\left[\left(A(i) - A(i-1)\right), 0\right]. \tag{B6}$$

The insurance contract is fair, if

$$\begin{aligned} P_0 &= E^{\mathbb{Q}}\left(e^{-rT} \cdot L(T)\right) = e^{-rT} \cdot E^{\mathbb{Q}}\left(P_0 + \alpha^* \cdot \sum_{i=1}^{T} \gamma \cdot max\left[\left(A(i) - A(i-1)\right), 0\right]\right) \\ &= e^{-rT} \cdot P_0 + \alpha^* \cdot \sum_{i=1}^{T} \gamma \cdot e^{-rT} \cdot E^{\mathbb{Q}}\left(max\left[\left(A(i) - A(i-1)\right), 0\right]\right), \end{aligned} \tag{B7}$$

which implies a fair annual surplus participation rate of

$$\alpha^* = \frac{P_0\left(1-e^{-rT}\right)}{\sum_{i=1}^{T}\gamma\cdot e^{-rT}\cdot E^{\mathbb{Q}}\left(max\left[\left(A(i)-A(i-1)\right),0\right]\right)}. \tag{B8}$$

The customer value for the fair α^* results in

$$\begin{aligned}
P_0^{\Phi} &= e^{-rT}E\left(L(T)\right)-e^{-rT}\frac{a}{2}\sigma^2\left(L(T)\right)\\
&= e^{-rT}E\left(P_0+\alpha^*\cdot\sum_{i=1}^{T}\gamma\max\left[(A(i)-A(i-1)),0\right]\right)\\
&\quad -e^{-rT}\frac{a}{2}\cdot\sigma^2\left(P_0+\alpha^*\cdot\sum_{i=1}^{T}\gamma\max\left[(A(i)-A(i-1)),0\right]\right)\\
&= e^{-rT}P_0+P_0\left(1-e^{-rT}\right)\frac{\sum_{i=1}^{T}\gamma\cdot e^{-rT}\cdot E\left(\max\left[(A(i)-A(i-1)),0\right]\right)}{\sum_{i=1}^{T}\gamma\cdot e^{-rT}\cdot E^{\mathbb{Q}}\left(\max\left[(A(i)-A(i-1)),0\right]\right)}\\
&\quad -e^{-rT}\frac{a}{2}\cdot\sigma^2\left(P_0\left(1-e^{-rT}\right)\frac{\sum_{i=1}^{T}\gamma\cdot\max\left[(A(i)-A(i-1)),0\right]}{e^{-rT}\sum_{i=1}^{T}\gamma\cdot E^{\mathbb{Q}}\left(\max\left[(A(i)-A(i-1)),0\right]\right)}\right)\\
&= e^{-rT}P_0+P_0\left(1-e^{-rT}\right)\frac{\sum_{i=1}^{T}E\left(\max\left[(A(i)-A(i-1)),0\right]\right)}{\sum_{i=1}^{T}E^{\mathbb{Q}}\left(\max\left[(A(i)-A(i-1)),0\right]\right)}\\
&\quad -e^{-rT}\frac{a}{2}\cdot\sigma^2\left(P_0\left(1-e^{-rT}\right)\frac{\sum_{i=1}^{T}\max\left[(A(i)-A(i-1)),0\right]}{e^{-rT}\sum_{i=1}^{T}E^{\mathbb{Q}}\left(\max\left[(A(i)-A(i-1)),0\right]\right)}\right).
\end{aligned} \tag{B9}$$

The formula shows that the ratio of the sum of the value of $\max\left[(A(i)-A(i-1)),0\right]$ under the real-world measure $\mathbb{P}$ and under the risk-neutral measure $\mathbb{Q}$ is an important factor in determination of customer value.

iii) $g = 0$, $\alpha = 0$, $\delta > 0$.

For the policy reserves, we adjust the up-front premium and the terminal bonus accordingly:

$$P(t) = P(t-1)\cdot(1+g) + max\left[\alpha\cdot\gamma\cdot\left(A(t)-A(t-1)\right) - g\cdot P(t-1),0\right] = P_0 \qquad \text{(B10)}$$

$$B(T) = max\left(\beta\cdot A(T) - P(T),0\right) = max\left(\beta\cdot A(T) - P_0,0\right). \qquad \text{(B11)}$$

Therefore, the policyholder payoff is given by

$$L(T) = P(T) + \delta\cdot B(T) - D(T) = P_0 + \delta\cdot max\left(\beta\cdot A(T) - P_0,0\right) \qquad \text{(B12)}$$

and the contract is fair, if

$$\begin{aligned} P_0 &= E^{\mathbb{Q}}\left(e^{-rT}\cdot L(T)\right) = e^{-rT}\cdot E^{\mathbb{Q}}\left(P_0 + \delta^* \, max\left(\beta\cdot A(T) - P_0,0\right)\right) \\ &= e^{-rT}\cdot P_0 + \delta^*\cdot e^{-rT}\cdot E^{\mathbb{Q}}\left(max\left(\beta\cdot A(T) - P_0,0\right)\right) \\ &= e^{-rT}\cdot P_0 + \delta^*\cdot\beta\cdot e^{-rT}\cdot E^{\mathbb{Q}}\left(max\left(A(T) - \frac{P_0}{\beta},0\right)\right) \\ &= e^{-rT}\cdot P_0 + \delta^*\cdot\beta\cdot e^{-rT}\cdot E^{\mathbb{Q}}\left(max\left(A(T) - A(0),0\right)\right). \end{aligned} \qquad \text{(B13)}$$

Hence,

$$\delta^* = \frac{P_0\left(1-e^{-rT}\right)}{e^{-rT}\cdot E^{\mathbb{Q}}\left(max\left(\beta\cdot A(T) - P_0,0\right)\right)}. \qquad \text{(B14)}$$

The customer value is given by

$$
\begin{aligned}
P_0^{\Phi} &= e^{-rT} \cdot E\left(L(T)\right) - e^{-rT}\,\frac{a}{2}\sigma^2\left(L(T)\right) \\
&= e^{-rT} \cdot E\left(P_0 + \delta^* \cdot \max\left(\beta \cdot A(T) - P_0, 0\right)\right) \\
&\quad - e^{-rT} \cdot \frac{a}{2} \cdot \sigma^2\left(P_0 + \delta^* \cdot \max\left(\beta \cdot A(T) - P_0, 0\right)\right) \\
&= e^{-rT} \cdot P_0 + P_0 \cdot \left(1 - e^{-rT}\right) \frac{e^{-rT} \cdot E\left(\max\left(\beta \cdot A(T) - P_0, 0\right)\right)}{e^{-rT} \cdot E^{\mathbb{Q}}\left(\max\left(\beta \cdot A(T) - P_0, 0\right)\right)} \\
&\quad - e^{-rT} \cdot \frac{a}{2} \cdot \sigma^2\left(P_0 \cdot \left(1 - e^{-rT}\right) \frac{\max\left(\beta \cdot A(T) - P_0, 0\right)}{e^{-rT} \cdot E^{\mathbb{Q}}\left(\max\left(\beta \cdot A(T) - P_0, 0\right)\right)}\right).
\end{aligned}
\tag{B15}
$$

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