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Dynamic Hybrid Products in Life Insurance: Assessing the Policyholders' Viewpoint

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DYNAMIC HYBRID PRODUCTS IN LIFE INSURANCE: ASSESSING THE POLICYHOLDERS' VIEWPOINT

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ABSTRACT

Dynamic hybrid life insurance products are intended to meet new consumer needs regarding stability in terms of guarantees as well as sufficient upside potential. In contrast to traditional participating or classical unit-linked life insurance products, the guarantee offered to the policyholders is achieved by a periodical rebalancing process between three funds: the policy reserves (i.e. the premium reserve stock, thus causing interaction effects with traditional participating life insurance contracts), a guarantee fund, and an equity fund. In this paper, we consider an insurer offering both, dynamic hybrid and traditional participating life insurance contracts and focus on the policyholders' perspective. The results show that higher guarantees do not necessarily imply a higher willingness-to-pay, but that in case of dynamic hybrid contracts, a minimum guarantee level should be offered in order to ensure that the willingness-to-pay exceeds the minimum premium the insurer has to charge when selling the contract. In addition, strong interaction effects can be found between the two products, which particularly impact the willingness-to-pay of the dynamic hybrids.

Keywords: Life insurance, guaranteed interest rates, dynamic hybrid, constant proportion portfolio insurance, customer value, mean-variance preferences, risk-return profiles

1. INTRODUCTION

Innovations in the life and pension industry have become increasingly important, especially against the background of demographic changes and as an alternative or supplement to public state-run pension schemes. However, the currently low interest rates and volatile capital markets make providing long-term guarantees increasingly difficult for insurers. In addition, the industry faces increasing regulation and cost pressure, and consumer preferences for stability, upside potential and flexibility must be taken into account when developing new contracts. In

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this context, dynamic hybrid life insurance products have recently been introduced in the German market.[1] Instead of explicitly (externally or internally) hedging the guarantees embedded in the contract or by means of capital, the guarantee is ensured implicitly by means of a dynamic reallocation of the dynamic hybrid account value between three funds: the policy reserves (i.e. the premium reserve stock), a guarantee fund, and a (risky) equity fund, following the idea of constant proportion portfolio insurance (see Bohnert and Gatzert, 2014). In this paper, our aim is to study these products in depth from the policyholders' perspective by taking into account the preferences and willingness-to-pay of consumers. We thereby also focus on the interaction effects that arise due to the fact that dynamic hybrid funds are periodically shifted to and from the conventional policy reserves, e.g., in times of adverse capital markets.

Dynamic hybrid products have first been modeled in the scientific literature by Kochanski and Karnarski (2011), who derive solvency capital requirements for static and dynamic hybrids using a rules-based shifting mechanism, but without focusing on possible interaction effects with other products. The latter has been studied in depth by Bohnert and Gatzert (2014), who present a comprehensive model framework to assess and demonstrate the (strong) interaction effects between dynamic hybrid products and traditional participating life insurance policies at the company level, thereby focusing on the insurer's risk situation and the policyholders' net present value. A comprehensive overview of the German market of dynamic hybrid products is further provided in Bohnert (2013), who shows the variety of options embedded in the contracts and the implications of different shifting mechanisms by studying risk-return profiles provided by the industry. Thus, the scientific literature on dynamic hybrid products is still rather scarce.

In contrast, the consumer perspective on guarantees embedded in life insurance contracts in general has received increasing attention in the literature. Gatzert, Holzmüller, and Schmeiser (2012), for instance, use a theoretical model and a simulation study to compare the perspective of the insurer and the policyholder. They derive the willingness-to-pay for participating life insurance contracts using mean-variance preferences for different assumptions regarding the diversification opportunities of the policyholder and identify contract specifications that—while keeping the contract value fixed for the insurer—maximize customer value. The authors show that increasing the guaranteed interest rates does not necessarily maximize customer value. Broeders, Chen, and Koos (2011) use a similar approach based on a power utility function for the policyholder and study two types of annuity providers (defined benefit pension funds and life insurers) that differ according to the extent of risk sharing between beneficiar-

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[1] Currently about 20 life insurance companies in Germany (out of roughly 100) provide dynamic hybrid products (see Bohnert, 2013). Life insurers in Japan are considering introducing dynamic hybrid products as well.

ies and shareholders, demonstrating the need for regulation to provide a level playing field for providers. Schmeiser and Wagner (2013) consider the consumers' perspective when deriving minimum solvency capital requirements, and thereby illustrate how minimum interest rates should be defined by the regulator in order to maximize the policyholders' utility level.

While these papers use theoretical models to study the consumers' perspective, Gatzert, Huber, and Schmeiser (2011), for instance, also conduct an empirical survey to study the willingness-to-pay for interest rate guarantees in unit-linked life insurance contracts. Their results indicate that customers may not be willing to pay the risk-adequate price for the valuable guarantees as, on average, the willingness-to-pay was significantly lower than the minimum prices derived based on option-pricing theory. At the same time, however, a substantial portion of participants were willing to pay a considerably higher price, thus indicating a higher degree of risk-aversion. Further literature also reveals the importance of such things as customer preferences (e.g., see Doskeland and Nordahl, 2006), demographic characteristics such as income, gender, and education (e.g., see Feldman and Schultz, 2004), and insurer characteristics and operations (e.g., see Marshall, Hardy, and Saunders, 2010) in the determination of willingness-to-pay.

In this paper, we explicitly focus on the policyholders' perspective, thereby studying the willingness-to-pay based on risk preferences as well as risk-return profiles. We thereby extend the model in Bohnert and Gatzert (2014) for a life insurer offering dynamic hybrids and participating life insurance contracts by focusing on different dynamic hybrid guarantee level, varying guaranteed interest rates (to be credited to the policy reserves). We further extend the previous setting by integrating different shifting mechanisms for the dynamic hybrid funds. This analysis is intended to provide insight into the impact of different types of long-term guarantees as well as features and characteristics of these life insurance financial products from the policyholders' viewpoint.

The remainder of the paper is structured as follows. Section 2 presents the model framework of the insurance company offering participating life insurance policies and the dynamic hybrid products including fair valuation and risk measurement as well as the derivation of the willingness-to-pay from the policyholders' perspective. Section 3 contains a numerical analysis and Section 4 provides concluding remarks.

2. MODEL FRAMEWORK

Modeling the insurance company - overview

In the following, we consider a life insurer offering two types of products: traditional participating life insurance policies (*PLI*) and dynamic hybrid products (*DHP*). The general model framework for the insurance company is based on the model presented by Bohnert and Gatzert (2014), which is then extended by taking the consumers' perspective, which is the focus of the present analysis. Table 1 shows the simplified balance sheet of the insurer.

Table 1: Balance sheet of the life insurer at time *t* (see Bohnert and Gatzert, 2014)

Assets	Liabilities	
$A_t^{long-term}$	PR_t^{PLI}	PR_t
$A_t^{short-term}$	PR_t^{DHP}	AV_t
GF_t^A	GF_t^L	
EF_t^A	EF_t^L	
	B_t	
A_t	A_t	

Regarding the liability side, policyholders of both contract types are assumed to pay a single up-front premium P^{PLI} and P^{DHP}, implying initial policy reserves (*PR*) of the participating life insurance contracts of

$$PR_0^{PLI} = P^{PLI}$$

and an initial account value (*AV*) of the dynamic hybrid products of

$$AV_0^{DHP} = P^{DHP}.$$

As exhibited in Table 1, the dynamic hybrid products' account value *AV* is thereby composed of up to three parts, including a part that is invested in the insurer's collective policy reserves *PR*, an equity fund (*EF*), and a guarantee fund (*GF*) as described in detail later. The portion of the total policy reserves coming from the dynamic hybrid products is denoted as PR^{DHP}, which, together with the part coming from the traditional participating life insurance contracts PR^{PLI}, sums up to the total policy reserves $PR_t = PR_t^{PLI} + PR_t^{DHP}$.

The contract term T is assumed to coincide with the lifetime of the considered insurance company. At inception of the contract, the buffer B_0, residually given by the difference between assets and liabilities, is filled by the initial contribution of the company's equityholders. The contracts are then calibrated to be fair from the equityholders' perspective to ensure risk-adequate compensation for their investment.[2]

A summary of the various guarantees involved in the following model description is given in Table 2.

Table 2: Overview of guarantees and guarantee notations involved in the model

Guarantee notation	*Description*
Interest rate guarantee	Guaranteed interest rate r^G, minimum interest rate applied to the policy reserves on an annual basis; applies to policies invested in the policy reserves (especially participating life insurance policies; also relevant for dynamic hybrids for the part invested in the policy reserves)
Maturity guarantee	Maturity guarantee of dynamic hybrid contracts, only promised at maturity of the contract term; $G_T^{DHP} = x \cdot P^{DHP}$ (x = 1 corresponds to a money-back guarantee)
Money-back guarantee	Guarantees the payback of the single up-front premium paid into the contract at maturity
Guarantee fund *GF*	Equity fund, which ensures a maximum loss of λ percent within one period Δt and thus guarantees that the fund drops at most to $(1-\lambda)\cdot GF_{t+\Delta t}$
Minimum dynamic hybrid account value $G_{t+\Delta t}^{DHP}$	Account value needed at time t to ensure that the guarantee promised to the dynamic hybrid policyholders G_T^{DHP} can be met at maturity; may vary depending on the concrete product design (especially the guarantee promised to the dynamic hybrids, x)

The participating life insurance contract

Participating life insurance contracts feature an annual guaranteed interest rate r^G and an annual surplus participation rate α. The annual policy interest rate r_t^P is declared in advance at

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[2] In the present setting, interaction effects between the two contracts are one main reason why the situation is not automatically fair for the policyholders as well. In fact, the value of the policies can considerably depend on the portfolio composition of the insurer, i.e. the portion of dynamic hybrid contracts in the portfolio (see Bohnert and Gatzert, 2014).

the beginning of each year (as is required in the German market, for instance) and given by the smoothing scheme (see Grosen and Jørgensen, 2000)

$$r_t^P = \max\left(r^G, \alpha \cdot \left(\frac{B_t}{PR_t} - \gamma\right)\right),$$

where γ is the target buffer ratio, i.e., the ratio of the buffer account to policyholder liabilities ($PR_t = PR_t^{PLI} + PR_t^{DHP}$). The buffer account thereby represents the free surplus, which can be used to absorb losses with respect to the guaranteed positions on the liability side of the balance sheet, i.e. the policy reserves. To ensure that surplus is smoothed over time and to reduce volatility of the policy interest rate, the proportion between the buffer account and the policy reserves must amount to at least the target buffer ratio γ before surplus is distributed to the policyholders. A second control parameter, the surplus distribution ratio α is used to control the fraction of the excess amount of the target buffer ratio that is actually credited to the policyholders.[3] The policy interest rate is then credited to the policy reserves at time *t*, i.e.,

$$PR_{(t+1)}^{PLI} = PR_t^{PLI} \cdot \left(1 + r_t^P\right)$$

The participating life insurance contracts remain invested in the policy reserves (premium reserve stock) during the whole contract term and at maturity *T*, the participating life insurance contracts receive their policy reserves PR_T^{PLI} given that the insurer remains solvent.

The dynamic hybrid life insurance contract

The policyholders with the dynamic hybrid product are promised a fraction *x* of their single premium at maturity *T*, i.e., the maturity guarantee amounts to $G_T^{DHP} = x \cdot P^{DHP}$ (*x* = 1 corresponds to a money-back guarantee).[4] During the contract term, as shown in Table 1, the dynamic hybrid products' account value (*AV)* is dynamically reallocated between the policy reserves (*PR*), an equity fund (*EF*), and a guarantee fund (*GF*), whereby the latter is equivalent to an equity fund with a hedge that ensures a maximum loss of λ percent within one period. Similar to a constant proportion portfolio insurance (CPPI) strategy, this dynamic reallocation (described in detail below) is intended to ensure the guarantee promised to the policy-

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[3] In Germany, for instance, regulators prescribe a maximum period of time (e.g. three years), during which the surplus can be kept in the buffer account before it has to be credited to the insureds (see, e.g., Schradin, Pohl, and Koch, 2006, p. 14).

[4] In general, policyholders can choose a guarantee level up to 100% (see Bohnert, 2013).

holders at maturity without additional guarantee costs or further comprehensive hedging activities.

According to regulatory requirements in Germany regarding the policy reserves, the part of the dynamic hybrid funds invested in the policy reserves at time *t* must thereby be compounded with the same policy interest rate credited to the participating life insurance contracts (even though the funds may only be invested short-term in the policy reserves),[5] i.e., $PR_{(t+1)}^{DHP} = PR_t^{DHP} \cdot \left(1 + r_t^P\right)$.

At maturity *T*, the dynamic hybrid products receive their account value ($AV_T^{DHP} = PR_T^{DHP} + GF_T^L + EF_T^L$), consisting of the total of the three funds, given that the insurer does not default during the contract term.

To secure the maturity guarantee of $G_T^{DHP} = x \cdot P^{DHP}$ promised to the dynamic hybrid policyholders, the account value AV_t^{DHP} is dynamically distributed between the policy reserves PR_t^{DHP}, the guarantee fund GF_t^L and equity fund EF_t^L as shown by Kochanski and Karnarski (2011) as follows[6]

$$PR_t^{DHP} = \begin{cases} \dfrac{G_{t+\Delta t}^{DHP} - (1-\lambda) \cdot AV_t^{DHP}}{\left(1+r^G\right)^{\Delta t} - 1 + \lambda}, & if \dfrac{G_{t+\Delta t}^{DHP}}{(1-\lambda) \cdot AV_t^{DHP}} > 1 \\ 0, & otherwise \end{cases}$$

$$GF_t^L = \begin{cases} AV_t^{DHP} - PR_t^{DHP}, & if \dfrac{G_{t+\Delta t}^{DHP}}{(1-\lambda) \cdot AV_t^{DHP}} > 1 \\ \dfrac{G_{t+\Delta t}^{DHP}}{1-\lambda}, & otherwise \end{cases}$$

$$EF_t^L = AV_t^{DHP} - PR_t^{DHP} - GF_t^L .$$

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[5] Substitution effects may arise due to the potentially different investment horizons. In particular, it can be shown that for higher portions of dynamic hybrid products in the portfolio, the guaranteed interest rate is more difficult to achieve, as it is derived based on an expected long-term investment (see also Bohnert and Gatzert, 2014). This also implies that the portfolio composition has an impact on the policyholders' willingness-to-pay, for instance, as with an increasing portion of dynamic hybrid products and thus an increasing share in short-term investments, the volatility of the payoff and the default risk generally increase (depending on the guarantees promised to the dynamic hybrid products).

[6] This mechanism invests the maximum proportion of the account value in the equity fund (and guarantee fund) along with ensuring that the guarantees can still be met. While this is a common practice in the market, an alternative strategy would be to balance the tradeoff between the number of shifts (i.e., transaction costs), and upside potential (high proportion in equity funds), which thus imply varied risk profiles.

The concrete shifting mechanism thereby depends on the assumptions regarding $G_{t+\Delta t}^{DHP}$ at the end of each period, for instance, which denotes the account value needed at time t to ensure that the guarantee can be met at maturity and which may vary depending on the concrete product design. In particular, in the following, we compare two types of shifting mechanisms, leading to different risk-return profiles from the policyholders' perspective. First, in case of the "less risky" shifting mechanism, we assume that the account value must be at least

$$G_{t+\Delta t}^{DHP} = G_T^{DHP} = x \cdot P^{DHP} \;\; \forall t \in \left\{0, \Delta t, \ldots, T\right\}, \tag{1}$$

i.e., the guarantee must be ensured at all times, implying a higher portion of "riskless assets" in the portfolio of the insurance company (i.e., policy reserves and guarantee fund) and thus generally a lower risk and return. Second, we consider a "more risky" shifting mechanism by discounting the maturity guarantee to the current date (as is usually done in case of constant proportion portfolio insurance (CPPI) strategy, see Black and Jones (1987), Leland (1980)), implying a higher portion "risky assets", i.e.,

$$G_{t+\Delta t}^{DHP} = G_T^{DHP} \cdot \left(1 + r^G\right)^{-(T-t-\Delta t)} = x \cdot P^{DHP} \cdot \left(1 + r^G\right)^{-(T-t-\Delta t)}. \tag{2}$$

When $G_{t+\Delta t}^{DHP}$ can be fulfilled by the guarantee fund only, the dynamic hybrid products' funds are distributed between the guarantee fund and equity fund.

Terminal bonus payments and total maturity payoff

In addition to the previously described payoffs, policyholders of both contracts receive a terminal bonus from the remaining buffer account after the equityholders have received adequate compensation for their initial contribution in case the buffer is positive. The buffer is given by

$$B_t = A_t - PR_t^{PLI} - PR_t^{DHP} - GF_t^L - EF_t^L,$$

and the buffer payback to the equityholders is determined by

$$BP_T = \max\left(\min\left(B_T, B_0 \cdot (1+b)\right), 0\right),$$

where b denotes the fair (risk-adequate) buffer interest rate paid on their initial contribution and represents the dividend for the entire length of the investment, which is then calibrated to be fair.

The policyholders receive the terminal bonus $TB_T = \max\left(0, B_T - BP_T\right)$, which in case of positive policy reserves (zero otherwise) is assumed to be distributed between the two types of contracts as

$$TB_T^{PLI} = TB_T \cdot \sum_{k=1}^{T/\Delta t} PR_{(k\cdot\Delta t)}^{PLI} \Bigg/ \left(\sum_{k=1}^{T/\Delta t} PR_{(k\cdot\Delta t)}^{PLI} + PR_{(k\cdot\Delta t)}^{DHP} \right),$$

$$TB_T^{DHP} = TB_T \cdot \sum_{k=1}^{T/\Delta t} PR_{(k\cdot\Delta t)}^{DHP} \Bigg/ \left(\sum_{k=1}^{T/\Delta t} PR_{(k\cdot\Delta t)}^{PLI} + PR_{(k\cdot\Delta t)}^{DHP} \right).$$

This distribution scheme thus uses a discrete time (Δt, later assumed to be 1/12) weighted average over time, which takes into account the investment in the policy reserves over the whole contract term. The intuition behind this terminal bonus distribution scheme is that the terminal accumulated surplus is generated by the investments in the policy reserves. These funds are collectively invested in the capital market by the insurer, while the dynamic hybrids' guarantee fund and equity fund are invested individually and are directly credited to the dynamic hybrid policyholders.

Hence, the total contract payoffs V_T are given by

$$V_T^{PLI} = \left(PR_T^{PLI} + TB_T^{PLI} \right) \cdot 1\left\{ T_s > T \right\} + RF_t^{PLI} \cdot 1\left\{ T_s = t \right\} \tag{3}$$

and

$$V_T^{DHP} = \left(AV_T^{DHP} + TB_T^{DHP} \right) \cdot 1\left\{ T_s > T \right\} + RF_t^{DHP} \cdot 1\left\{ T_s = t \right\}, \tag{4}$$

where T_S denotes the time of default with $T_s = \inf\left\{ t : A_t^{long-term} + A_t^{short-term} < PR_t \right\},\ t = 1,...,T$, and *RF* refers to the remaining funds in case the insurer defaults during the contract term, i.e. if $B_{(t+\Delta t)} < 0$, which are distributed analogously to the terminal bonus according to the policy reserves over the contract term (as is shown in Bohnert and Gatzert, 2014).

The asset side

As described before, the policy reserves at time t (PR_t) are composed of funds stemming from the participating life insurance contracts (PR_t^{PLI}) and the dynamic hybrid products (PR_t^{DHP}). This (synthetic) separation allows us to account for the different asset investment maturities, as funds from the dynamic hybrid products may be shifted to the policy reserves for a short period only, and are then shifted back to the guarantee fund or equity fund. The funds of participating life insurance policies, in contrast, are generally invested long-term. This is reflected on the asset side in Table 1, where the company's assets are split into long-term investments $A_t^{long-term}$ (e.g. long-term bonds) and short-term investments (e.g. bills and short-term bonds), $A_t^{short-term}$, whereby the buffer account is also invested short-term due to its smoothing function.

All three asset investment types (long-term assets, short-term assets, and equity fund) are assumed to evolve according to a geometric Brownian motion

$$dI_t^i = \mu_i \cdot I_t^i \cdot dt + \sigma_i \cdot I_t^i \cdot dW_{t,i}^P,\ i = 1,2,3$$

with constant drift μ_i and volatility σ_i, P-Brownian motions $dW_{t,i}^P$ defined on the probability space $(\Omega, \mathcal{F}, P)$ with correlations $dW_{t,i}^P \cdot dW_{t,j}^P = \rho_{i,j}$, i.e. (see Björk, 2009)

$$I_t^i = I_0^i \cdot \exp\left(\left(\mu_i - \frac{1}{2}\cdot\sigma_i^2\right)\cdot t + \sigma_i \cdot dW_{t,i}^P\right).$$

At the beginning of period t, the company's asset investments are thus given by $A_t^{long-term} = PR_t^{PLI}$, $A_t^{short-term} = PR_t^{DHP} + B_t$, $GF_t^A = GF_t^L$, $EF_t^A = EF_t^L$, and at the end of period t, one obtains

$$A_{(t+\Delta t)}^{long-term} = A_t^{long-term} \cdot \frac{I_{t+\Delta t}^1}{I_t^1},$$

$$A_{(t+\Delta t)}^{short-term} = A_t^{short-term} \cdot \frac{I_{t+\Delta t}^2}{I_t^2},$$

$$EF_{(t+\Delta t)}^A = EF_t^A \cdot \frac{I_{t+\Delta t}^3}{I_t^3},$$

$$GF_{(t+\Delta t)}^A = GF_t^A \cdot \max\left(1-\lambda; y \cdot \frac{I_{t+\Delta t}^3}{I_t^3}\right),$$

where the guarantee fund is given by a fraction y of the equity fund's return, since the periodic downside protection has to be financed. We thereby assume a put option on the equity fund GF_t^A with strike price $(1-\lambda)\cdot GF_t^A$ ($0 \le \lambda \le 1$) and maturity Δt, which is purchased at the beginning of each period as proposed by Bohnert and Gatzert (2014). In this case, Bohnert and Gatzert (2014) show that the price of the put option depends only on the given set of parameters λ, r_f, σ_3 and Δt and is thus given by a constant fraction y of the guarantee fund, i.e. $P_t = (1-y)\cdot GF_t^A$.[7] The total assets are thus given by

$$A_{(t+\Delta t)} = A_{(t+\Delta t)}^{long-term} + A_{(t+\Delta t)}^{short-term} + EF_{(t+\Delta t)}^{A} + GF_{(t+\Delta t)}^{A}.$$

The equityholders' perspective

To ensure a fair situation for the equityholders, the buffer interest rate b is calibrated such that the value of the payout to equityholders is equal to their initial contribution, i.e.

$$B_0 = E^Q\left(BP_T \cdot e^{-T\cdot r_f}\right), \tag{5}$$

where E^Q denotes the expected value under the risk-neutral pricing measure Q and r_f is the constant risk-free interest rate. Under the risk-neutral measure Q, the drift of the investment processes changes to the risk-free rate (see Björk, 2009).

The policyholder's perspective – willingness-to-pay

For fairly calibrated contract parameters from the equityholders' viewpoint, we next focus on the policyholders' perspective and determine the maximum willingness-to-pay for the given contract design. As the relevant preference function, we use mean-variance preferences (Berketi, 1999; Gatzert, Holzmüller, and Schmeiser, 2012; Mayers and Smith, 1983), which implies that the order of preferences is given by the difference between expected payoff (wealth) and the variance of the payoff in case insurance is purchased (the participating life insurance policy (*PLI*) or the dynamic hybrid product (*DHP*), respectively) or in the case without insurance under the real-world measure P, multiplied by the policyholder's individual risk aversion coefficient a times one half, i.e.,

$$\Phi^j = E\left(V_T^j\right) - \frac{a}{2}\cdot\sigma^2\left(V_T^j\right),\ j = PLI,\ DHP,\ no\ insurance.$$

To derive the willingness-to-pay, the preference function for the case with and without insurance must be compared, whereby the maximum willingness-to-pay $WTP_0^{\Phi,j}$ is the amount at which the customer is indifferent between the two cases, i.e. where $\Phi^{no\ insurance} = \Phi^{PLI,DHP}$. The willingness-to-pay must then exceed the single premiums P^{PLI} and P^{DHP} that the insurer must charge for the contracts, respectively, as otherwise the contract will not be taken out.

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[7] Alternatively, a constant-proportion portfolio insurance strategy can be used.

In the following, to keep calculations simply, we focus on the case of deterministic wealth, where the policyholder can choose to invest part of his or her wealth in the risk-free asset r_f and the respective insurance policy, and cannot further diversify. Let W_0 denote the initial wealth of the customer. In case no insurance is purchased, one obtains

$$\Phi^{no\ insurance} = W_0 \cdot e^{r_f \cdot T}$$

and in case a participating life insurance or dynamic hybrid contract is purchased (willingness-to-pay for the contract denoted by $WTP_0^{\Phi,j}$),

$$\Phi^j = E\left(\left(W_0 - WTP_0^{\Phi,j}\right) \cdot e^{r_f \cdot T} + V_T^j\right) - \frac{a}{2} \cdot \sigma^2\left(\left(W_0 - WTP_0^{\Phi,j}\right) \cdot e^{r_f \cdot T} + V_T^j\right), j = PLI, DHP.$$

Equating the two conditions implies that the maximum willingness-to-pay is given by

$$WTP_0^{\Phi,j} = e^{-r_f \cdot T} \cdot \left(E\left(V_T^j\right) - \frac{a}{2} \cdot \sigma^2\left(V_T^j\right)\right), j = PLI, DHP, \tag{6}$$

which does not depend on the policyholder's initial wealth, W_0 (see Gatzert, Holzmüller, and Schmeiser, 2012, p. 652).

As an alternative to mean-variance preferences, certainty equivalents could be derived to obtain an impression of the utility level in case the premium volume is the same (see Schmeiser and Wagner, 2013; Broeders, Chen, and Koos, 2011). However, since we consider the impact of different portfolios and thus vary the respective premium volumes, results would no longer be comparable.

3. NUMERICAL ANALYSIS

In the following tables, we provide several numerical examples to illustrate the impact of different portfolio compositions, contract designs, and shifting mechanisms on the policyholders' willingness-to-pay. The input parameters are displayed in Table 3 and are based on those used by Bohnert and Gatzert (2014), which were subject to sensitivity analyses. Monte Carlo simulation with 50,000 latin hypercube samples was used to derive the results.

Table 3: Input parameters for the numerical analyses

Parameter	*Notation*	*Value*
Single premiums of participating life insurance contracts	P^{PLI}	100
Single premiums of dynamic hybrid products	P^{DHP}	100
Contract duration	T	10
Guarantee of dynamic hybrid products	x	1
Initial buffer[8]	B_0	6
Guaranteed interest rate (p.a.)	r^G	0.0175
Surplus distribution ratio	α	0.30
Target buffer ratio	γ	0.10
Drift of long-term investments	μ_1	0.045
Volatility of long-term investments[9]	σ_1	0.04
Drift of short-term investments	μ_2	0.035
Volatility of short-term investments	σ_2	0.03
Drift of equity fund	μ_3	0.08
Volatility of equity fund	σ_3	0.20
Linear correlation of long-term and short-term investments	$\rho_{1,2}$	0.2
Linear correlation of long-term investments and equity fund	$\rho_{1,3}$	0.2
Linear correlation of short-term investments and equity fund	$\rho_{2,3}$	0.2
Maximal loss of the guarantee fund per period	λ	0.20
Risk-free interest rate	r_f	0.03
Length of a period[10]	Δt	1/12

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[8] Note that in this setting, the initial buffer ratio is 6/200 = 0.03, which is below the target buffer ratio of g = 0.10, meaning that the insurer first needs to build up the buffer account by means of surplus before the surplus can be distributed to the policyholders' accounts.

[9] Input parameters are chosen for illustration purposes and were subject to robustness tests to make sure that the general findings are stable. For the set of investment parameters, the Sharpe ratios of the three types of investments are not equal, but with respect to these parameters, we conducted a sensitivity analysis by varying the volatilities of the different investments to ensure the stability of the results.

[10] Funds are shifted once per month, which resembles the typical approach in the market; only a few insurers also shift daily (see Bohnert, 2013).

The impact of different types and levels of guarantees on policyholders' willingness-to-pay

Figure 1 displays the impact of different guarantee features on the policyholders' willingness-to-pay for dynamic hybrids as well as for traditional participating life insurance contracts for different degrees of risk aversion. In particular, we study the impact of different levels of the guaranteed interest rate r^G as well as the design with respect to the shifting mechanisms (i.e., less risky with a higher safety level and a more risky strategy with a higher upside potential, see Equations (1) and (2)). The dynamic hybrids are assumed to feature a full money-back guarantee.[11]In the left column in Figure 1, the shifting mechanism of the dynamic hybrid product is set according to Equation (1), i.e., G_t^{DHP} (reflecting the minimum account value at time t) is kept constant during the contract term, always requiring at least the maturity guarantee $G_t^{DHP} = G_T^{DHP}$ and thus implying less risk with a higher safety level. The right column shows the case with a shifting strategy that is more risky and with a higher upside potential based on Equation (2), i.e. G_t^{DHP} is given by the discounted value of the maturity guarantee in each period. With respect to the varying degrees of risk aversion, three cases are considered. The case of a less risk averse policyholder is shown in the first row of Figure 1 (with a = 0.001), while the third (second) row demonstrates the case for a policyholder that is more (medium) risk averse with a = 0.01 (a = 0.005). In all cases, the buffer interest rate (equityholders' dividend) is calibrated to ensure that the situation is fair from the equityholders' viewpoint, i.e. Equation (5) is satisfied (see Figure A.1 in the Appendix).[12]

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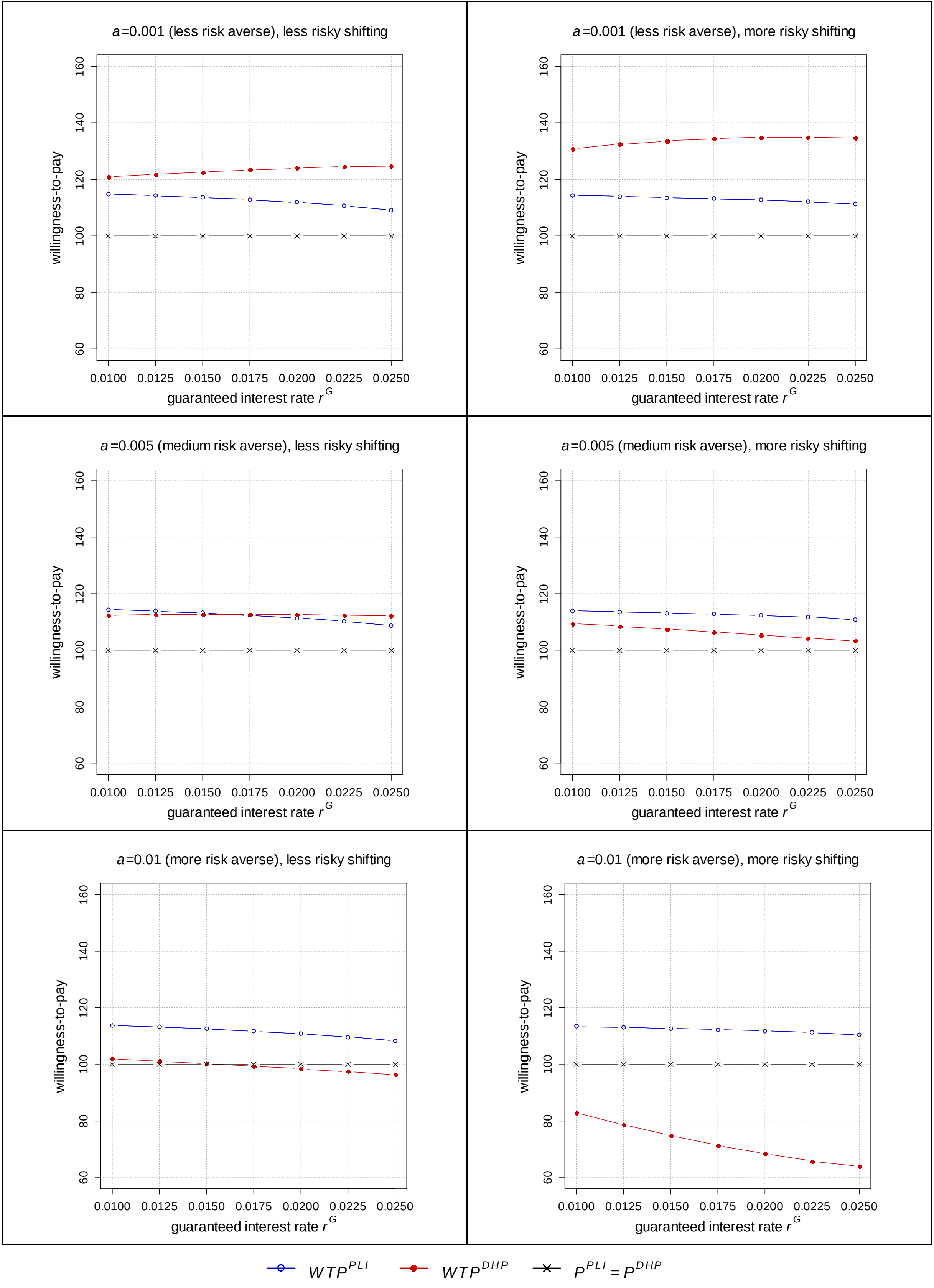
[11] Figure A.2 in the Appendix shows the results when assuming a 50% money-back guarantee.

[12] The fair buffer interest rate represents the return on the equityholders' investment over the entire contract duration (i.e., 10 years in the example here), given that the insurer does not default during the contract term. Since the default risk is generally increasing for higher guaranteed interest rates, for instance, the fair buffer interest rate increases as well (see Figure A.1).

Figure 1: Willingness-to-pay for the participating life insurance (*PLI*) and dynamic hybrid (*DHP*) contracts for different degrees of risk aversion a = 0.001, 0.005, 0.01 (less / medium / more risk averse) as well as differing shifting mechanisms for a money-back guarantee of the dynamic hybrids for varying guaranteed interest rates for fair contracts (see Figure A.1)



One can observe that in the cases of a medium and a more risk averse policyholder considered in Figure 1, the policyholders' willingness-to-pay for both life insurance products (sometimes substantially) decreases when *increasing the guaranteed interest rate*, which mainly stems from an increase in shortfall risk (see Bohnert and Gatzert, 2014, for this result) and a higher volatility of the payoff (see Equation (6)). However, this increase in the payoff volatility and the interaction effects of the participating life insurance contracts with the dynamic hybrids cause an increase of the willingness-to-pay for higher guaranteed interest rates among less risk averse policyholders. When comparing the right column with the left column in Figure 1, it can be seen that the *design of the dynamic hybrids' shifting mechanism* has a considerable impact on dynamic hybrid policyholders' willingness-to-pay. In particular, a more risky shifting system (right column in Figure 1) leads to a higher volatility of the dynamic hybrid products' final payoff, which thus results in a lower willingness-to-pay for the dynamic hybrids (WTP^{DHP}) for more risk averse policyholders as compared to the case of the less risky shifting mechanism in the left column of Figure 1. This is especially pronounced in the last row and also (to a considerable lesser extent) in the second row of Figure 1. while the *WTP* of dynamic hybrids is affected in the opposite way by the shifting mechanisms in the first row, where the willingness-to-pay for dynamic hybrids is even increased for less risk averse policyholders (as compared to the left graph in the first row with a less risky shifting mechanism).

The willingness-to-pay for participating life insurance contracts (WTP^{PLI}) within the same portfolio of a life insurance company is also affected by the different shifting mechanisms due to interaction effects between the two types of policies. However, in contrast to the dynamic hybrid products, this effect is considerably reduced and the participating life insurance policyholders' *WTP* remains fairly stable in all cases. When using a more risky shifting mechanism for the dynamic hybrids, the *WTP* even slightly increases (at least for higher guaranteed interest rates, see first row in Figure 1), as the more risky shifting reduces the guarantees for the *DHP* products, thus benefiting the participating life insurance policyholders.

In addition to a full money-back guarantee in Figure 1, we further study the case with a reduced guarantee level of x = 50% of the single up-front premium (see Figure A.2 in the Appendix). The results show that lowering the *dynamic hybrid products' guarantee level* from a full (100%) to a 50% money-back guarantee either implies a considerable reduction in the dynamic hybrid policyholders' willingness-to-pay for more risk averse policyholders, or a considerable increase in the WTP^{DHP} in case of policyholders with lower risk aversion. These effects are especially pronounced in case of the less risky shifting mechanism (compare left columns in Figures 1 and A.2). In the case of the more risky shifting mechanism (right column, first and second row), results are ambiguous and strongly depend on the guaranteed interest rate credited on the policy reserves. In particular, reducing the *DHP* guarantee level to a

50% money-back guarantee implies a lower WTP^{DHP} for lower guaranteed interest rates, but a higher WTP^{DHP} for guaranteed interest rates above 1.7% (approximately).

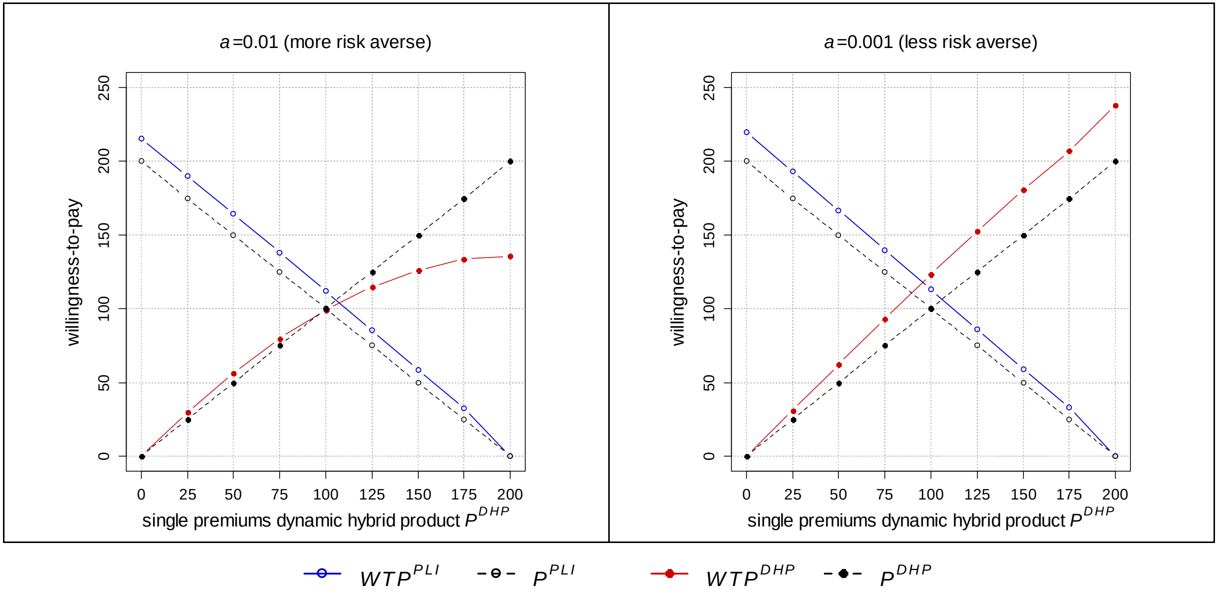
To ensure that an insurance contract is actually purchased by customers, the policyholders' willingness-to-pay for the contract has to exceed the required premium assumed when calculating the contracts' payoffs, i.e., in our setting a single premium of $P^{PLI} = P^{DHP} = 100$ for both products. While the WTP^{PLI} exceeds the required premium in all considered cases in Figure 1, the situation for dynamic hybrids depends heavily on the specific contract design and especially the guarantee features. In particular, results depend on the guaranteed interest rate, the shifting mechanism and the policyholders' risk aversion. For a more risk averse policyholder as given in the last row in Figure 1, the policyholders' willingness-to-pay is below the required premium of 100 in case of the risky shifting mechanism (right graph). This stems from the fact that only a minor amount of the dynamic hybrids' account value is invested in the policy reserve stock leading to a more volatile final payoff, for which the willingness-to-pay is low for a more risk averse policyholder. In case of the more conservative shifting mechanism (left graph in the last row), the dynamic hybrid policyholders' willingness-to-pay is above 100 for guaranteed interest rates below $r^G = 1.5\%$ and lower than 100 if the guarantee interest rate exceeds this value in the present example. The higher the guaranteed interest rate, the higher the shortfall risk and the higher the payoff volatility, which is valued as negative by a risk averse policyholder. This situation changes considerably for a less risk averse policyholder (see first row in Figure 1). In this case, the willingness-to-pay for dynamic hybrids is strictly above the required premium of 100 and exceeds the willingness-to-pay for the participating life insurance contracts, since the upside potential and higher volatility is valued positively. In contrast, the policyholders' willingness-to-pay for participating life insurance contracts can be either higher or lower than the willingness-to-pay for dynamic hybrids in the case of a medium risk averse policyholder and the less risky shifting mechanism (see left graph in the second in Figure 1).

The impact of portfolio composition on the policyholders' willingness-to-pay

We now study the impact of the *portfolio composition* on the policyholders' willingness-to-pay for dynamic hybrid products and participating life insurance contracts within a portfolio of a life insurance company as exhibited in Figure 2. We thereby fix the total premium volume to 200. The premium of the dynamic hybrids is then given on the *x*-axis, while the single premium for the participating life insurance contracts is residually given by $P^{PLI} = 200–P^{DHP}$.

Figure 2 shows the willingness-to-pay for participating life insurance contracts and dynamic hybrid products for different portfolio compositions for a more risk averse policyholder (left graph in Figure 2) and a less risk averse one (right graph). In line with the results in Figure 1, the policyholders' willingness-to-pay for participating life insurance contracts is higher than the corresponding required premium (black dotted line) in all considered cases. The willingness-to-pay for dynamic hybrid contracts, in contrast, depends greatly on the portfolio composition and the risk aversion parameter. For a more risk averse policyholder (left graph in Figure 2), the willingness-to-pay for dynamic hybrids only exceeds the required single premium for a portfolio with less than about 50% dynamic hybrid products in the insurer's portfolio (i.e. $P^{PLI} = P^{DHP} = 100$). Here, the contracts' stability and relatively low volatility of the payoffs to a large part stems from the participating life insurance contracts in the portfolio, which allows the corresponding asset base to be invested long-term with stable returns for the policy reserve stock. The upside potential of the dynamic hybrids' fund investments are added to this. In turn, for a portfolio composition with more than about 50% dynamic hybrids in the portfolio, the corresponding investments of the policy reserve stock cannot be invested long-term, since funds may be shifted to the guarantee fund or equity fund in a subsequent period. Thus, the policy reserve stock generates less stable returns with fewer surplus (due to higher default risk), which results in a reduced willingness-to-pay for dynamic hybrids in this setting.

Figure 2: Willingness-to-pay for the participating life insurance and dynamic hybrid contracts and the contracts' single premiums when varying the portfolio composition ($P^{PLI} = 200 – P^{DHP}$) for fair contracts (see Figure A.3) (with 100% money-back guarantee of the dynamic hybrids as well as the less risky shifting mechanism)

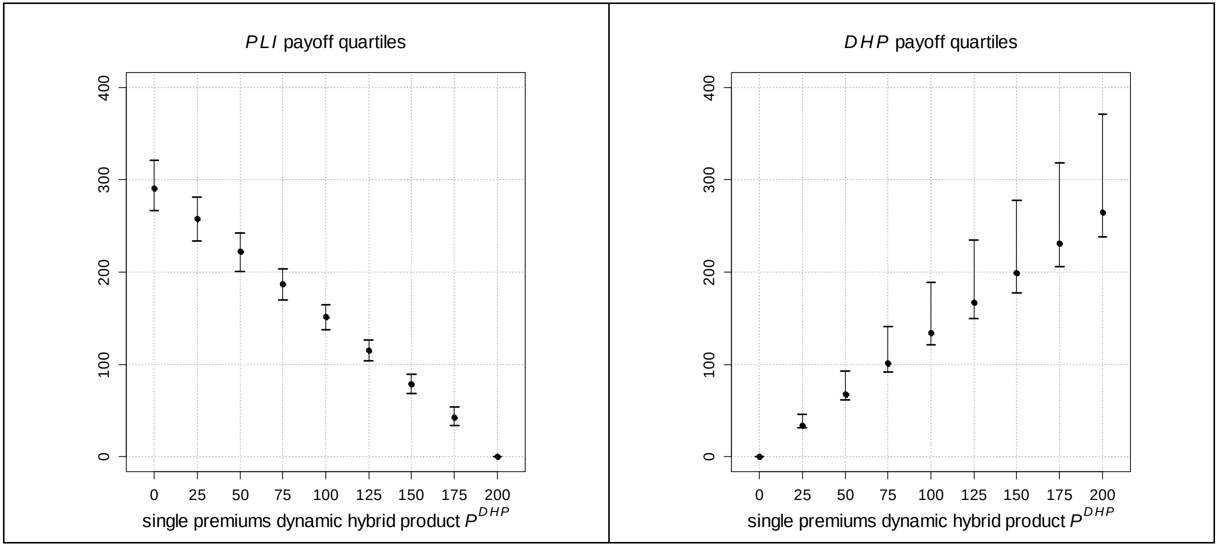


The right graph in Figure 2 further reveals that for a less risk averse policyholder, the willingness-to-pay for dynamic hybrids is relatively higher, the higher the portion of dynamic hybrid products is within the portfolio. Here, the policyholders prefer the upside potential of a more volatile payoff, which results from a higher portion of dynamic hybrids in the portfolio (see

Figure 3), instead of the more stable payoff that results from a higher portion of participating life insurance policies in the portfolio.

To obtain further insight, Figure 3 additionally illustrates the corresponding quartiles of the payoff distributions of the participating life insurance contracts (left graph) and the dynamic hybrid products (right graph) for the various portfolio compositions exhibited in Figure 2, where a money-back guarantee for the dynamic hybrids as well as the less risky shifting mechanism is applied. In particular, the upper and lower quartiles are shown along with the median indicated as a black dot. When considering the *PLI* payoff quartiles, it can be seen that the median and the *PLI* payoffs' interquartile range is increasing with an increasing portion of participating life insurance contracts in the portfolio, but at a relatively low level as compared to the *DHP* payoff quartiles. In case of the dynamic hybrid products, the results show a strong increase of the interquartile range, i.e., an increase in the payoff volatility for a higher portion of dynamic hybrids in the portfolio. In addition to this, the *DHP* payoffs exhibit a positive skew, which illustrates the much higher upside potential of dynamic hybrids as compared to participating life insurance contracts, which is preferred by less risk averse policyholders (see Figure 2).

Figure 3: Lower quartile, median, and upper quartile (i.e. 25th, 50th, and 75th percentile) of the payoff distributions for *PLIs* and *DHPs* when varying the portfolio composition (P^{PLI} = 200–P^{DHP}) for a money-back guarantee of the dynamic hybrids as well as the less risky shifting mechanism (corresponding to Figure 2)



The impact of different types and levels of guarantees on the contracts' payoff distribution

To further study the dynamic hybrid policyholders' perspective, we next analyze the contracts' payoff distributions, i.e., their risk-return profiles. Toward this end, we derive the real-world distributions of the *PLI* and *DHP* payoffs, i.e. V_T^{PLI} and V_T^{DHP} (see Equations (3) and (4)), for varying levels of guaranteed interest rates and *DHP* guarantees.

Figure 4: Payoff distributions for *PLIs* and *DHPs* (risk-return profiles) for different guaranteed interest rates and different *DHP* guarantee levels (less risky *DHP* shifting mechanism; corresponding to the left columns in Figures 1 and A.2)

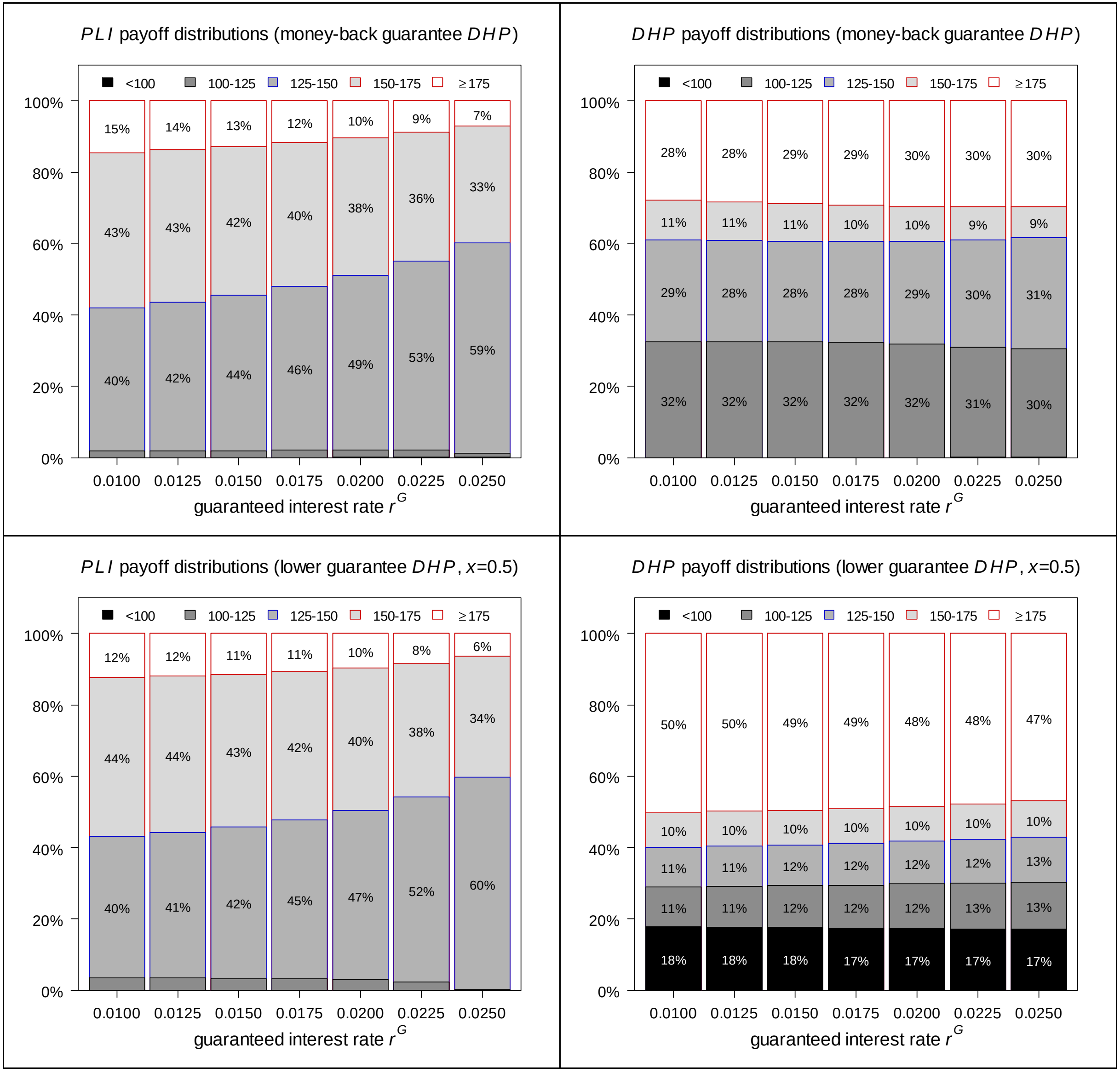


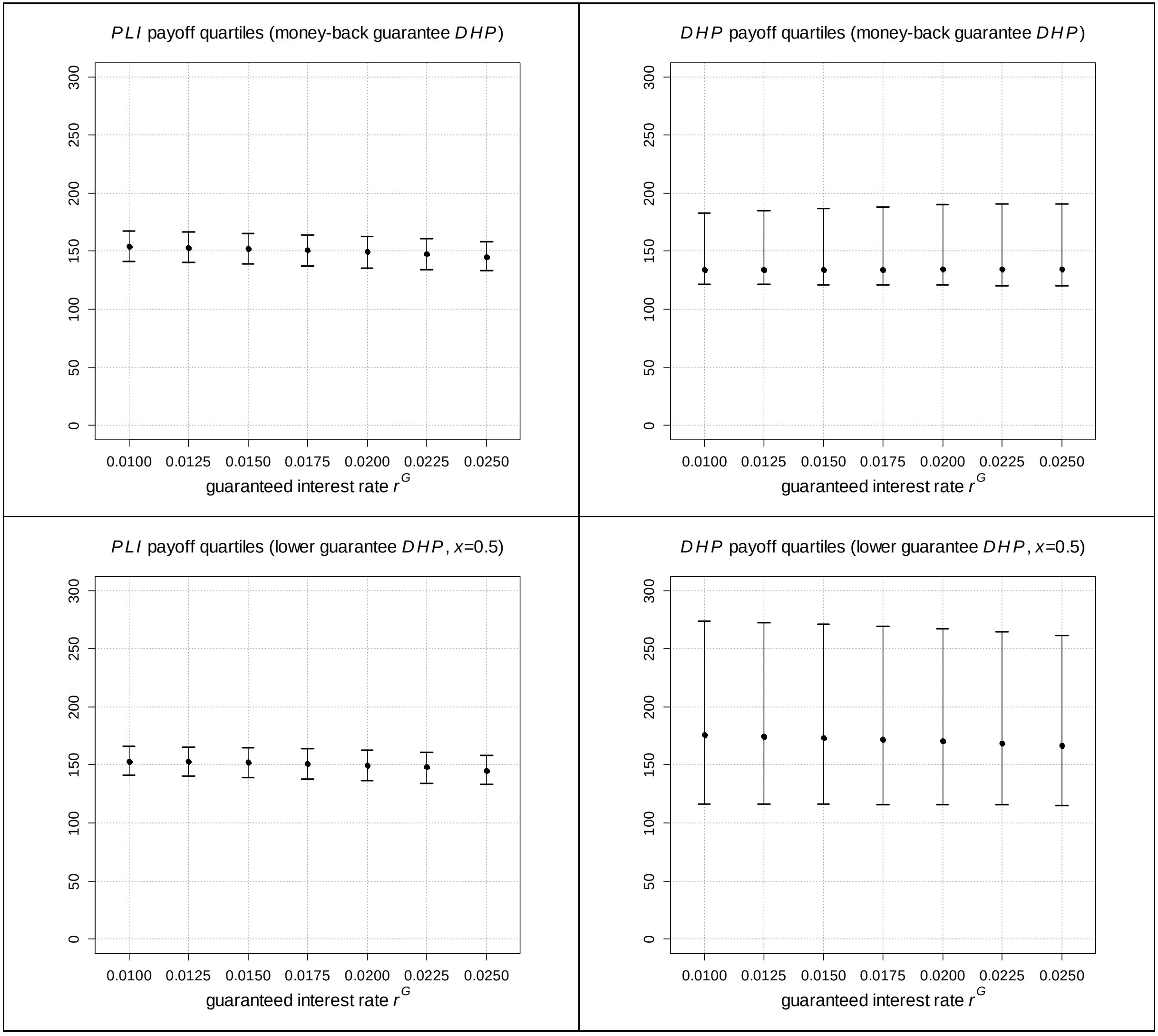
Figure 4 displays risk-return profiles for the less risky shifting mechanism of the dynamic hybrids corresponding to the left columns of Figures 1 and A.2, which are the basis for deriving the willingness-to-pay. In particular, the first and second row show the case of a full

(100%) and 50% money-back guarantee for the *DHPs*, respectively. In the left column of Figure 4, the payoff distributions for the participating life insurance contracts are exhibited, whereas the case of the dynamic hybrids is shown in the right column.

When comparing the left column to the right column, i.e., the case of *PLIs* to *DHPs,* the results show that the participating life insurance contracts' payoffs are relatively stable with a low volatility, whereas the dynamic hybrids' payoffs exhibit a considerably higher volatility, in line with the previous findings (see also Figure 5). While the participating life insurance payoffs' upside potential (payoffs above 175) remains almost unchanged when reducing the guarantee level of the dynamic hybrids from a full money-back guarantee (first row) to a 50% money-back guarantee (second row), the dynamic hybrids' payoff volatility considerably increases. In particular, the upside potential as well as the downside potential of the payoffs of the dynamic hybrid products is considerably higher for a partial money-back guarantee as compared to a full money-back guarantee (compare right graphs in Figure 4). This is based on the fact that in case of a 50% money-back guarantee, the dynamic hybrid shifting mechanism invests a higher proportion of the contracts' account value in the equity fund (and the guarantee fund). This results in higher payoff volatilities compared to a full money-back guarantee, where the available funds are, to a larger extent, invested in the stable and less volatile policy reserve stock. These findings are in line with previous results showing that the willingness-to-pay for dynamic hybrids with a full money-back guarantee is higher than for a partial money-back guarantee for more risk averse policyholders, i.e. for a relatively less volatile payoff compared to a more volatility payoff (see Figures 1 and 5). Furthermore, this result shows the great flexibility of dynamic hybrids, which can be adjusted to various customers' needs to achieve different payoff distributions without changing the contracts' basic setting.

Figure 5 additionally shows the corresponding quartiles (upper and lower quartile, median) for the payoff distributions of the *PLIs* and *DHPs* presented in Figure 4. The left column in Figure 5 illustrates the relatively stable payoffs of the participating life insurance contracts, while the right column again shows the right-skewed and volatile dynamic hybrid products' payoffs. The quartiles illustrate the sensitivity of the dynamic hybrids' payoff with respect to their guarantee features. The results also show that an increase in the guaranteed interest rate implies a decrease in the median payoff and reduction in the upside potential of the participating life insurance contracts' payoffs. In contrast to this, the dynamic hybrids' payoff (and thus the willingness-to-pay) is almost not affected by a change in the guaranteed interest rates for the less risky shifting mechanism (see also left column in Figure 1). Results for the more risky shifting mechanism are in line with previous findings and are thus omitted here.

Figure 5: Lower quartile, median, and upper quartile (i.e. 25th, 50th, and 75th percentile) of the payoff distributions for *PLIs* and *DHPs* for different guaranteed interest rates and different *DHP* guarantee levels (less risky *DHP* shifting mechanism; corresponding to the left columns in Figures 1 and A.2)



Further findings reveal that the willingness-to-pay for the participating life insurance contracts and dynamic hybrids can be more extreme in a positive or negative way depending on the degree of risk aversion and depending on a risk measure that takes downside risk and skewness into account.[13]

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[13] We further studied the first-order lower partial moment (LPM_1) with reference point $z = E\left(V_T^j\right)$, also referred to as lower semi-absolute deviation measure (*LSAD*) (see Gustafsson and Salo, 2005) to measure downside risk in a general mean-risk preference model (see Fishburn, 1977) and found the results to be robust.

4. CONCLUDING REMARKS

This paper assesses the interaction effects when insurers offer dynamic hybrid policies in addition to participating life insurance contracts and explicitly focuses on the policyholders' perspective for the first time. We consider a 3-fund dynamic hybrid account whose value is periodically reallocated between the conventional premium reserve stock (corresponding to the policy reserves), a guarantee fund (which loses at most a certain percentage of its value in each period), and a risky equity fund, following a mathematical shifting mechanism that is based on the concept of constant proportion portfolio insurance (CPPI). To assess the policyholders' perspective, mean-variance preferences were used to derive the willingness-to-pay (*WTP*) for different degrees of risk aversion of the policyholders.

Our results show that higher guarantees (e.g., the *DHP* guarantee level, riskiness of shifting mechanisms, guaranteed interest rates) do not necessarily imply an increase in consumers' willingness-to-pay. In contrast, in the examples considered here, which are based on fair contracts from the equityholders' perspective, the willingness-to-pay for both types of policyholders clearly decreases for higher guaranteed interest rates. These findings are consistent with findings in several other studies that evaluate the willingness-to-pay for guarantees. Higher guarantees within insurance products generally do not necessarily imply an increase in consumers' willingness-to-pay (e.g., Gatzert, Holzmüller, and Schmeiser (2012). Furthermore, consumers are also willing to pay a substantially higher price for guarantees when the option is provided in a simple “all else equal” context (e.g., see Gatzert, Huber, and Schmeiser, 2011), primarily because they are risk averse, while on average, the willingness-to-pay is significantly below the insurer's risk-adequate premium. Further relevant impact factors regarding the determination of willingness-to-pay include customer preferences (e.g., see Doskeland and Nordahl, 2006), demographic characteristics such as income, gender, and education (e.g., see Feldman and Schultz, 2004), and insurer characteristics and operations (e.g., see Marshall, Hardy, and Saunders, 2010).

The demand for guarantees has important implications on the supply side, encouraging insurers to constantly look for opportunities to innovate in their product offerings. We show here, for example, that in the case of the dynamic hybrid products, a certain minimum guarantee level must be offered by the insurer in order to ensure that the contract is purchased in the first place, i.e., that the willingness-to-pay of the dynamic hybrid policyholders exceeds the required minimum premium that the insurer must charge when selling the contract. For instance, while the willingness-to-pay was sufficient for a full money-back guarantee and guaranteed interest rates up to around 1.5% in case of a less risky shifting mechanism, the willingness-to-pay considerably decreased when reducing the dynamic hybrid guarantee level to a

50% money-back guarantee or when using the more risky shifting algorithm to reallocate the dynamic hybrid funds. Thus, the willingness-to-pay by far did not exceed the necessary premium in order to close the contract. The same holds true when keeping the guarantee level at a full money-back guarantee but using a more risky shifting mechanism for distributing the dynamic hybrid account value between the three funds. In the latter cases, contracts could only be sold when consumers exhibited a rather lower risk aversion. Thus, the attractiveness of these products strongly depends on the consumers’ preferences and varies considerably depending on the contract design, including the level of guarantee, the shifting mechanism and the degree of risk aversion. At the same time, this result also emphasizes the great flexibility of these products, which can be easily adjusted in order to meet different consumers’ needs for stability or upside potential. Given consumers’ sensitivity to particular contract features, it is important that insurers are transparent in the marketing of their products.

Our analysis also emphasizes the need for insurers to regularly (re)evaluate their mix of products with different types of guarantees, recognizing that the attractiveness of any one of product may be affected by the insurer’s product mix. Here we found that the willingness-to-pay for a participating life insurance policy remained almost unchanged when varying the risk aversion parameter, which emphasizes the low volatility and stable payoff of these traditional products. The interaction effects that may arise in the portfolio, which differ depending on the dynamic hybrid contract features, may impact the willingness-to-pay or preference level of the participating life insurances, although we do not find this effect to be substantial for the cases considered. The potential for interaction effects raises concerns that customers may inadvertently choose a life insurance policy that, subsequently, is not suitable for their situation, or purchase inadequate amounts of coverage. The present results certainly depend on the assumed mean-variance preferences used to derive the willingness-to-pay, but they are generally consistent with previous theoretical and empirical work with respect to the willingness-to-pay of consumers for guarantees.

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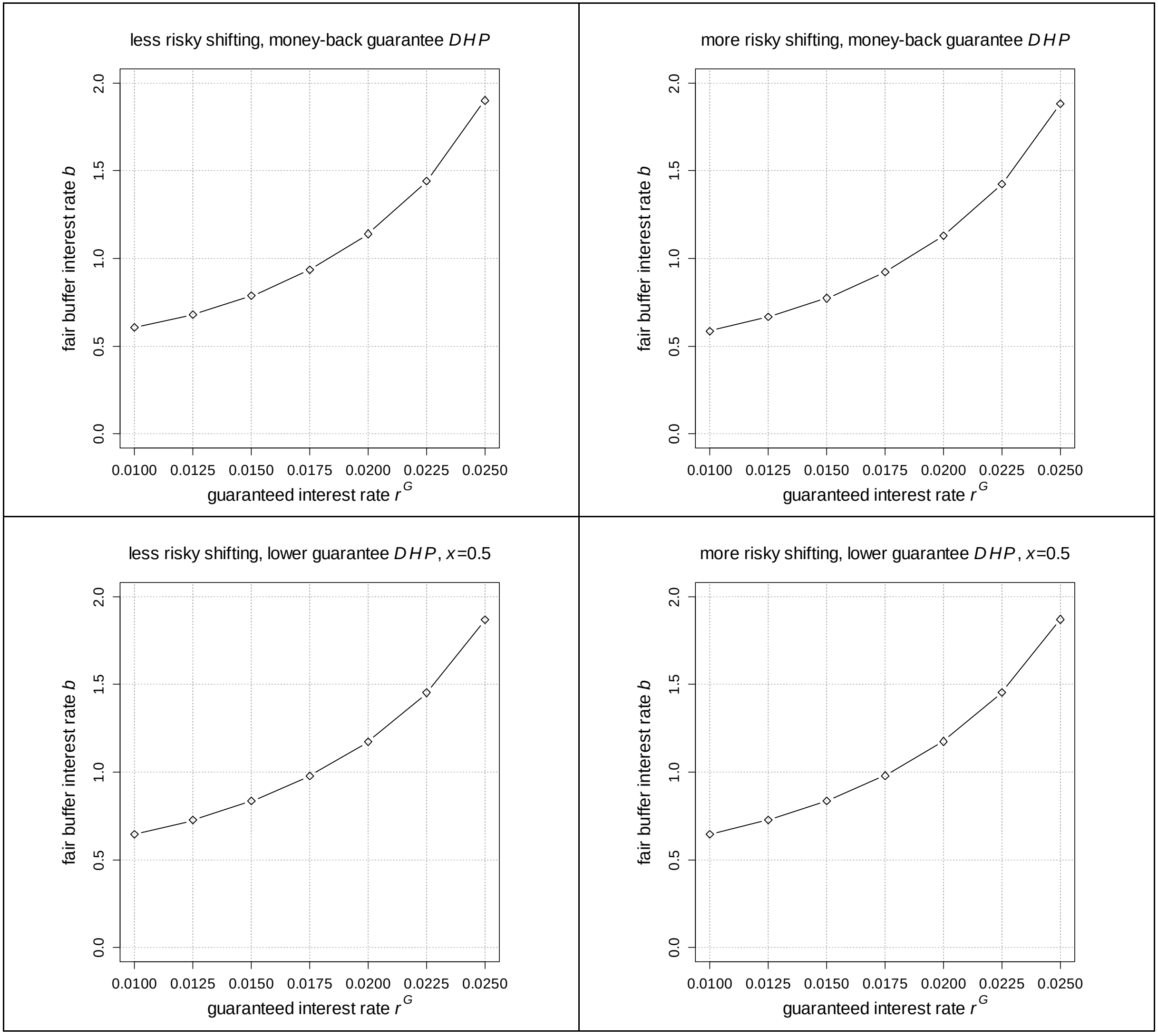
APPENDIX

Figure A.1: Fair buffer interest rate (equityholders' dividend) for the participating life insurance (*PLI*) and dynamic hybrid (*DHP*) contracts for different shifting mechanisms as well as different guarantee levels of the dynamic hybrid products for varying guaranteed interest rates



less risky shifting, money-back guarantee DHP

more risky shifting, money-back guarantee DHP

less risky shifting, lower guarantee DHP, x=0.5

more risky shifting, lower guarantee DHP, x=0.5

Figure A.2: Willingness-to-pay for the participating life insurance (*PLI*) and dynamic hybrid (*DHP*) contracts for different degrees of risk aversion a = 0.001, 0.005, 0.01 (less / medium / more risk averse) as well as differing shifting mechanisms for a lower guarantee of the dynamic hybrids (x = 0.5) for varying guaranteed interest rates for fair contracts (see Figure A.1)

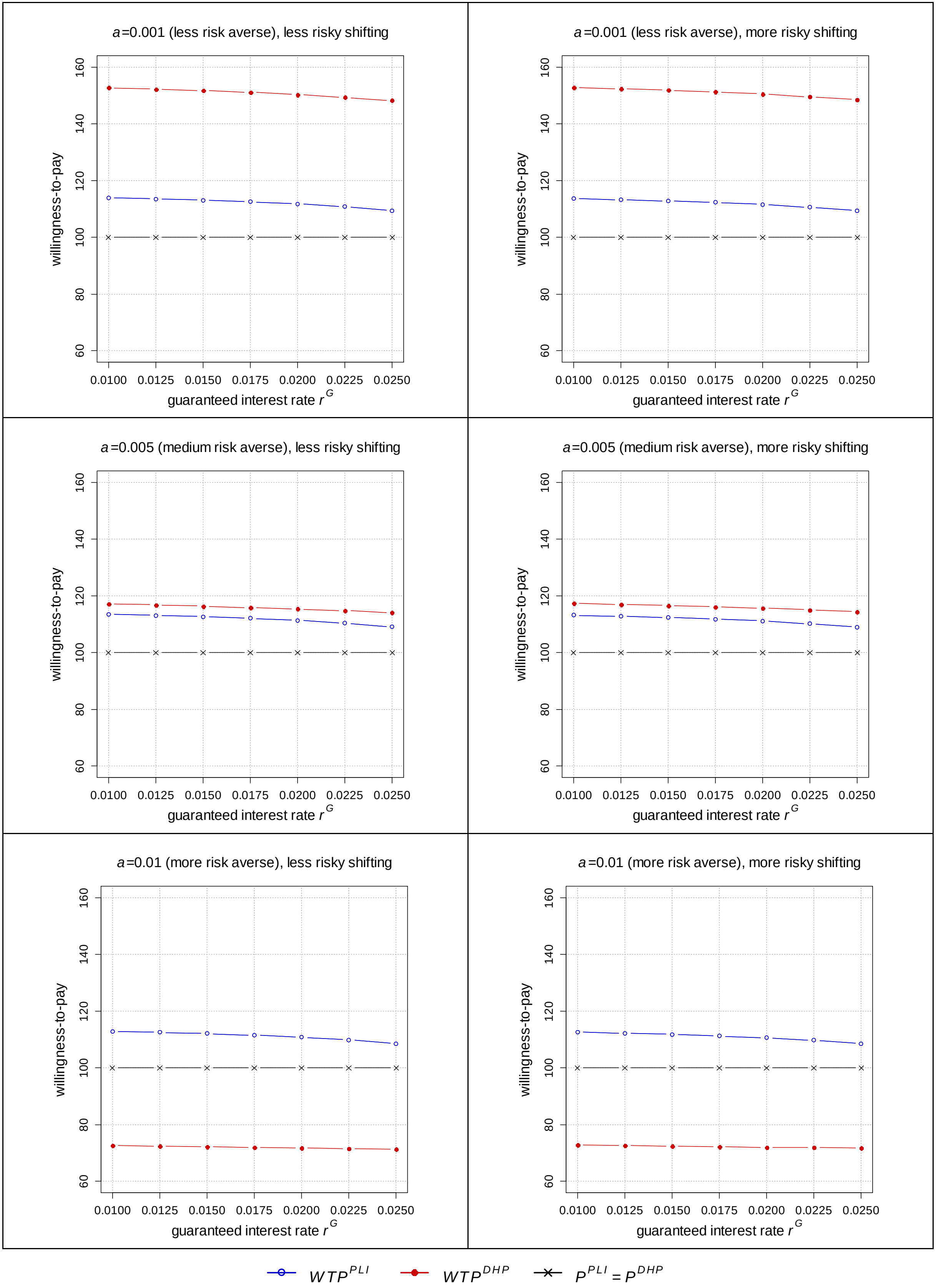


Figure A.3: Fair buffer interest rate (equityholders' dividend) for the participating life insurance (*PLI*) and dynamic hybrid (*DHP*) contracts for different shifting mechanisms as well as different guarantee levels of the dynamic hybrid products for varying the portfolio composition (P^{PLI} = 200–P^{DHP}) (see Figure 2)