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THE IMPACT OF DISABILITY INSURANCE ON A PORTFOLIO OF LIFE INSURANCES

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ABSTRACT

The aim of this paper is to study the impact of disability insurance on an insurer's risk situation for a portfolio also consisting of annuity and term life contracts. We provide a model framework using discrete time non-homogeneous Markov renewal processes and focus on diversification benefits as well as potential natural hedging effects that may arise within the portfolio due to the different types of biometric risks. Our analyses emphasize that disability insurances are a less efficient tool to hedge shocks to mortality and that their high sensitivity towards shocks to disability risks cannot be easily counterbalanced by other life insurance products. However, the addition of disability insurance can still considerably lower the overall company risk.

Keywords: Disability insurance, life insurance, mortality risk, natural hedging, non-homogenous Markov renewal model

JEL classification: G22, G23, G32, J11

1. INTRODUCTION

Due to an increasing social relevance and demand of disability insurance, the management of disability risk within life insurance portfolios is becoming increasingly important. The importance of disability insurance has also been pointed out by Chandra and Samwick (2005), for instance, who concluded that precautionary saving is a less useful hedge against disability risk. At the same time, the risk associated with offering these products is increasingly focused by insurers and regulators due to the development of new risk-based regulatory frameworks such as Solvency II in the European Union and the introduction of the own risk and solvency assessment (ORSA) in the United States (see Wicklund and Christopher, 2012). In this context, especially diversification benefits that may arise within an insurance portfolio as a whole play an important role to reduce the overall risk level. Due to the specific exposure of

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disability insurance towards disability and mortality risks, in this paper we extend previous analysis and specifically focus on the effect of disability insurance policies in a life insurer's portfolio also consisting of annuities and term life policies. We thereby also study the effectiveness of natural hedging in case of shocks to mortality and disability risk.

Previous literature has already emphasized the relevance of diversification benefits in regard to mortality risk by means of natural hedging, which can help immunizing an insurer's risk situation against changes in mortality, as well as other hedging approaches (see, e.g., Blake et al., 2006; Cox, Lin, and Pedersen, 2010; Gatzert and Wesker, 2012a; Gatzert and Wesker, 2012b; Wang et al., 2010; Wetzel and Zwiesler, 2008; Wong, Sherris, and Stevens, 2013). In general, in the literature concerning mortality risk, one objective is to hedge this risk which comprises all forms of uncertainty that are related to future mortality rates (e.g., Cairns, Blake, and Dowd, 2006a). Mortality risk can generally be decomposed into systematic and unsystematic risk as well as basis risk (see Gatzert and Wesker, 2012a). In contrast to unsystematic mortality risk, which refers to the randomness of deaths in a life insurance portfolio given a fixed mortality intensity, systematic mortality risk is not diversifiable but may be reduced either by including a safety loading on premiums or by transferring part of it to the insured, e.g. by offering mortality-linked contracts (see Dahl, 2004; Dahl, Melchior, and Møller, 2008; Richter and Weber, 2011). Systematic mortality risk itself can be further divided into process risk, parameter risk and model risk (see, e.g., Levantesi and Menzietti, 2012). Basis risk may arise due to adverse selection effects from the difference between the mortality of policyholders and the population mortality (see Gatzert and Wesker, 2012a). A partition of disability risk into systematic risk, unsystematic risk and basis risk can be defined analogously.

The management of biometric risk including mortality and disability risk plays a major important role in risk management (see, e.g., Cairns, Blake, and Dowd, 2008). The mortality risk inherent to life insurance or annuity contracts may be hedged using financial instruments such as longevity or mortality bonds, for instance (see Dowd et al., 2006). As an alternative, natural hedging effects can be exploited as described above to reduce mortality risk by stabilizing the aggregated cashflows resulting from an insurance product portfolio making use of the opposed development of the value of liabilities due to changes in mortality (see Cox and Lin, 2007). Several papers have studied and shown the effect of natural hedging between annuity and term life insurance and proposed different natural hedging techniques (see, e.g., Bayraktar and Young, 2007; Cox and Lin, 2007; Gatzert and Wesker, 2012a; Gatzert and Wesker, 2012b; Gründl, Post, and Schulze, 2006; Luciano, Regis, and Vigna, 2011; Luciano, Regis, and Vigna, 2012; Tsai, Wang, and Tzeng, 2010; Wang et al., 2010; Wetzel and Zwiesler, 2008). While a relevant amount of research has been conducted in regard to mortality risk associated with life insurance and annuities, the literature concerning disability insurance so far has not focused on studying disability insurance in a portfolio context, but rather on the adequate modeling and evaluation of disability insurance risk and policies using several state models and processes (see, e.g., Haberman and Pitacco, 1999; D'Amico, Guillen, and Manca, 2009; D'Amico, Guillen, and Manca, 2013; Helwich, 2008; Janssen and Manca, 2007; Maegebier, 2013; Stenberg, Manca, and Silvestrov, 2007).

In this paper, we aim to combine the two strands of the literature and incorporate disability insurances in a life insurance portfolio proposing a multi-period framework to analyze and quantify the effectiveness of hedging effects between annuity, disability and term life insurance. Toward this end, we model the insurance company as a whole and calibrate discrete time non-homogeneous Markov renewal processes for all three insurance types following the approach in Stenberg, Manca, and Silvestrov (2007). Our results show that, although disability insurance is less efficient to hedge mortality risk, and term life insurances in general are not well suited to limit the impact of disability risk, an optimal (in the sense of risk-minimizing or risk-immunizing) product mix may nevertheless considerably decrease the shortfall risk inherent in the insurance portfolio.

The remainder of this article is structured as follows. In Section 2, the model of the insurance company along with assets and liabilities is presented. Section 3 presents results of the numerical analyses and Section 4 concludes.

2. MODEL FRAMEWORK

This section first presents the model for mortality rates and survival probabilities. Then, we introduce general Markov renewal models as a framework for life and disability insurance. Afterwards, the model for the insurance company is laid out based on Gatzert and Wesker (2012a) along with the relevant risk measures.

2.1 The modeling of mortality rates

We use the model by Brouhns, Denuit, and Vermunt (2002a), which assumes that the number of deaths at age x during period t (D_{xt}) is Poisson distributed, i.e.,

$$D_{xt} \sim Poisson\left(E_{xt} \cdot \mu_x(t)\right).$$

with an exposure-to-risk E_{xt} and the force of mortality $\mu_x(t)$ that in the following is assumed to be defined according to the Lee and Carter (1992) model (see Brouhns, Denuit, and Vermunt, 2002b), where the natural logarithm of the force of mortality $\mu_x(t)$ at age x during period t is split into age-specific components α_x and β_x as well as a time-varying parameter κ_t that describes the time trend of mortality

$$\ln\left(\mu_x(t)\right) = \alpha_x + \beta_x \cdot \kappa_t + \varepsilon_x(t) \Leftrightarrow \mu_x(t) = e^{\alpha_x + \beta_x \cdot \kappa_t + \varepsilon_x(t)}.$$

and the homoskedastic centered error terms of the model are denoted by $\varepsilon_x(t)$ ($\sum_t \kappa_t = 0$ and $\sum_x \beta_x = 1$). The model is calibrated based on a uni-dimensional Newton method as proposed by Goodman (1979). Based on the projected force of mortality $\mu_x(t)$, the one-period survival probability for a person aged x in period t can be computed by $p_x(t) = \exp\left(-\mu_x(t)\right)$ (see Brouhns, Denuit, and Vermunt, 2002a).

In the following, we apply the notation for Markov renewal processes as laid out in D'Amico, Guillen, and Manca (2009), where the one-period survival probability is included as the associated waiting time distribution ${}_xF_{ij}(s,t)$. In general, the waiting time distribution denotes the probability that a transition from an alive state i to the dead state j occurs up to time t given that an x-year old individual entered state i at time s and that j is the next state. It can be calculated by

$${}_xF_{ij}(s,t) = \sum_{\vartheta=0}^{t-s} {}_x f_{ij}(s, s+\vartheta) \text{ with}$$

$${}_x f_{ij}(s, s+\vartheta) = \begin{cases} \left(1 - p_{x+\vartheta-1}\left(s+\vartheta-1\right)\right) \cdot \prod_{\vartheta'=1}^{\vartheta-1} p_{x+\vartheta'-1}\left(s+\vartheta'-1\right), & if \ s \le T-1, \vartheta \in [1, T-s], \\ 0, & if \ s = T \ or \ \vartheta = 0, \end{cases}$$

where T denotes the time horizon and ${}_xf_{ij}(s,s+\vartheta)$ depicts the probability that a transition from an alive state i to the dead state j takes place exactly at time $s+\vartheta$, given that j is the next state and that an individual aged x entered state i at time s. Both, active and disabled states are considered as alive states. Since homogeneous age groups will be studied in the subsequent analysis, the index x is omitted in the following.

2.2 The discrete time non-homogeneous Markov renewal process

Based on the notations and definitions given in D'Amico, Guillen, and Manca (2009), the model framework applied to disability insurance, term life insurance and annuity policies is presented in this subsection. This general framework consists of a discrete time non-homogeneous bivariate Markov renewal process.

The random variables J_n and T_n are defined on a probability state (Ω,Σ,P) as $J_n:\Omega\to I$ and $T_n:\Omega\to\mathbb{N}$, where Ω is the sample space and $I=\{1,\dots,m\}$ is the state space including the active and the (absorbing) dead state m as well as all potential disability levels. These two random variables run together and the corresponding stochastic process (J_n,T_n), $n\in\mathbb{N}$, is described as a non-homogeneous bivariate Markov renewal process, where J_n denotes the state occupied at the n-th transition, T_n the time of the n-th transition and $F_n=\sigma(J_n,T_n;u\le n)$ the natural filtration. In the context of disability insurance, each transition may represent changes of the disability level or the event of death and the time of each transition corresponds to the time when the change in the health state is registered by the insurer. Then, for the process (J_n,T_n), the information regarding the n-th transition is sufficient to state the conditional distribution of the successive state J_{n+1} entered at time T_{n+1}, $\forall\, i,j\in I,\ \forall\, s,t\in\mathbb{N},\ s\le t$,

$$P[J_{n+1}=j,T_{n+1}\le t\,|\,\sigma\left(J_u,T_u\right),0\le u\le n,J_n=i,T_n=s]$$
$$=P[J_{n+1}=j,T_{n+1}\le t\,|\,J_n=i,T_n=s]=:Q_{ij}(s,t)$$

as well as the transition probability p

$$p_{ij}(s)=\lim_{t\to\infty}Q_{ij}(s,t)\,.$$

The first transition probability Q thereby describes the probability that the successive state j is entered up to time t and the second transition probability p denotes that state j is the next state occupied, regardless of the time of the associated transition. Both transition probabilities Q and p are conditional upon the state i being entered at time s. The matrix $\mathrm{P}(s)=[\,p_{ij}(s)\,]$ is introduced as the transition matrix, which corresponds to the embedded non-homogeneous Markov chain in the process, and the probability Q is constrained by the following assumptions:

1. $Q_{ij}(s,s)=0,\ \forall\, i,j\in I,\quad \forall\, s\in\mathbb{N}$ and
2. $Q_{ii}(s,t)=0,\ t-s>0,\quad \forall\, s\in\mathbb{N}$.

The first assumption forbids multiple transitions at any time s and the second restriction excludes virtual transitions from a state to itself. Based on the previously defined transition probabilities and on condition that the successive state j is known, the distribution function of the waiting time in the current state i is defined as

$$F_{ij}(s,t) = P\left[T_{n+1} \le t \mid J_n = i, J_{n+1} = j, T_n = s\right] = \begin{cases} Q_{ij}(s,t)/p_{ij}(s), & if\ p_{ij}(s) \ne 0, \\ 1, & if\ p_{ij}(s) = 0, \\ 0, & if\ i = j = m. \end{cases}$$

This waiting time distribution specifies, e.g., by what time an active policyholder will die, given that this policyholder will die without becoming disabled prior to death. Therefore, this distribution is crucial in the context of disability insurance as well as term life insurance and annuity products as it takes the duration in the states into account and, thus, determines the time of disability, of death and of potential recoveries. Furthermore, with m being the number of states in the considered model, the following probabilities can be defined:

$$b_{ij}(s,t) = P[J_{n+1} = j, T_{n+1} = t \mid J_n = i, T_n = s] = \begin{cases} Q_{ij}(s,t) - Q_{ij}(s,t-1), & if\quad t > s, \\ 0, & if\quad t = s, \end{cases} \text{ and}$$

$$d_{ij}(s,t) = P[T_{n+1} > t \mid J_n = i, T_n = s] = \begin{cases} 1 - \sum_{j=1}^{m} Q_{ij}(s,t), & if\ i = j, \\ 0, & if\ i \ne j. \end{cases}$$

Probability $b_{ij}(s,t)$ is almost equivalent to probability Q except that the transition to state j takes place exactly at time t. The distribution $d_{ij}(s,t)$ represents the probability that the current state i, which was entered at time s, will not be left up to time t. For notational reasons, this definition makes sense iff $i = j$. The previously defined probabilities $b_{ij}(s,t)$ and $d_{ij}(s,t)$ are extended by conditioning them on the time already spent the present state, i.e. the initial backward recurrence time (see, e.g., D'Amico, Guillen, and Manca, 2009; Stenberg, Manca, and Silvestrov, 2007):

$$b_{ij}(l,s;t) = P[J_{n+1} = j, T_{n+1} = t \mid J_n = i, T_n = l, T_{n+1} > s] = \begin{cases} 0, & if\ d_{ii}(l,s) = 0\ or\ t = s, \\ \dfrac{b_{ij}(l,t)}{d_{ii}(l,s)}, & otherwise, \end{cases}$$

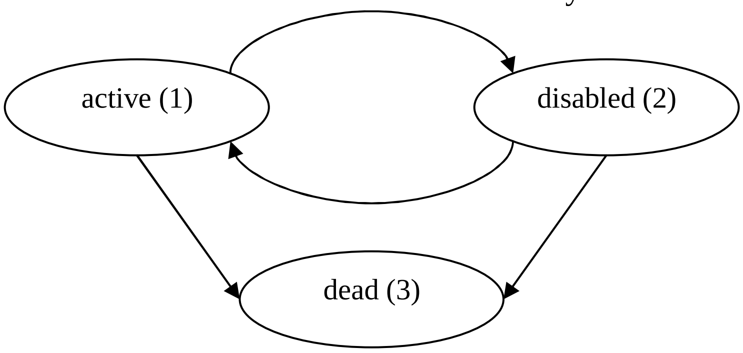
$$d_{ij}(l,s;t) = P[T_{n+1} > t \mid J_n = i, T_n = l, T_{n+1} > s] = \begin{cases} \dfrac{d_{ii}(l,t)}{d_{ii}(l,s)}, & if\ i = j, \\ 0, & if\ i \ne j\quad or\quad if\ d_{ii}(l,s) = 0. \end{cases}$$

The difference between the original and extended probabilities is that $b_{ij}(s,t)$ and $d_{ij}(s,t)$ are conditional upon state i being entered at time s, while $b_{ij}(l,s;t)$ and $d_{ij}(l,s;t)$ assume that the health state i did not change after time l up to time s. These probabilities are of importance for the computation of the book values as defined below because the time already spent in the current state i may affect the next transition, i.e. the waiting time distribution may not be memoryless.

2.3 Multiple state models in disability and life insurance

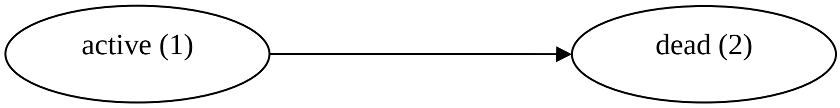
In the context of disability insurances, several state models have been suggested and implemented. Generally, a three-state-model is utilized to display the health states of the policyholder: active (1), disabled (2) and dead (3). In the case of permanent disability, recoveries, i.e. transitions from the disabled state to the active state, are not allowed (see, e.g., Pitacco, 2004), while in the case of potentially temporary disability, recoveries are considered. In addition, the disabled state can be further split according to the duration of the disability and further states can be added to account for lapses and pensioners (see, e.g., D'Amico, Guillen, and Manca, 2009; Haberman and Pitacco, 1999). In this paper, we will employ a three-state-model with recoveries to model the disability insurance contract. The corresponding set of states and set of transitions are exhibited in Figure 1.

Figure 1: Set of states and set of transitions for the disability insurance model



Term life insurances and annuity policies can be modeled using a two-state-model with transitions only from the active state (1) to the dead state (2) (see Macdonald, 1996). To ensure comparability between the model used for the disability insurance on one hand and the term life insurance as well as annuity contract on the other hand, we will apply a Markov renewal model to all three contract types. In case of annuities and term life, the transition probability from the active state to the dead state is equal to one and the waiting time distribution is only affected by the rate of mortality and thus specifies when the transition takes place. The associated set of states and set of transitions are depicted in Figure 2.

Figure 2: Set of states and set of transitions for the term life insurance and annuity model



2.4 Modeling the insurance company

The considered insurance company is assumed to offer disability insurances, term life insurances and annuity policies with a simplified balance sheet as laid out in Table 1.

Table 1: Balance sheet of the insurance company at time *t*

Assets	**Liabilities**
$A(t)$	$B^A(t)$
	$B^D(t)$
	$B^L(t)$
	$E(t)$
$A(t)$	$A(t)$

The market value of the assets of the insurance company at time t is denoted by $A(t)$. Furthermore, the book value of liabilities is referred to by $B^k(t)$ with superscript k specifying the type of insurance, i.e. annuity policies (A), disability insurance (D) and term life insurance (L), which are sold with the same time to maturity T. $E(t)$ is the insurer's equity at time t and residually given by the difference between assets $A(t)$ and liabilities $L(t) = B^A(t) + B^D(t) + B^L(t)$. The equityholders are assumed to make an initial investment $E(0)$ and in return obtain an annual dividend, which is given as a constant fraction r_e of the difference of the surplus,

$$div(t) = r_e \cdot \max\left(E^*(t) - E(t-1); 0\right),$$

with $E^*(t)$ being the value of equity at time t before the dividend is paid (see, e.g., Gatzert and Wesker, 2012a).

We assume that the insurer's assets yield a continuous one-period return ε_t at time t, which is normally distributed with an expected value μ_ε and a standard deviation σ_ε, i.e.

$$A(t) = A(t-1) \cdot \exp\left(\varepsilon_t\right) \text{ with } \varepsilon_t \sim N\left(\mu_\varepsilon, \sigma_\varepsilon^2\right).$$

The development of the asset base is also influenced by premiums, benefits and the previously described dividends. At the beginning of the contract term, i.e. at time $t = 0^+$, the asset base of the insurance company consists of the initial equity and the premiums received in t = 0. Any shocks to input parameters such as changes in mortality or disability rates are assumed to occur after the contracts are closed and do not affect the initial balance sheet. From the insurer's perspective and for all insurance contracts, the premiums are received in advance, while the benefits and dividends are paid in arrears.

The book values of liabilities of an insurance contract at time t, i.e. the prospective reserve for all cash flows after time t, are computed based on the respective Markov renewal process, which is extended by taking into account premiums and benefits (see Table 2). In this paper, we follow the approach in Stenberg, Manca, and Silvestrov (2007) to incorporate premiums and benefits as "rewards" in the Markov renewal process and assume that the market risk, the disability risk as well as the mortality risk are independent.

Table 2: Description of rewards (premiums and benefits)

Parameter	Description
SP	Single premium (annuity) (paid at contract inception)
ψ_1^A	Annual benefit (annuity), also: annuity (paid in arrears)
ψ_1^D	Annual premium (disability insurance) (received in advance)
ψ_2^D	Annual benefit (disability insurance) (paid in arrears)
$\gamma_{13}^D = \gamma_{23}^D$	Death benefit (disability insurance) (paid in arrears)
ψ_1^L	Annual premium (term life insurance) (received in advance)
γ_{12}^L	Single death benefit (term life insurance) (paid in arrears)

The risk-free interest rate is denoted by r. Let $B_i^A(t)$ be the prospective reserve for an *annuity* contract at time t being in state i at time t and let reward ψ_1^A be the annuity paid in arrears. $B_i^A(t)$ is computed as the difference between future expected discounted benefits and premium payments, i.e.

$$B_1^A(t) = d_{11}^A(0,t;T) \cdot \sum_{\vartheta=1}^{T-t} \psi_1^A \cdot e^{-r\cdot\vartheta} + \sum_{\vartheta=1}^{T-t} b_{12}^A(0,t;t+\vartheta) \cdot \left[\sum_{\vartheta'=1}^{\vartheta-1} \psi_1^A \cdot e^{-r\cdot\vartheta'} \right]$$

$$= \sum_{\vartheta=1}^{T-t} \left(\frac{1-F_{12}^A(0,t+\vartheta)}{1-F_{12}^A(0,t)} \right) \cdot \psi_1^A \cdot e^{-r\cdot\vartheta}.$$

The first summand considers the case that the policyholder stays in the initial active state, which implies the reception of annuities throughout the considered time horizon. The second summand accounts for death occurring during the contract duration inferring a stop of annuities at and after the time of death. As dead policyholders do not receive further annuities, being in state 2 implies zero further liabilities, i.e. $B_2^A(t)=0, \forall t$. Hence, the total book value of liabilities resulting from the portfolio of annuities at time *t* is given by

$$B^A(t)=n_1^A(t)\cdot B_1^A(t),$$

with $n_1^A(t)$ denoting the number of policyholders still alive at time *t*. The prospective reserve of a *disability insurance* contract at time *t*, given that the individual has been in health state *i* since *s* time periods, is denoted by $B_i^D(s,t)$, *i* = 1, 2. In this case, ψ_1^D describes the premium and ψ_2^D the disability benefit, while the death benefit is denoted by γ_{13}^D and γ_{23}^D, paid upon transition from the active (1) or disabled state (2) to the dead state (3).[1] Moreover, $B_3^D(s,t)=0, \forall s,t$. The number of active policyholders, who have been active since exactly *s* periods at time *t*, is described with $n_1^D(s,t)$ and the number of disabled policyholders, who have been disabled since exactly *s* periods at time *t*, is specified with $n_2^D(s,t)$. The prospective reserves for the active (1) and the disabled (2) policyholders are thus calculated as

$$\begin{aligned} B_1^D(s,t) &= d_{11}^D(t-s,t;T)\cdot\sum_{\vartheta=0}^{T-t-1}-\psi_1^D\cdot e^{-r\cdot\vartheta} \\ &+\sum_{\vartheta=1}^{T-t} b_{12}^D(t-s,t;t+\vartheta)\cdot\left[\sum_{\vartheta'=0}^{\vartheta-1}-\psi_1^D\cdot e^{-r\cdot\vartheta'}+\psi_2^D\cdot e^{-r\cdot\vartheta}+B_2^D(0,t+\vartheta)\cdot e^{-r\cdot\vartheta}\right] \\ &+\sum_{\vartheta=1}^{T-t} b_{13}^D(t-s,t;t+\vartheta)\cdot\left[\sum_{\vartheta'=0}^{\vartheta-1}-\psi_1^D\cdot e^{-r\cdot\vartheta'}+\gamma_{13}^D\cdot e^{-r\cdot\vartheta}\right] \end{aligned}$$

and

$$\begin{aligned} B_2^D(s,t) &= d_{22}^D(t-s,t;T)\cdot\sum_{\vartheta=1}^{T-t}\psi_2^D\cdot e^{-r\cdot\vartheta} \\ &+\sum_{\vartheta=1}^{T-t} b_{21}^D(t-s,t;t+\vartheta)\cdot\left[\sum_{\vartheta'=1}^{\vartheta-1}\psi_2^D\cdot e^{-r\cdot\vartheta'}+B_1^D(0,t+\vartheta)\cdot e^{-r\cdot\vartheta}\right] \\ &+\sum_{\vartheta=1}^{T-t} b_{23}^D(t-s,t;t+\vartheta)\cdot\left[\sum_{\vartheta'=1}^{\vartheta-1}\psi_2^D\cdot e^{-r\cdot\vartheta'}+\gamma_{23}^D\cdot e^{-r\cdot\vartheta}\right]. \end{aligned}$$

[1] Disability insurances typically provide coverage for disability only and, therefore, do not include death benefits. However, the policy may be sold as a rider to term life insurance, for instance, such that the resulting product covers disabilities as well as death. For simplicity, we will refer to this combined product as a disability insurance contract with death benefit.

In the first equation, the prospective reserve when being in the active state is determined. The first summand describes that the initial active state is not left until contract maturity and hence, only premiums are received. The second summand considers a transition to the disabled state during the contract duration. Here, premiums are received up to the beginning of the disability and the disability benefit ψ_2^D is added. The prospective reserve for the upcoming disability is then assessed by $B_2^D(0,t+\vartheta)$. At last, direct transitions from the active state to death are included in the third summand. In this case, premiums are received up to the time of death and the death benefit γ_{13}^D is paid upon death. The second equation computes the prospective reserve for a disabled policyholder at time *t*. The disability benefits are either paid for the remaining contract duration, as described in the first summand, or paid up to the time of recovery or death, as depicted in the second and third summand respectively. In the second summand, the future premium payments due to recovery are assessed by $B_1^D(0,t+\vartheta)$ and, in the third summand, the death benefit γ_{23}^D is included. Hence, the total book value for disability insurances at time *t* is computed by

$$B^D(t) = \sum_{s=0}^{t} n_1^D(s,t) \cdot B_1^D(s,t) + \sum_{s=0}^{t} n_2^D(s,t) \cdot B_2^D(s,t).$$

The book value for a *term life insurance* contract at time *t*, $B_i^L(t)$, is computed as

$$B_1^L(t) = d_{11}^L(0,t;T) \cdot \sum_{\vartheta=0}^{T-t-1} -\psi_1^L \cdot e^{-r\cdot\vartheta} + \sum_{\vartheta=1}^{T-t} b_{12}^L(0,t;t+\vartheta) \cdot \left[\sum_{\vartheta'=0}^{\vartheta-1} -\psi_1^L \cdot e^{-r\cdot\vartheta'} + \gamma_{12}^L \cdot e^{-r\cdot\vartheta} \right]$$

$$= \sum_{\vartheta=0}^{T-t-1} \left(\gamma_{12}^L \cdot \frac{F_{12}^L(0,t+\vartheta+1) - F_{12}^L(0,t+\vartheta)}{1-F_{12}^L(0,t)} \cdot e^{-r\cdot(\vartheta+1)} - \psi_1^L \cdot \frac{1-F_{12}^L(0,t+\vartheta)}{1-F_{12}^L(0,t)} \cdot e^{-r\cdot\vartheta} \right)$$

$$B_2^L(t) = 0.$$

Here, the first summand in the first equation refers to the case that the policyholder stays in the initial active state and hence, premiums are received for each point in time during the contract duration. The second summand considers the case that the death of the policyholder occurs before the end of the duration and therefore, premiums are paid up to the time of death and then, the death benefit is paid. As described by $B_2^L(t)$, the book value of the liabilities is equal to zero after the policyholder's death. Overall, with $n_1^L(t)$ being the number of policyholders alive at time *t*, the total book value resulting from term life insurances at time *t* is given by $B^L(t) = n_1^L(t) \cdot B_1^L(t)$.

To ensure comparability in the subsequent numerical analysis, the three insurance types are calibrated to have the same individual contract volume *V*, which here refers to the present value of benefit payments. Thus, based on the previous formulas for the book values, premiums and benefits are computed according to the actuarial equivalence principle. The potential default of the insurance company is not considered in the premium calculation and external institutions are assumed to fulfill remaining contractual obligations in the case of insolvency (see, e.g., Gatzert and Wesker, 2012b).

2.5 Risk measurement

As the relevant risk measure, we first consider the probability of default (*PD*), which is defined as the probability that the assets $A(t)$ are not sufficient to cover the liabilities $L(t)$ during the contract duration, i.e.,

$$PD = P\left(T_d \leq T\right) \text{ with } T_d = \begin{cases} \inf\left\{t \in [0,T] : A(t) < L(t)\right\}, & if\ \exists t \in [0,T] : A(t) < L(t), \\ T+1, & otherwise. \end{cases}$$

In addition, the mean loss is derived, which measures the discounted expected loss in the case of default,

$$ML = E\left[\left(L\left(T_d\right) - A\left(T_d\right)\right) \cdot e^{-r \cdot T_d} \cdot \mathrm{I}\left\{T_d \leq T\right\}\right],$$

where $\mathrm{I}\{\bullet\}$ stands for an indicator function. In addition, the standard deviation of liabilities at each time *t* is calculated, i.e.,

$$\sigma\left(L(t)\right) = \sigma\left(B^A(t) + B^D(t) + B^L(t)\right).$$

3. NUMERICAL ANALYSIS

In this section, after introducing the mortality estimation and the input parameters, numerical examples regarding the risk situation of an insurance company offering annuity policies, disability and term life insurances are presented including comprehensive sensitivity analyses.

3.1 Mortality estimation and projection

The general mortality rates for all regarded groups of insured are based on the number of deaths and the exposure-to-risk in the German male population from 1956 to 2011 from the Human Mortality Database.[2] With 100 being the maximum age attainable, the demographic parameters, as defined in Lee and Carter (1992), were estimated with the regression model in Brouhns, Denuit, and Vermunt (2002a) and the results of α_x and β_x are displayed in Appendix A.1. Because of its non-stationarity, the first differenced series of the estimated mortality trend κ_t was modeled with an ARIMA model. Even though the Akaike information criterion indicated a more complex model, the Box-Ljung Test and, using a significance level of 5%, the autocorrelation function as well as the partial autocorrelation function showed no significant residual autocorrelation for the applied ARIMA(2,1,1) model. This ARIMA model has the following parameters with their respective standard error in parentheses: an autoregressive part with parameters ϕ_1 = 0.6915 (0.1400), ϕ_2 = 0.2987 (0.1382) and a moving average part with parameter θ_1 = -0.8350 (0.0665). The estimated as well as forecasted mortality trend κ_t is shown in Appendix A.1.

3.2 Input parameters

We consider male policyholders aged 35 purchasing disability or term life insurance contracts and male policyholders of age 75 who purchase annuity policies. For both age groups, the contract duration is thus T = 25 years for contracts closed in the year 2012.

The calibration of the disability insurance model is challenging due to the scarceness of available information and empirical data. However, the empirical analysis of Spanish disability data in D'Amico, Guillen, and Manca (2009) suggests that the transition probability from the active state to the disabled state ($p_{12}^{D}(s)$) is approximately 20% across the ages 35 to 60 years, i.e. within each age group, 20% of the policyholders have an actual chance to become disabled, whereas the remaining 80% certainly die without becoming disabled during the remaining lifetime.[3] Therefore, we assume an (average) disability risk group with a

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[2] See www.mortality.org. For the years 1956 to 1990, the available data for East and West Germany are combined.

[3] In their analysis, D'Amico, Guillen, and Manca (2009) study Spanish disability insurance policies, where disability corresponds to a serious dependence level due to a policyholder's inability to perform daily life activities without the help of other individuals. In addition, blindness and dismemberment are covered by the regarded insurance company. This definition is not identical to the one in Germany where the permanent inability to practice an occupation due to diseases, personal injury or decomposition is covered (see Voit and Neuhaus, 2009). However, the disability tables of the German Actuarial Association also consider long-term

transition probability $p_{12}^{D}(s)=20\%$, $\forall s$, i.e. for all age groups, and additionally study groups with higher disability risk by increasing the transition probability $p_{12}^{D}(s)$ to 40%. For all risk groups considered, we consider permanent disability, where the transition probability from the disabled state back to the active state (in the following probability of recovery, $p_{21}^{D}(s)$) is 0%. Moreover, we further examine the impact of potential recoveries within the group with the highest disability risk by setting the probability of recovery to 40% and, thus, we explicitly analyze temporary disability.

The mortality rate for disability insurance policyholders is derived from the mortality of the whole population. While the mortality rates for active policyholders of disability insurance is assumed to be the same as for the term life insurance policyholders, Segerer (1993) points out that the mortality rates for disabled policyholders are higher than the active life mortality and that this difference is decreasing in age and in duration of the current disability. Therefore, we compute the ratio of the probability mass functions for the mortality of active and disabled policyholders based on the actuarial tables of the German Actuarial Association (DAV)[4] via

$$ratio=\frac{F_{23}\left(s,s+\vartheta\right)-F_{23}\left(s,s+\vartheta-1\right)}{F_{13}\left(s,s+\vartheta\right)-F_{13}\left(s,s+\vartheta-1\right)}$$

and approximate this ratio by the function $\max\{\alpha\cdot\beta^{s}\cdot\gamma^{\vartheta};1\}$. This function follows the empirical results in Segerer (1993) by implying a simple non-linear relationship between the factor on the one hand and the age as well as duration on the other. The application of the method of least-squares yields an initial value α of 49.56,[5] which is decreasing by β = 7.4% each year of age and by γ = 32% each period spent in the current disability state.[6] The modeled ratio is never less than 1, implying that the mortality of disabled policyholders is always at least as high as the one for active policyholders. Based on the modeled ratio and the mortality of active policyholders, the waiting time distribution from the disabled state to the dead state is constructed. As the actuarial tables do not include the future advances in medical

care as a disability claim and, thus, apply a similar definition of disability as compared to the Spanish definition. Due to this comparability, we use empirical results from D'Amico, Guillen, and Manca (2009) in combination with the actuarial tables and, thus, mix results from Spanish and German disability data, as no further data was available.

4 In this numerical analysis, the German mortality and disability tables DAV1997I, DAV1997RI, DAV1997TI and DAV2008T were used, but without the safety loadings.

5 This implies that for a 35-year old policyholder who just became disabled, the one-year probability of death is 49.56 higher as compared to the mortality probability of a 35-year old active policyholder.

6 The coefficient of determination, R^2, is equal to 98.1%.

sciences during the last two decades, these assumptions may not fully reflect the current reality and are thus subject to sensitivity analyses.

Following the empirical results in D'Amico, Guillen, and Manca (2009), the waiting time distribution from the active state to the disabled state is assumed to be logistically distributed with a truncation at 0 and, via the method of least-squares, the (discretized) logistic distribution is fitted to the probability mass function, which is based on the actuarial tables of DAV. For a 35-year-old policyholder, the corresponding mean and standard deviation are 33.69 years and 11.03 years, respectively, and still active policyholders and recovered ones with the same age show the same disability rates. In addition, it can be observed that the disability rates are increasing in age. Moreover, the waiting time distribution for recoveries, i.e., for transitions from the disabled state back to the active state, is assumed to be a geometric distribution and is fitted to the DAV recovery tables by applying the method of least-squares again. The parameter of the resulting geometric distribution, i.e. the inverse of its expected value, is equal to 9.29% for all policyholders aged 35 years at the inception of disability and for each year of age at disability inception, this parameter is reduced by 11.59%.[7] The resulting waiting time distribution reflects the reality in that recoveries become less likely with a higher duration of disability and a higher age. The transition probabilities, the mean of the waiting time distribution to the disabled state as well as the mortality rates are subject to variation.

Independent of the resulting portfolio composition, to ensure comparability, the total amount of contracts sold is set to 10,000 and the individual contract volume *V*, defined as the present value of future benefit payments, is set to 10,000 to ensure the comparability between the three regarded insurance types (see Gatzert and Wesker, 2012a) and to isolate potential hedging effects. In addition, the risk-free interest rate is fixed to r = 3%. Based on this risk-free interest rate and contract volume, the annuity ψ_1^A is equal to 1,139.59 for a single premium of *V*. The resulting annual premium for the term life insurance ψ_1^L is equal to 570.58 and the death benefit γ_{12}^L is 245,390.60. In case of disability insurance, the annual premium ψ_1^D, the annual disability benefit ψ_2^D as well as the death benefit $\gamma_{13}^D = \gamma_{23}^D$ are given in Table 3 depending on the respective disability insurance design (with or without death benefit and for different transition probabilities for disability and recovery). For instance, in order to keep the volume of the contracts equal to 10,000, in case a death benefit of 100,000 is paid, the annual disability benefit must be reduced from 77,337.31 (No. 1) to 50,272.77 (No. 2) when keeping the annual premium unchanged. For a given disability transition probability,

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7 The regarded geometric distribution is supported on the set {1,2,3,…}.

the inclusion of recoveries increases the disability benefit, e.g., from 38,668.66 (No. 3: probability of recovery 0%) to 38,981.26 (No. 4; probability of recovery 40%), but the effect is not substantial.

Table 3: Premiums and benefits for different designs of the disability insurance contract

No.	Disability	Recovery	Premium ψ_1^D	Benefit ψ_2^D	Death benefit $\gamma_{13}^D = \gamma_{23}^D$
1	$p_{12}^D(s) = 20\%$	$p_{21}^D(s) = 0\%$	572.52	77,337.31	0
2	$p_{12}^D(s) = 20\%$	$p_{21}^D(s) = 0\%$	572.52	50,272.77	100,000
3	$p_{12}^D(s) = 40\%$	$p_{21}^D(s) = 0\%$	574.47	38,668.66	0
4	$p_{12}^D(s) = 40\%$	$p_{21}^D(s) = 40\%$	574.08	38,981.26	0

To analyze the risk situation of the insurance company, a Monte Carlo simulation with 100,000 simulation paths is used and, for each simulation run, 10,000 contracts of each insurance type are simulated based on the underlying Markov renewal model. For each contract sample, the total number of policyholders in each state at any time is determined. The actual number of each insurance contract in the regarded portfolio is then considered by multiplying the total number of policyholders in each state at any time with the share of the contract in the portfolio. Thus, the sub-stock of insurance contracts is determined by means of the maximum total stock. The shareholders make an initial investment of 10 million at the beginning and receive a constant fraction of r_e = 25% of the earnings as dividends.[8] At each point in time, the book values for each type of contract are computed with the risk-free rate (3%). In addition, assets are calculated with an annual expected rate of return of μ_ε = 5% and a volatility of σ_ε = 8%. The risk measures are then computed based on the determined liabilities and assets at each time. The input parameters are chosen for illustration purposes and are subject to robustness tests.

3.3 The impact of disability insurance on a life insurer's risk situation: The impact of diversification effects

To examine the initial risk situation of the fictive insurance company, the parameters used for the calculation of premiums and benefits are first assumed to coincide with the actually realized parameters, especially the mortality rate and the transition probability from the active to the disabled state.

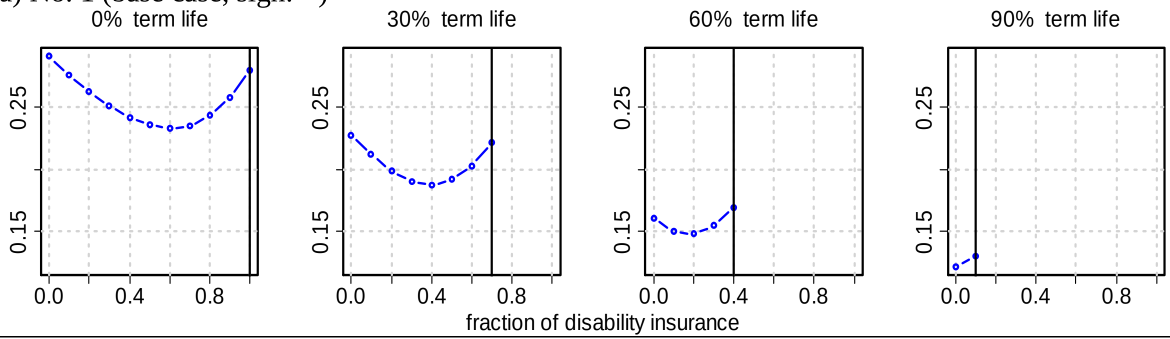
[8] Note that from the equityholders' perspective, the dividends (or the insurance contracts) are not computed to be fair.

In general, the total payout resulting from the portfolio of annuities declines over time, whereas the payout for the term life insurance portfolio increases from the insurer’s perspective. In case of disability insurance, the cash inflow due to premium payments gradually declines as active policyholders either die or become disabled, which is analogous to term life insurances with annual premiums where the portfolio of active policyholder declines. The benefits comprise disability benefits, which are paid as long as the disabled policyholder is alive. Once the disabled policyholder dies during the contract term, benefit payments are stopped and a death benefit may be paid. Hence, depending on the chosen disability rates and mortality rates for disabled policyholders, the payment of benefits may fluctuate over time, which is in contrast to term life insurance and resembles the payout structure of an annuity with uncertain starting date and uncertain time to maturity. In our analysis, the number of policyholders who become disabled exceeds the number of dying disabled policyholders at any time during the considered time horizon and therefore the cash outflow is steadily increasing. Overall, these different payment structures provide opportunities for counterbalancing the payments and can thus contribute to reduce the overall risk resulting from the portfolio.

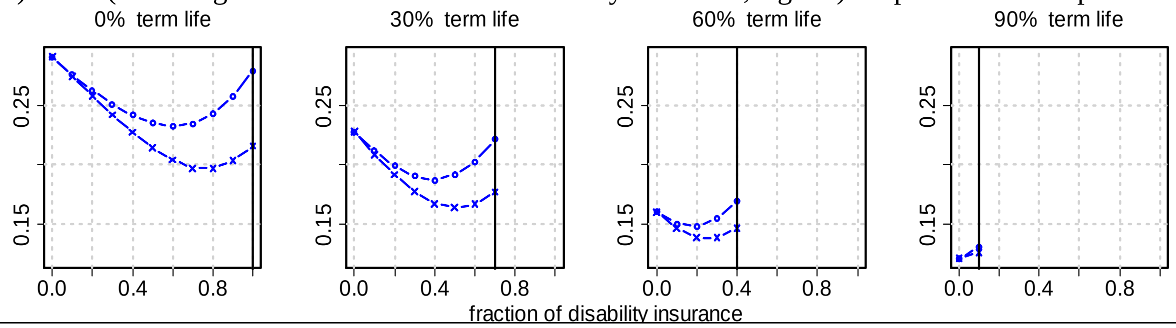
Figure 3 displays the shortfall risk of an insurance company using the probability of default depending on the portfolio structure, where the vertical line in each plot represents the maximum portion of disability insurances in the considered portfolio (e.g. first row, second figure: for an insurance portfolio with 30% term life insurances, at most 70% disability insurances can be sold). Figure A.2 in the Appendix additionally exhibits results when using the mean loss as the relevant risk measure.

Figure 3: Probability of default for different portfolio compositions and disability insurances for different disability insurance designs (No. in Table 3)

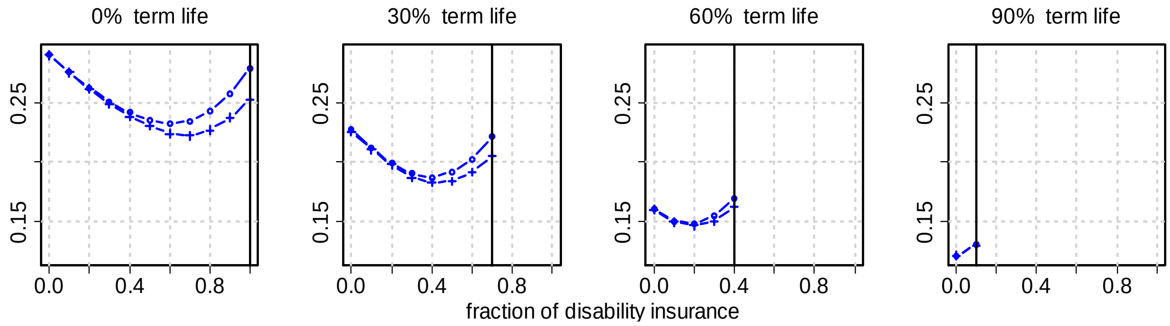
a) No. 1 (base case, sign: ○)



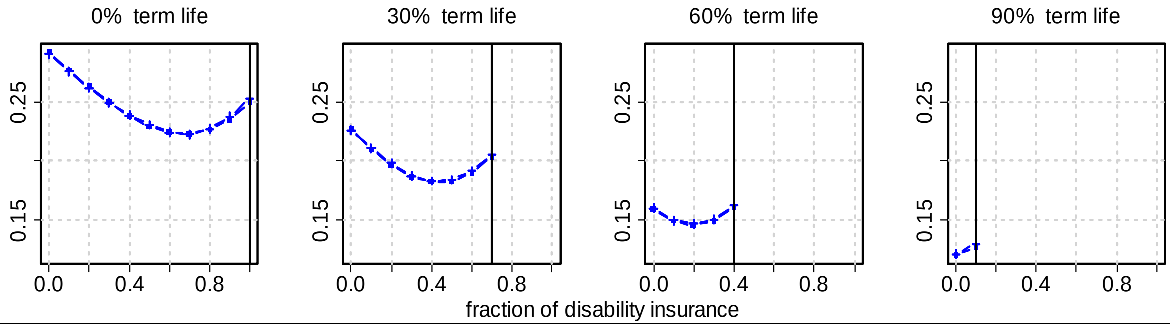
b) No. 2 (including death benefit in case of disability insurance, sign: x): Improvement compared to a)



No. 3 (higher disability transition probability, 40%, sign: +): Improvement compared to a)



d) No. 4 (higher disability transition probability, 40%, and with recovery transition probability, 40%, sign: ▪): Improvement compared to c)



Notes: The vertical line in each plot represents the maximum portion of disability insurances in the considered portfolio (total number of contracts = 10,000; volume of each contract (present value of benefit payments) = 10,000). Example: first row, second figure from the left: for an insurance portfolio with 30% term life insurances, at most 70% disability insurances can be sold. In case of 30% term life insurances (3,000 contracts) and a fraction of 40% disability insurances (4,000 contracts), 100%-30%-40% = 30% annuity policies (3,000 contracts) are sold.

Figure 3 shows that portfolios solely consisting of annuity policies exhibit the highest risk in the considered setting, which also holds for the mean loss (see Figure A.2 in the Appendix). The risk can be reduced by either adding term life or disability insurances to the portfolio, thus generating diversification effects due to uncorrelated biometric risks. The portion of disability insurances that minimizes the company's default risk in the base case (Figure 3a) lies between 0% (see right graph in Figure 3a) and 60% (see left graph in Figure 3a) depending on the risk measure and the fraction of term life insurances in the portfolio. In addition, one can observe that increasing the portion of term life insurances lowers the risk-minimizing proportion of both disability insurance and annuity contracts because of their comparable structure of annual benefit payments that are paid as long as the disabled policyholder or the annuitant is alive. Regardless of the risk measure (see also Figure A.2 in the Appendix), the portfolios with the least risk in the present setting feature a high percentage of term life insurances and, depending on the initial portfolio composition and the risk measure, the overall risk can be reduced up to 85% (see right graph in Figure 3a). Because the differences between the three insurance types cannot arise from different contract values (in the sense of the present value of benefit payments at inception of the contracts, which is calibrated to 10,000 for all contracts), the timing of the payouts matters and a balanced portfolio composition may smooth the cash flows from the various insurance types.

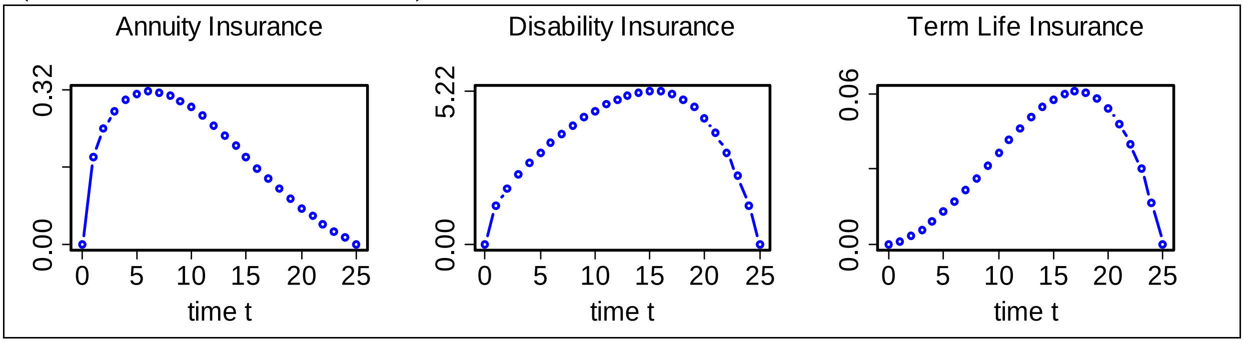
Figure 3b) displays the probability of default if a death benefit of 100,000 is paid in case the policyholder dies during the disability contract term (e.g. in case of a standalone disability insurance contract in contrast to a supplementary disability insurance that is attached to a term life contract, for instance). The results show that the risk inherent to disability insurance contracts is reduced by the inclusion of death benefits (compare to Figure 3a), see case of highest disability insurance portion) because the combined product now includes the less risky term life insurance component and a reduced disability benefit payment (see No. 2 in Table 3). Thus, the risk-minimizing fraction of disability insurances is slightly increasing.

Figure 3c) shows the case where the transition probability from the active to the disabled state ($p_{12}^{D}(s)$) is increased from 20% to 40% (No. 3 in Table 3). Due to the calibration of the contracts to ensure the same volume at contract inception, the disability benefit is reduced by about 35% (while annual premiums remain approximately unchanged), which overall implies a slight reduction in shortfall risk. Further analyses also showed that the standard deviation is considerably reduced in this case due to the lower disability benefit.

In Figure 3d), recovery is included in the model with a transition probability ($p_{21}^{D}(s)$) from the disabled to the active state of 40% (instead of 0%) for a disability transition probability of 40% as in Figure 3c), i.e. for the case of the highest risk group. In this case, the impact of recoveries is higher, but still negligible, as very few policyholders actually recover after becoming disabled during the contract term. Overall, the different effects are rather small and do not substantially impact the results, including the risk-minimizing portfolio. Thus, in the following, we focus on the base case as displayed in Figure 3a).

Figure 4 displays the development of the standard deviation of the liabilities over the considered time horizon of 25 years for a portfolio of one single contract type, respectively. In general, the standard deviation is affected by time, the distribution of the policyholders within the state model, the benefit level as well as the type of benefit, and these factors may potentially counterbalance each other. In particular, the bow-shaped curve follows from the standard deviation being zero at time zero and at the end of the time horizon. In addition, a higher benefit level and a smoother allocation of policyholders across the state model, i.e., policyholders are not concentrated in a single state, imply a greater standard deviation. Moreover, annuities yield a higher standard deviation of liabilities because of the prospective reserve being greater. The standard deviation of the liabilities resulting from annuity products reaches its maximum in the first half of the time horizon, whereas the variation of the liabilities arising from disability insurances and term life insurances have their maximum in the second half on the time horizon. As shown in this figure, disability insurances have the highest variation followed by annuity policies because even though the disability policyholders are more concentrated in the active state compared to the annuity policyholders, the disability benefit level is much higher than the annuity. In contrast, term life insurances reveal the lowest variation of liabilities because of the relatively highest concentration in the alive state (the vast majority of policyholder survives the duration of the contract) and due to the death benefit being a single payment. As a result, the addition of term life insurances to the insurance portfolio may considerably lower the overall standard deviation over the time horizon in the considered setting. Figure 4 also reveals that a disadvantage of disability insurances as a potential hedging tool may be the relatively high standard deviation of liabilities.

Figure 4: Standard deviation of liabilities (in million) over time for a single contract type (10,000 contracts for each case)



3.4 Natural hedging in a life insurance portfolio with disability insurance: The impact of shocks to mortality

As indicated in the analyses in the previous section, strong diversification benefits may arise in a portfolio of different life insurance contracts depending on the respective portfolio composition. In the following, we extend our analysis and study the effectiveness of natural hedging in the presence of disability insurance, first focusing on shocks to mortality. To illustrate central effects, we model simple shocks to mortality by multiplying the time trend of mortality κ_t with a constant factor *e* and assume that these shocks are not taken into account by the insurer when calculating benefits and premiums.[9] This constant shock for all κ_t implies a non-identical change of the mortality rates across ages because of the age-specific sensitivity parameter b_x.

Figure 5 displays the results for different shocks to mortality *e* corresponding to the setting in Figure 3a) for the probability of default (see Figure A.2 in the Appendix for the mean loss). Since the forecasted time trend is negative during the time horizon (see Figure A.1), a factor *e* less than one describes an increase in mortality (see lines with symbols ‘square’ and ‘triangle facing downwards’ in Figure 5) and *e* greater than one implies a decrease in mortality (see lines with symbols ‘triangle facing upwards’ and ‘cross’ in Figure 5).

In the case of annuities, shortfall risk is increased if mortality rates decrease ($e < 1$) because annuities need to be paid out longer than expected. In the left graph with 0% term life insurances and 0% disability insurances (i.e. 100% annuities), for example, a decrease in

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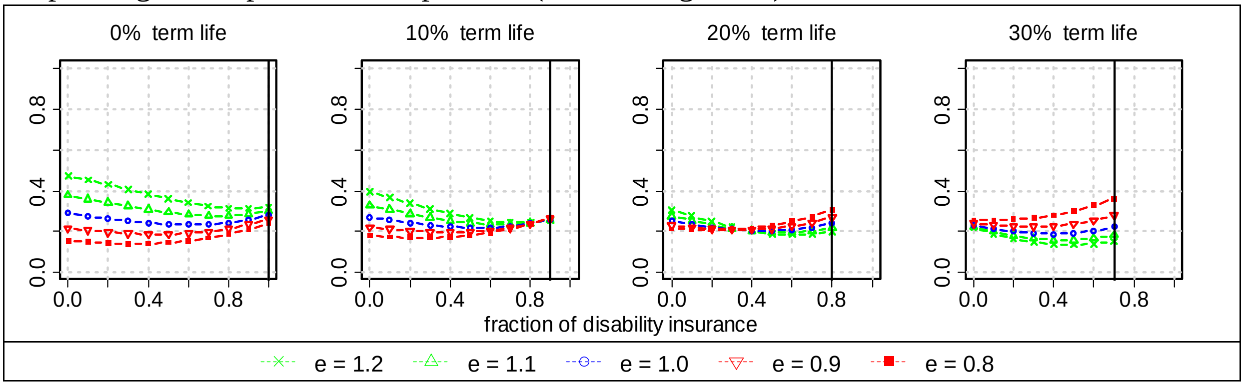
9 See, e.g., Wang et al. (2010), Gatzert and Wesker (2012a). Note that in general, insurers may impose a premium loading to account for this risk of misassessing mortality risk, which can alter the results. For a detailed analysis of premium loadings in the context of term life and annuities, we refer to Wong, Sherris, and Stevens (2013).

mortality rates (e < 1) causes a higher probability of default as compared to the setting without shock (e = 1). This effect is reversed when considering a portfolio that comprises term life insurances (see, e.g., right graph with 30% term life, 70% disability insurances, and 0% annuities).

The numerical results further show that disability insurances are rather inefficient to counterbalance shocks to mortality rates in other policies, as these shocks have only a minor influence on the (generally low) mortality risk inherent to disability insurance contracts. In particular, as can be seen in the left graph in Figure 5, the shortfall risk for various shocks to mortality converges for an increasing fraction of disability insurances in the portfolio. Hence, in contrast to the other two insurance types, disability insurances are considerably less sensitive to changes in mortality risk because shifts in mortality rates are counterbalanced among the disability insurance contracts. For instance, a higher mortality rate implies that fewer premiums are paid by active policyholders, but at the same time, fewer benefits have to be paid to disabled policyholders. Thus, mortality risk plays a minor role for this type of contract, which is in contrast to annuities and term life insurances.

Figure 5: The impact of shocks to mortality on a life insurer's probability of default depending on the portfolio composition (see also Figure 3a)



Notes: The vertical line in each plot represents the maximum portion of disability insurances in the considered portfolio (total number of contracts = 10,000; volume of each contract (present value of benefit payments) = 10,000). Example: first row, second figure from the left: for an insurance portfolio with 30% term life insurances, at most 70% disability insurances can be sold. In case of 30% term life insurances (3,000 contracts) and a fraction of 40% disability insurances (4,000 contracts), 100%-30%-40% = 30% annuity policies (3,000 contracts) are sold. A factor e greater (less) than 1 *implies a decrease (increase) in mortality rates.*

Natural hedging effects between annuities and disability insurances in regard to mortality risk are also limited because of the similarities in the payout structure, as analogous to annuity contracts, an increase in mortality rates generally results in reduced disability insurance liabilities. While there is no intersection point for the different shocks to mortality e in the case without term life (left graph), including term life insurance leads to an intersection point, where the shortfall risk remains unchanged for a given shock to mortality and a given risk measure, thus representing the risk-immunizing portfolio in the considered setting. This intersection point is shifted to the left when increasing the portion of term life insurance (from left to right graph in Figure 5). This implies that more annuities (and less disability insurances) are needed for immunizing a portfolio against shocks to mortality, which further emphasizes the considerably stronger reaction of annuities towards shocks to mortality as opposed to disability insurances.

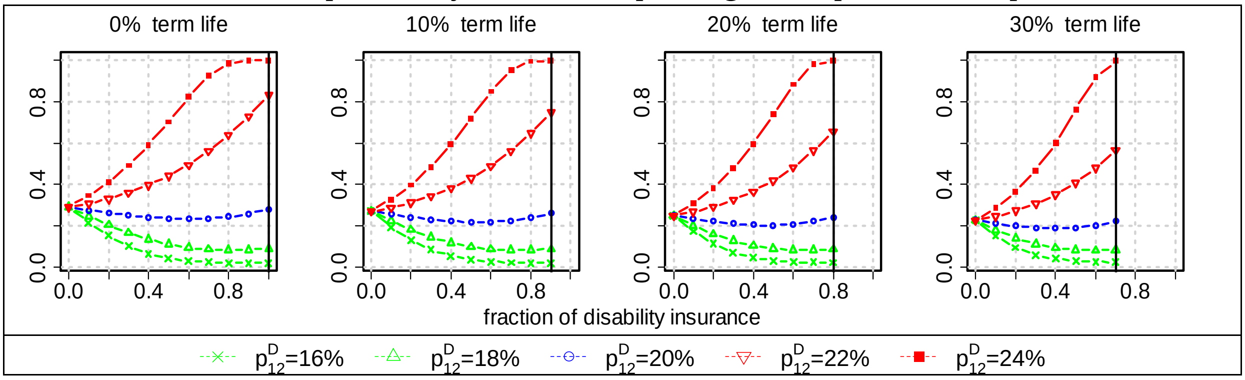
Although this result may partly depend on the calibration, it must be noted nonetheless that the hedge ratio between disability and term life insurance, which immunizes the portfolio against shocks to morality, is much higher than the one between annuity policies and term life insurances. Thus, annuity policies are considerably more efficient to hedge mortality risk inherent in life insurances. Overall, the portfolio can be arranged either to minimize the overall risk inherent in the portfolio or to immunize the portfolio against shocks to mortality. Hence, there is a trade-off between the risk level and the immunization, which can be addressed by including further risk management instruments to reduce the risk level and immunize the portfolio at the same time (see Gatzert and Wesker, 2012b).

In the previous analysis, we have focused on shifts in the general mortality. As described before, the mortality rate of disabled policyholders can be decomposed into general mortality and a specific factor that describes the relationship between active life mortality and the mortality of disabled insured. Additional analyses show that a higher mortality of disabled policyholders (in the sense of different shocks) has a positive effect on the risk situation of the disability insurance provider, since benefit recipients die earlier than assumed, and, as a result, actual liabilities resulting from this contract are reduced. The opposite holds for a lower mortality. These specific shocks only apply to disability insurances and thus have no effect on the risk inherent to annuity contracts and term life insurance. Therefore, these insurance types cannot immunize the portfolio against this shock. However, as only a minority of policyholders actually becomes disabled, shocks to the specific mortality of disabled policyholders have only a minor influence, which is slightly larger in case death benefits are included in the policy.

3.5 Natural hedging in a life insurance portfolio with disability insurance: The impact of shocks to disability risk

The disability risk is influenced by the transition probability $p_{12}(s)$ from the active to the disabled state as well as by the waiting time $F_{12}(s,t)$, which describes when the transition to the disabled states occurs. Figure 6 shows results when varying the transition probability from the active to the disabled state and its impact on the probability of default (see Figure A.4 in the Appendix for the mean loss). A decrease in the transition probability implies that fewer policyholders become disabled during the time horizon (e.g., from 20% to 16%; see lines with symbols 'cross' and 'triangle facing upwards' in Figure 6). This is favorable for the insurer because fewer benefits will be paid than assumed in the calculation of the benefit, thus reducing the shortfall risk. The opposite can be found when considering the upper lines (with symbols 'triangle facing downwards' and 'square') with higher transition probabilities of 22% and 24%, respectively, which exhibit a strong sensitivity of disability insurances and result in a considerably worsened overall company risk level.

Figure 6: The impact of changes of the transition probability from the active to the disabled state on a life insurer's probability of default depending on the portfolio composition



Notes: The vertical line in each plot represents the maximum portion of disability insurances in the considered portfolio (total number of contracts = 10,000; volume of each contract (present value of benefit payments) = 10,000). Example: first row, second figure from the left: for an insurance portfolio with 30% term life insurances, at most 70% disability insurances can be sold. In case of 30% term life insurances (3,000 contracts) and a fraction of 40% disability insurances (4,000 contracts), 100%-30%-40% = 30% annuity policies (3,000 contracts) are sold.

Term life and annuities are not affected by these changes. In a portfolio only consisting of disability insurance (0% term life insurance, 100% disability insurance), a decrease of the underlying transition probability by 10% (20%) to $p_{12}(s)$ =18% (16%) implies a decrease in the probability of default by 68% (93%), whereas an increase by 10% (20%) to $p_{12}(s)$ =22% (24%) results in an increase of the probability of default by 198% (258%).

The mean loss was found to be even more sensitive to such changes than the probability of default (see Figure A.4). Therefore, using a single risk measure may lead to a severe underestimation of the impact of disability risk on an insurer's risk situation. In addition, estimating and forecasting the transition probability based on empirical data appears to be a critical step in the evaluation of disability insurance contracts. However, challenges arise due to continuously changing working conditions for instance, and as future trends of disability risk are difficult to predict. The exposure to this risk may be reduced by means of increasing the amount of other insurance types and by better balancing the insurance portfolio, using e.g. annuity policies. With 30% term life insurances, for instance, the sensitivity of the probability of default is at least slightly reduced, whereas the impact on the mean loss can be considerably lowered.

The risk measures are also observed to be highly sensitive to changes of the expected waiting time from the active to the disabled state. In general, the waiting time distribution describes when a policyholder transfers to another state. Hence, a lower expected waiting time implies that policyholders become disabled earlier than assumed, which has a negative impact on the insurer's risk situation because benefits are paid earlier and less premiums are received. Further analyses showed that a decrease of the expected waiting time by 3.3% results in a 258% to 844% higher risk depending on the respective risk measure. An increase by the same percentage yields a risk reduction up to 84% in case of the mean loss, whereas the probability of default is less sensitive. In comparison with changes to the transition probability, shocks to the expected waiting time from the active to the disabled state have an even higher impact. Thus, with the expected waiting time being a representative for the waiting time distribution, this distribution must also be selected and calibrated with care as well as regularly checked and possibly updated. Analogous to the risk resulting from changes of the underlying transition probability, the exposure to risk due to unexpected variations of the expected waiting time can either be reduced by transferring parts of it to the capital market via derivatives or by adding other insurance types to the portfolio and thus improving the general diversification benefits.

4. CONCLUSION

In this paper, an insurance portfolio consisting of annuity contracts as well as disability and term life insurances is modeled, calibrated and studied. Specifically, we consider assets and liabilities of an insurance company to quantify the effect of mortality and disability risk on the insurer's overall risk situation and to study risk-minimizing as well as risk-immunizing portfolios. Within our framework, we aim to examine the diversification benefits in the insurance portfolio and potential natural hedging effects that may arise in the portfolio with focus on the role of disability insurances. Diversification benefits play an important role in the context of risk management and are of high relevance in the context of Solvency II when deriving solvency capital requirements for different risks, including mortality and disability risks.

The analysis of the portfolio composition shows that in the considered setting, especially the addition of term life insurances can considerably reduce the overall risk and thus potentially represent a major component for reducing the risk inherent in a portfolio. Our numerical examples illustrate that disability insurances are not very sensitive to shocks in mortality rates and thus represent a less efficient hedging instrument for the mortality risk (especially inherent in term life insurances) as opposed to annuity contracts. In addition, disability insurances exhibit the highest standard deviation of liabilities in the considered case, which is a disadvantage in regard to the overall risk situation and the application as a hedging tool. Furthermore, the numerical analysis emphasizes the (naturally) strong sensitivity of disability insurance contracts in regard to disability risk and that this risk cannot be easily hedged by other types of life insurance contracts. However, adding disability insurance to a portfolio of life insurances can still considerably lower the company risk due to natural diversification effects arising from uncorrelated biometric risks in the considered setting.

Thus, an adequate portfolio composition on the liability side that systematically exploits diversification benefits and potential natural hedging effects does represent an important tool for insurers to reduce their overall risk situation. However, altering the portfolio composition may also introduce additional expenses and operational risk (see Wang et al., 2010). In addition, specific risks such as disability risk may not always be hedgeable in a similar manner as mortality risk using annuities and term life policies as demonstrated in this paper. Therefore, instead of or in addition to adjusting the portfolio composition and besides the usage of reinsurance, derivatives such as swaps may be of interest to balance the exposure to disability and mortality risk and more efficiently utilize existing natural hedging opportunities (see, e.g., Dowd et al., 2006).

Further studies should examine the influence of adverse selection, premium loadings and distinct disability levels on the insurer’s risk situation. In addition, the independence of mortality risk and disability risk on the one hand and disability risk and market risk on the other hand requires further theoretical and empirical analyses.

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APPENDIX

Figure A.1: Estimated values of exp(α_x) and β_x over all ages, estimated and predicted mortality time trend

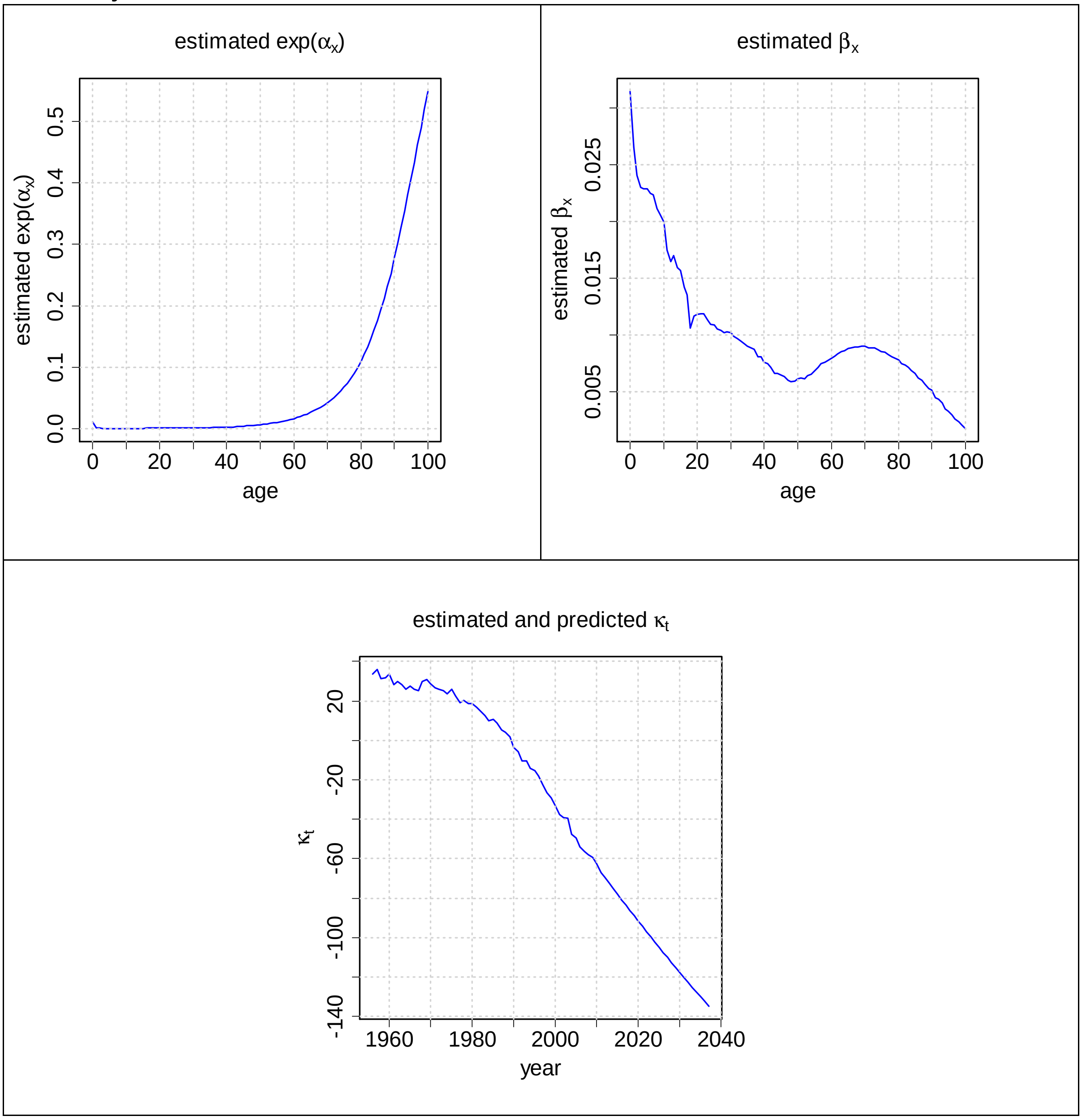
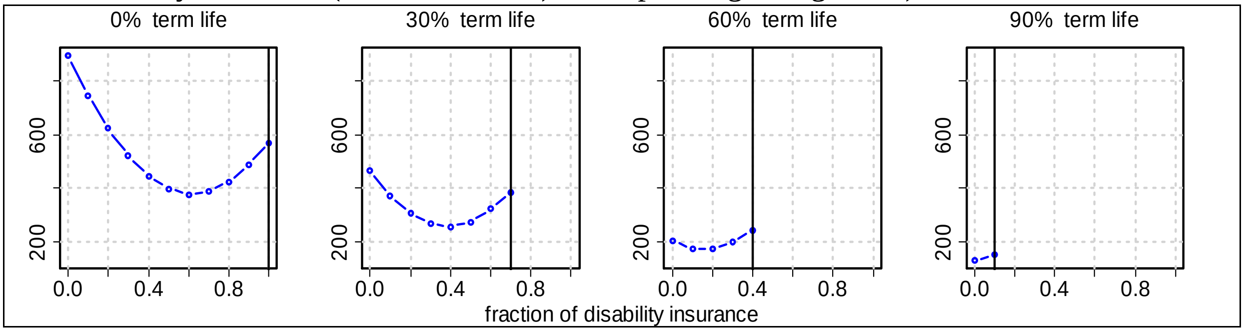


Figure A.2: Mean loss for different portfolio compositions consisting of annuities, term life, and disability insurance (No.1 in Table 3) corresponding to Figure 3a)



Notes: The vertical line in each plot represents the maximum portion of disability insurances in the considered portfolio.

Figure A.3: The impact of shocks to mortality on a life insurer's risk situation depending on the portfolio composition – mean loss (in thousand) (see also Figure 5)

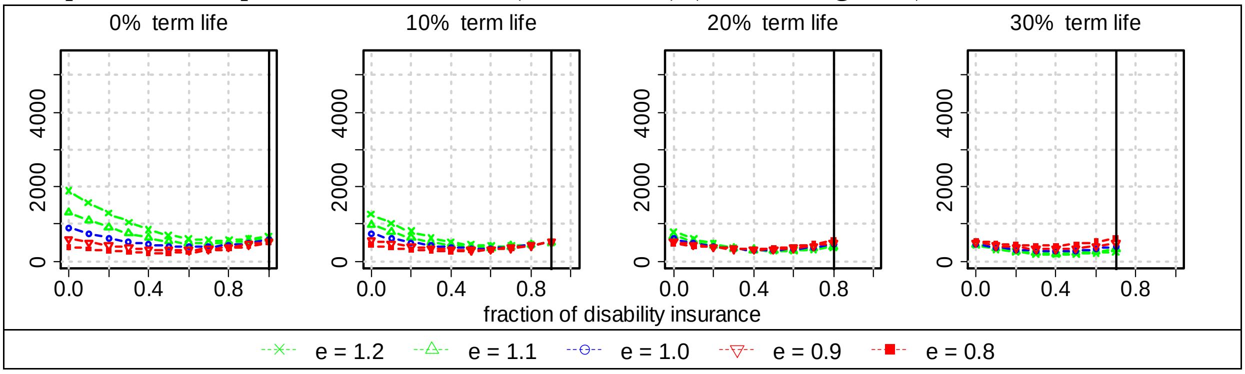


Figure A.4: The impact of changes of the transition probability from the active to the disabled state on a life insurer's risk situation – mean loss (in thousand) (see also Figure 6)