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ABSTRACT

The purchase of index-linked alternative risk transfer instruments can lead to basis risk, if the insurer's loss is not fully dependent on the index. One way to reduce basis risk is to additionally purchase gap insurance, which fills the gap between an insurer's actual loss and the index-linked instrument's payout. Previous literature detects gains in the effectiveness of this hedging strategy in a mean-variance framework. The aim of this paper is to extend this analysis and to examine the effectiveness of gap insurance in a shareholder value maximization framework under solvency constraints. Our results show that purchasing gap insurance can generally increase the hedging effectiveness in multiple ways by reducing basis risk, thus increasing shareholder value and, at the same time, lowering shortfall risk.

1. INTRODUCTION

The increasing number and magnitude of catastrophic events in recent years emphasized the potential stress on insurance and traditional reinsurance markets' capacities. To overcome these capacity constraints, alternative risk transfer (ART) instruments such as cat bonds, cat options or industry loss warranties (ILWs) have been introduced in the past decades. These instruments often feature a contract design that links their payoff to the development of an index. Thus, they come along with benefits such as higher transparency, lower transaction costs than, e.g., traditional reinsurance, and a reduction of moral hazard (see, e.g. Gatzert and Schmeiser, 2011). However, at the same time, basis risk can occur, as the insurer's exposure is usually not fully dependent on the index (see, e.g. Harrington and Niehaus, 1999; Zeng, 2000), thus implying that the index-linked product does not pay off, even though the buying insurer has a high loss. A potential strategy to overcome basis risk is to additionally purchase so-called gap insurance, thus filling the gap between an insurer's actual loss and the index-linked instrument's payout. In previous work, Doherty and Richter (2002) demonstrate potential gains in the effectiveness in a mean-variance framework if the index-based hedge is replenished through a fraction of an indemnity-based instrument. The aim of this paper is to

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take this analysis further and to analyze whether gains in the effectiveness from gap insurance can also be observed in a comprehensive shareholder value maximization framework under solvency constraints.

There has been steady growth in research on index-linked cat instruments. Even though ILWs were the first traded index-linked instruments in the 1980s (see SwissRe, 2009), the literature only began to focus on these instruments with the implementation of insurance futures based on catastrophic loss indices in 1992 by the Chicago Board of Trade. Early articles analyze the usage of these instruments (see, e.g. D'Arcy and France, 1992; Harrington, Mann and Niehaus, 1995) and discuss possible impediments for their success (see, e.g. Cox and Schwebach, 1992; Hoyt and Williams, 1995). The impact of basis risk is measured in several articles by means of the hedging effectiveness. Major (1999), for instance, determines an insurer's loss volatility reduction through a linear hedge in a simulation analysis, comparing attained volatilities through hedging strategies using statewide and zip-based indices. Harrington and Niehaus (1999) and Cummins, Lalonde and Phillips (2004) empirically analyze basis risk of insurance derivatives and find basis risk not to be a significant impediment for hedging strategies that are based on state-specific indices. However, they detect that using state-wide indices leads to substantial basis risk, especially for insurance companies whose underwriting business is not diversified across the country. An extension of existing basis risk definitions is introduced by Zeng (2000, 2003, 2005), who compares the hedging effectiveness of index-linked instruments to traditional reinsurance. Lee and Yu (2002) develop a model to price cat bonds, incorporating moral hazard and basis risk, while Gatzert, Schmeiser and Toplek (2011) simultaneously examine basis risk and pricing of ILWs based on various measures of basis risk and a comparison of different actuarial and financial pricing approaches. An analysis of contract features, pricing and central demand factors of ILWs is conducted in Gatzert and Schmeiser (2011), and a comparison of existing basis risk definitions and the impact of non-linear dependencies on basis risk in the context of ILWs are studied in Gatzert and Kellner (2011).

Besides the examination of basis risk, interaction effects between traditional reinsurance and ART instruments are analyzed in several articles. Doherty and Richter (2002) examine a hedging strategy that combines an index-linked instrument and indemnity-based (gap) reinsurance and find potential gains in the effectiveness in a mean-variance framework, while Nell and Richter (2004) analyze interactions between reinsurance and cat bonds in an expected utility approach, thereby identifying substitution effects between cat bonds and the demand for reinsurance for large losses. Lee and Yu (2007) study how the value of a reinsurer's contract can be increased by means of issuing cat bonds, whereas Song and Cummins (2008) observe substitution effects between derivative hedging and traditional reinsurance. One example of a transaction that combines a reinsurance structure (i.e. comparable to gap insurance) and a parametric cat bond is a transaction sponsored by the Mexican Government, which is examined by Härdle and López-Cabrera (2010), who find that the coverage of this transaction is received for lower costs and lower exposure at default than a "pure" reinsurance transaction.

The impact of risk management on shareholder value maximization has also been considered in previous literature. Yow and Sherris (2008), for instance, study an insurer's optimal capital and structure with respect to maximum net shareholder value in the presence of frictional costs and policyholders' risk sensitivity with respect to the insurer's solvency situation and pricing policy. In their analysis, the risk management strategy depends on holding more or less economic capital and their results show a tradeoff between costs for economic capital and improving the insurance company's solvency situation. Furthermore, Krvavych and Sherris (2006) examine the demand for a change-loss reinsurance contract and risk capital provided by shareholders when aiming to maximize shareholder value in the presence of frictional costs and under a solvency constraint. They thereby distinguish between the two cases that reinsurance can or cannot be used to reduce capital requirements. Their results show that if reinsurance is not taken into account to reduce capital requirements, risk management does only increase shareholder value in the presence of frictional costs. Otherwise, reinsurance is purchased if it can be used to decrease capital requirements, whereby a tradeoff between improving the solvency situation and higher costs of risk management exists. Hence, in both works, focus was neither laid on the impact of basis risk associated with index-linked instruments and its impact on shareholder value nor on solutions using gap insurance, which is the main focus of the present analysis.

In addition, while the basis risk associated with index-linked contracts has been studied before, the effectiveness of combined risk management strategies consisting of an index-linked instrument and gap insurance (to account for basis risk) has only partly been focused in the literature to date. Thus, we extend previous literature in the following way. In contrast to, e.g., Doherty and Richter (2002), we focus on a shareholder value maximization setting under solvency constraints and include a reaction of the policyholder's willingness to pay with respect to the insurer's solvency situation. We then examine whether an optimal risk management strategy consisting of gap insurance and an index-linked instrument can increase the net shareholder value (and reduce shortfall risk) as compared to a hedging strategy that only includes an index-linked instrument without gap insurance. We thereby assess different types of gap insurance structures and show how they differ in their effectiveness. The optimization problem is solved using differential evolution. Our results show that an optimal use of gap insurance improves the effectiveness of the hedging strategy compared to the case of a hedge with an index-linked instrument only. In particular, by means of gap insurance, the insurer's solvency situation can be improved and basis risk can be lowered while, at the same time, the maximum net shareholder value can be increased.

The remainder of this paper is structured as follows. In Section 2, the model framework is presented, including the model of a non-life insurer and the definition of basis risk. Section 3 contains numerical analyses and Section 4 concludes.

2. MODEL FRAMEWORK OF A NON-LIFE INSURER

Modeling the asset side and the relevant risk management instruments

We consider a given time horizon T inside a dynamic setting, where at time t = 0, the insurer receives premiums π^{S_T} paid by policyholders for stochastic insured losses S_T at time T, and shareholders make an initial contribution E_0. In addition, the insurer can choose to purchase among five types of risk management instruments at time t = 0 to hedge against losses at time T: A binary industry loss warranty contract ('*ILW*'), a proportional reinsurance contract ('*re*'), an aggregate excess of loss reinsurance contract ('*XL'*) and two indemnity-based gap insurance instruments ('*gap*','*gap-XL'*) with resulting payoffs X_T^i, i = *ILW*, *re*, *XL*, *gap*, *gap-XL* at time T. Each of the risk management instruments' contract parameters are fixed at time t = 0 and cannot be adjusted during the contract term (i.e. we consider one period). Furthermore, at time T, entrepreneurial activities are closed down and the remaining funds are distributed to shareholders.

The payoff of the binary ILW contract X_T^{ILW} only depends on the development of an industry loss index I_T at time T and pays a fraction α (determined by the insurer) of the ILW's limit L^{ILW} if the index exceeds a certain trigger level Y:

$$X_T^{ILW}(\alpha) = \alpha \cdot L^{ILW} \cdot 1\{I_T > Y\},$$

where $1\{I_T > Y\}$ represents the indicator function, which is equal to 1 if the industry loss I_T at time T is greater than the trigger Y and 0 otherwise.[1] An insurer purchasing solely an ILW for risk management faces basis risk, in that the industry loss index may not be triggered, even though the insurer's losses S_T exceed a critical loss level (see, e.g. Zeng, 2000). To reduce basis risk, the hedging strategy can be extended such that the ILW is combined with an instrument whose payment is independent of the index's development. In this context, we analyze four different indemnity-based instruments that can be combined with the ILW. Doherty and Richter (2002) show that purchasing proportional gap insurance ('*gap*'), which pays a fraction β (determined by the insurer) of the difference between the insurer's actual losses S_T and the ILW's payment[2] X_T^{ILW}

$$X_T^{gap}(\beta) = \max\left\{\beta \cdot \left(S_T - X_T^{ILW}(\alpha)\right), 0\right\} = \max\left\{\beta \cdot \left(S_T - \alpha \cdot L^{ILW} \cdot 1\{I_T > Y\}\right), 0\right\},$$

can lead to gains in the effectiveness of the hedging strategy. The payment structure within the maximum operator $\left(\beta \cdot \left(S_T - X_T^{ILW}(\alpha)\right)\right)$ can be constructed by a swap contract between the insurer and a reinsurance company, which exchanges a fraction β of an ILW's payment with a proportional reinsurance contract, paying $\beta \cdot S_T$. Thus, the swap also induces the possibility that the insurer has to make a payment to the reinsurer that exceeds the received payment. As the analysis is intended to examine the impact of exclusively reinsuring the gap between an insurer's loss and the index-linked instrument, the payment structure is limited through the maximum operator. However, apart from the index-linked instrument's payment part, the structure of proportional gap insurance is similar to a proportional reinsurance contract ('*re*'), which pays a fraction λ of the insurance company's losses

$$X_T^{re}(\lambda) = \lambda \cdot S_T.$$

Thus, the proportional reinsurance contract can be considered as a natural benchmark for the proportional gap insurance contract. Nevertheless, the basic idea of gap insurance can be adapted to further reinsurance-like contract structures of different risk management instruments. Both, the proportional reinsurance and the proportional gap insurance contract, provide coverage over the entire range of losses, which might be a disadvantage for contracts that reinsure losses above a certain threshold, as coverage for low losses might increase costs for

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[1] Note that other indices, e.g. a parametric index, could be used to structure the ILW. While parametric indices tend to exhibit a higher degree of standardization to investors, the associated basis risk is usually also higher (see SwissRe, 2009).

[2] As an alternative to this illustration, the ILW's limit can be adjusted to the desired level.

risk management, but do little improve the insurer's solvency situation. Thus, we further consider an aggregate excess of loss reinsurance contract (‘*XL*’), whose payment is given through

$$X_T^{XL}(\omega) = \omega \cdot \min\left(\max\left(S_T - A^{XL}, 0\right), L^{XL}\right),$$

where ω denotes the fraction that is purchased, A^{XL} the attachment of the company loss and L^{XL} the contract’s layer limit. Similar to the case of proportional reinsurance, the aggregate excess of loss reinsurance contract represents a natural benchmark for a corresponding excess of loss gap insurance contract (‘*gap-XL*’), which is structured according to

$$\begin{aligned} X_T^{Gap-XL}(\vartheta) &= \max\left\{\vartheta \cdot \min\left(\max\left(S_T - A^{gap-XL} - X_T^{ILW}, 0\right), L^{gap-XL}\right), 0\right\} = \\ &= \max\left\{\vartheta \cdot \min\left(\max\left(S_T - A^{gap-XL} - \alpha \cdot L^{ILW} \cdot 1\left\{I_T > Y\right\}, 0\right), L^{gap-XL}\right), 0\right\}, \end{aligned}$$

where ϑ denotes the fraction that is purchased, A^{gap-XL} the attachment of the company loss and L^{gap-XL} the contract’s layer limit. Summing up, we consider five possible hedging strategies in the model framework as displayed in Table 1.

Table 1: Analyzed hedging strategies

No.	*Hedging strategy*	*Hedge parameters*	*Payoff*
HS^{I}	ILW only	$\alpha \geq 0, \beta = 0, \lambda = 0, \omega = 0, \vartheta = 0$	$X_T^{ILW}(\alpha)$
HS^{II}	ILW + proportional reinsurance	$\alpha \geq 0, \beta = 0, \lambda \geq 0, \omega = 0, \vartheta = 0$	$X_T^{ILW}(\alpha) + X_T^{re}(\lambda)$
HS^{III}	ILW + proportional gap insurance	$\alpha \geq 0, \beta \geq 0, \lambda = 0, \omega = 0, \vartheta = 0$	$X_T^{ILW}(\alpha) + X_T^{gap}(\beta)$
HS^{IV}	ILW + excess of loss reinsurance	$\alpha \geq 0, \beta = 0, \lambda = 0, \omega \geq 0, \vartheta = 0$	$X_T^{ILW}(\alpha) + X_T^{XL}(\omega)$
HS^{V}	ILW + excess of loss gap insurance	$\alpha \geq 0, \beta = 0, \lambda = 0, \omega = 0, \vartheta \geq 0$	$X_T^{ILW}(\alpha) + X_T^{gap-XL}(\vartheta)$

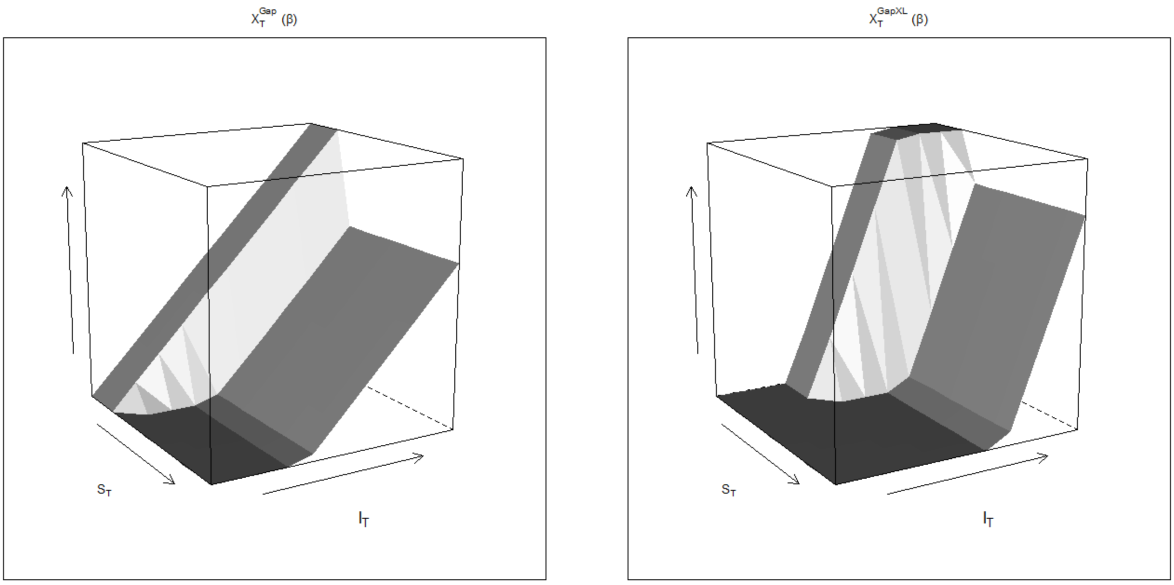
All hedging instruments are considered to be asset hedges (see Doherty, 2000) with respect to financial accounting. The ILW’s design equals a derivative structure, as no indemnity[3] trigger is involved in the transaction (see SwissRe, 2009), and the gap insurance contracts feature a reinsurance-like structure. Accordingly, all five instruments (ILW, proportional reinsurance, proportional gap insurance, excess of loss reinsurance and excess of loss gap insurance) are accounted for on the asset side. By combining an ILW and an indemnity-based instrument ($HS^{II} - HS^{V}$), advantages and drawbacks of the individual hedging instruments can be coun-

[3] ILWs can also be structured as double-trigger contracts, such that besides the industry trigger, an indemnity-based trigger is included in the contract design (see, e.g. SwissRe, 2006; Gatzert and Schmeiser, 2011).

terbalanced and offset to some extent. In particular, indemnity-based contracts increase the protection seller's costs to control for moral hazard (see Doherty and Richter, 2002), while the index-based ILW exhibits a high degree of standardization achieved through the use of an index and implies a less complex underwriting process that is less costly (see Gatzert and Schmeiser, 2011), but in turn faces basis risk.

Note that while transactions that combine traditional reinsurance and index-linked instruments already exist,[4] gap insurance contracts typically depend on the buyer's individual situation and the respective index-linked instrument and may thus require an individual contract design, which are hardly published. Figure 1 exemplarily illustrates the coverage provided by proportional and excess of loss gap insurance for a given level of β and ϑ, respectively.[5] It can be seen that the payment of each contract can be represented as a function of the insurer's loss S_1 and the index value I_1. This and the similarity to traditional reinsurance contracts shows that reinsurance coverage as given through gap insurance should be available for purchase, especially as the development of alternative risk transfer instruments made reinsurers more flexible in providing new coverages and contract designs (see Doherty and Richter, 2002).

Figure 1: Coverage provided through proportional gap insurance (left side) and excess of loss gap insurance (right side)



[4] In 2006, for instance, Mexico's Fund for Natural Disasters entered into an insurance contract with European Finance Reinsurance, covering a total volume of 450 Mio.$ in which 160 Mio.$ were issued in a cat bond transaction based on a parametric index (see Härdle and López-Cabrera, 2010).

[5] Note that changes in β and ϑ impact the steepness of the coverage surface, but not it's piecewise linear progression.

The premiums for these instruments are paid at time t = 0 and consist of the discounted expected payment under the risk-neutral pricing measure Q and an additional loading, i.e. $\pi^i = E^Q\left(e^{-r_f \cdot T} \cdot X_T^i\right) \cdot \left(1+\delta^i\right)$, i = *ILW*, *re*, *XL*, *gap*, *gap-XL*, where r_f stands for the risk-free interest rate. Depending on the concrete hedging strategy and the composition of the hedging instruments, the total initial capital thus sums up to

$$A_0 = E_0 + \pi^{S_T} - \sum_{i \in \{ILW, re, gap, XL, gap-XL\}} \pi^i , \tag{1}$$

where E_0 represents shareholders' initial contribution, π^{S_T} are the premiums paid by the policyholders for insured claims S_T, and $\pi^i = 0$ if $\alpha = 0$ (i = *ILW*), $\lambda = 0$ (i = *re*), $\omega = 0$ (i = *XL*), $\beta = 0$ (i = *gap*) or $\vartheta = 0$ (i = *gap-XL*), respectively. A fraction γ of the initial capital is invested in a risky asset $A_{0,high} = \gamma \cdot A_0$, and the remaining part $(1-\gamma)$ is invested risk-free $A_{0,risk-free} = (1-\gamma) \cdot A_0$. Concerning the fraction invested in the risky asset, the Heston (1993) model is used, given by

$$dA_{t,high} = A_{t,high}\left(\mu_{high} dt + \sqrt{V_t} dW^P_{A_{t,high}}\right),$$

where μ_{high} denotes the drift of the risky assets' process. The instantaneous variance V_t follows the square-root process (see Cox, Ingersoll and Ross, 1985) and is described by

$$dV_t = \kappa(\theta - V_t) dt + \sigma_{V_t} \sqrt{V_t} dW^P_{V_t} ,$$

which reverts to the long-term variance θ with a speed of mean reversion κ, where σ_{V_t} is the variance's volatility and $W^P_{A_{t,high}}$, $W^P_{V_t}$ are standard P-Brownian motions on a probability space $(\Omega, \mathcal{F}, P)$ with $dW^P_{A_{t,high}} dW^P_{V_t} = \rho_{A_{t,high},V_t} dt$, $\rho_{A_{t,high},V_t}$ denoting the linear coefficient of correlation, and where P represents the objective real-world measure. Under the risk-neutral pricing measure Q, the risky asset process is given through

$$dA_{t,high} = A_{t,high}\left(r_f dt + \sqrt{V_t} dW^Q_{A_{t,high}}\right),$$

$$dV_t = \tilde{\kappa}\left(\tilde{\theta} - V_t\right) dt + \sigma_{V_t} \sqrt{V_t} dW^Q_{V_t} ,$$

and $\tilde{\kappa} = \kappa + \lambda_{V_t} \cdot \sigma_{V_t}$, $\tilde{\theta} = \kappa \cdot \tilde{\theta} / \tilde{\kappa}$, λ_{V_t} denoting the market price of risk for the volatility process, and $W^Q_{A_{t,high}}$, $W^Q_{V_t}$ standard Q-Brownian motions. The asset portfolio A_T at time T depends on the chosen hedging strategy and is determined by

$$A_T = \gamma \cdot A_{T,high} + (1-\gamma) \cdot A_{T,risk-free} + \sum_{i \in \{ILW,re,gap,XL,gap-XL\}} X_T^i ,$$

with $i = ILW, re, gap, XL, gap - XL$ and $X_T^i = 0$ if α = 0 (*ILW*), λ = 0 (*re*), β = 0 (*gap*), ω = 0 (*i* = *XL*) or ϑ = 0 (*i* = *gap*-*XL*) respectively. The value of the assets A_0^V at time *t* = 0 is thus given by calculating the expected value under the risk-neutral measure *Q* and discounting to zero,

$$A_0^V = E^Q\left(e^{-r_f \cdot T} \cdot A_T\right) = A_0 + \sum_{i \in \{ILW,re,gap,XL,gap-XL\}} E^Q\left(e^{-r_f \cdot T} \cdot X_T^i\right), \tag{2}$$

which thus depends on the type of premium calculation for the risk management instruments (see Equation (1)).

Modeling the liability side

The policyholders' claims S_T and industry losses I_T are assumed to follow a geometric Brownian motion, such that

$$dS_t = \mu_S S_t \, dt + \sigma_S S_t \, dW_{S_t}^P ,$$

and

$$dI_t = \mu_I I_t \, dt + \sigma_I I_t \, dW_{I_t}^P ,$$

with drift μ_i , standard deviation σ_i , and $W_{i_t}^P$ denoting a standard *P*-Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$ with filtration $\mathcal{F}$, i = *S*, *I*. The solutions of these stochastic differential equations under the real-world measure *P* for *t* = *T* are given by (see, e.g. Björk, 2009)

$$I_T = I_0 \cdot e^{\left(\mu_I - 0.5 \cdot \sigma_I^2\right) \cdot T + \sigma_I \cdot W_{I_T}^P} ,$$

and

$$S_T = S_0 \cdot e^{\left(\mu_S - 0.5 \cdot \sigma_S^2\right) \cdot T + \sigma_S \cdot W_{S_T}^P} ,$$

thus implying a lognormal distribution for losses at time T. By changing the probability measure to the risk-neutral measure Q, the stochastic processes of the company loss and the industry loss at time T are given by

$$I_T = I_0 \cdot e^{\left(r_f - 0.5 \cdot \sigma_I^2\right) \cdot T + \sigma_I \cdot W_{I_T}^P}$$

and

$$S_T = S_0 \cdot e^{\left(r_f - 0.5 \cdot \sigma_S^2\right) \cdot T + \sigma_S \cdot W_{S_T}^P} ,$$

respectively, where $W_{S_T}^Q$ and $W_{I_T}^Q$ are standard Q-Brownian motions.[6] The geometric Brownian motion is used for two reasons. First, it can be used to describe the development of the loss estimate between time $t = 0$ and T (see, e.g. Braun, 2011; Loubergé, Kellezi and Gilli (1999); Cummins and Sommer, 1996; Cummins and Geman, 1994). Second, the resulting lognormal distribution of the loss estimate is in line with empirical findings of Burnecki, Kukla and Weron (2000) for the PCS index in the United States and further allows an easier analysis in an otherwise complex setting (see, e.g. Cummins and Geman, 1994). The type and degree of dependence between the index and the insurer's losses is thereby assumed to be constant over time, which allows isolating the impact of different types and degrees of dependence on shareholder value and shortfall risk. Alternatively, dynamic dependence models can be taken into account (see, e.g. Hafner and Manner, 2012; Patton, 2006), which, however, do not allow an isolated assessment of the type and degree of dependence (as in case of, e.g., autoregressive process).

If the value of assets A_T at time T is not sufficient to cover the policyholders' claims, i.e. if $S_T > A_T$, the insurer becomes insolvent. Due to the shareholders' limited liability, no additional equity capital is provided at time T, and, thus, only the remaining funds are distributed to the policyholders. Hereby, one has to take into account that market frictions such as agency costs, taxation or costs of financial distress (see, e.g. Yow and Sherris, 2008; Krvavych and Sherris, 2006) can play an important role for a firm's financing policy including decisions for risk management (see, e.g. Modigliani and Miller, 1958; Mayers and Smith, 1982). In the present analysis, we thus incorporate costs of financial distress,[7] which can occur due to, e.g.

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[6] Even though the insurer's losses are non-tradable and frictional costs, which are introduced in this section, exist, we assume the market to be arbitrage-free, such that the risk-neutrale Q exists but may not be unique (see Yow and Sherris, 2008).

[7] Further analyses regarding taxation and agency costs in a shareholder value maximization setting can be found in Yow and Sherris (2008) and Krvavych and Sherris (2006).

bankruptcy costs in case of insolvency and are thus defined as a fraction τ_{FD} of losses in case of default at time *T*, $\tau_{FD} \cdot \max\left(S_T - A_T, 0\right)$ (see Yow and Sherris, 2008).

Hence, the value of policyholders' payoff V_T is determined by (see, e.g. Yow and Sherris, 2008; Zanjani, 2002)

$$V_T = \min\left(A_T - \tau_{FD} \cdot \max\left(S_T - A_T, 0\right), S_T\right) = S_T - \max\left(S_T - A_T, 0\right) - \tau_{FD} \cdot \max\left(S_T - A_T, 0\right) = \\ = S_T - \left(1 + \tau_{FD}\right) \cdot \max\left(S_T - A_T, 0\right),$$

which is composed of the policyholders' claims less the so-called default put option, which is increased through bankruptcy costs and represents the loss in case of insolvency. If the insurance company is solvent, the remaining surplus, i.e., the difference between assets and liabilities, is paid out to the shareholders, and their payoff E_T at time *T* residually given by

$$E_T = \max\left(A_T - S_T, 0\right) = A_T - S_T + \max\left(S_T - A_T, 0\right).$$

Hence, the net present value of the shareholder's investment (net shareholder value SHV_0) under the risk-neutral measure *Q* is given by (see, e.g. Zimmer, Gründl and Schade, 2009)[8]

$$SHV_0 = E^Q\left(e^{-r_f \cdot T} \cdot E_T\right) - E_0. \tag{3}$$

Determination of premiums, loadings, and policyholders' willingness to pay

To determine the insurer's premium income, we adapt and adjust a procedure used in Gründl, Post and Schulze (2006). We thereby assume that the insurer's premium income consists of the value of payments to policyholders at time *t* = 0, V_0, where an additional loading δ^{S_T} is imposed that takes into account the policyholders' risk aversion. V_0 is derived using risk-neutral valuation by calculating the discounted expected value under the risk-neutral measure *Q*

$$V_0 = E^Q\left(e^{-r_f \cdot T} \cdot V_T\right) = E^Q\left(e^{-r_f \cdot T} \cdot S_T\right) - E^Q\left(e^{-r_f \cdot T} \cdot \left(1 + \tau_{FD}\right) \cdot \max\left(S_T - A_T, 0\right)\right) = S_0 - \left(1 + \tau_{FD}\right) \cdot DPO,$$

where $DPO = E^Q\left(e^{-r_f \cdot T} \cdot \max\left(S_T - A_T, 0\right)\right)$, which depends on the actual premium payment by the policyholders. Furthermore, we assume that besides amortizing costs for moral hazard

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[8] Note that in a financially fair situation without premium loadings and without assumptions on policyholders' willingness to pay, SHV_0 would be equal to zero.

and transaction costs, the insurer additionally aims to increase the loading to enhance its premium income from insuring losses S_T. As the insurance company is not in a monopolistic market position, it cannot fix the loading arbitrarily. Instead, we assume that the loading that policyholders are willing to pay is endogenous, and, in the presence of policyholders' risk aversion, depends on the insurer's shortfall probability, which is calculated under the real-world measure P (see, e.g. Gründl, Post and Schulze, 2006; Zimmer, Gründl and Schade, 2009) through

$$SP_T = P\left(A_T < S_T\right).$$

This assumption is based on experimental results by Wakker, Thaler and Tversky (1997) and Zimmer, Gründl and Schade (2009), who find that people demand premium reductions for insurance with default risk, such that the willingness to accept higher loadings on the premium decreases with an increase in the insurer's shortfall probability (see also Gründl, Post and Schulze, 2006; Yow and Sherris, 2008).[9] Thus, the loading δ^{S_T} reflects the policyholders' willingness to pay and is modeled by

$$\delta^{S_T} = \left(1 - q \cdot SP_T\right) \cdot \delta^{\max,S_T},$$

where $\delta^{\max,S_T}$ represents the maximum loading that policyholders would accept in the case of an insurer without default risk and q[10] stands for the policyholders' sensitivity towards insolvency risk.[11] Thus, the insurer's premium income π^{S_T} is given by[12]

$$\pi^{S_T} = V_0 \cdot \left(1 + \delta^{S_T}\right) = \left(S_0 - \left(1 + \tau_{FD}\right) \cdot DPO\right) \cdot \left(1 + \delta^{S_T}\right),$$

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9 Furthermore, similar results can be found in an empirical analysis by Epermanis and Harrington (2006) who detect that rating downgrades come along with premium declines in the year of the downgrade and the following years after the downgrade.

10 Policyholders' sensitivity towards insolvency risk might change over time. Here, we assume a homogenous group of policyholders purchasing insurance at the initial time $t = 0$. Moreover, the impact of different degrees of risk sensitivity is analyzed in more depth in Section 3.

11 This approach is similar to the one used in Gründl, Post and Schulze (2006), who model the quantity of policyholders demanding insurance as a decreasing function of the insurer's shortfall probability. Note that this assumes that the insurer can fully observe the policyholders' risk sensitivity when determining the premium. If this is not the case (e.g. if it is revealed ex post), the solution to the optimization problem might be suboptimal.

12 Note that the purchased amount of risk management at $t = 0$ affects the value of the default put option and the shortfall probability at $t = T$, which in turn influences the willingness to pay at $t = 0$. Due to this interaction, the fractions α, β, λ, ω and ϑ have to be adjusted such that the "promised" shortfall probability, used to determine the policyholder's willingness to pay at $t = 0$ and influenced by the policyholders' premiums, equals the shortfall probability actually realized at $t = T$.

where V_0 can be considered as the "fair basic premium", which is increased by a loading that is determined under the real-world measure P. Hence, actions taken by the insurer to improve its solvency situation generally have two positive effects on its premium income: A reduction in the shortfall probability decreases the default put option value, which increases V_0 and, in addition, raises policyholders' willingness to pay by means of δ^{S_T}.

Premiums for the risk management instruments consist of their discounted payment's expected value under the risk-neutral measure Q and loadings $\delta^i \ (\geq 0)$, *i = ILW, re, XL, gap, gap-XL*, representing costs usually associated with these instruments. In general, the loading for indemnity-based contracts can be assumed to be higher than the ILW's loading, since the binary ILW exhibits a high degree of standardization and is not exposed to moral hazard. Contrariwise, indemnity-based transactions demand higher expenses to monitor an insurer's business operations and to control for moral hazard. We thus take into account that the loading for the indemnity-based risk management contracts (*i = re, XL, gap, gap-XL*) should reflect the costs associated with moral hazard, which tend to increase for higher rates of (re-) insurance (i.e. the higher the portion of the insurer's loss that is reinsured, see Doherty and Richter, 2002) and is determined by

$$\delta^i = \max\left\{ v^l \cdot \left(\frac{E\left(X_T^i\right)}{E\left(S_T\right)} \right)^k, \delta^{\min} \right\}, \ i = re, XL, gap, gap - XL, \ l = 0,1, \ v, k \in \mathbb{R}. \tag{4}$$

Here, the rate of reinsurance is given by the proportion of the expected payment from the respective risk management instrument to the expected loss of the insurer (both under the objective measure P). While the shape of the cost function δ^i can be controlled through k, the steepness can be adapted through v and l, where $\delta^{\min}$ represents a minimum loading, which we assume to be higher or equal to the loading of the industry loss warranty contract. Thus, the corresponding prices π^i of the risk management instruments *i = re, XL, gap, gap-XL* result from

$$\pi^i = E^Q\left(e^{-r_f \cdot T} \cdot X_T^i\right) \cdot \left(1 + \delta^i\right), \tag{5}$$

where a constant loading for the ILW contract $\delta^{ILW} \leq \delta^{\min}$ is assumed.

Optimization problem

The insurer's objective in risk management is to create value through entrepreneurial activities.[13] In the present setting, we consider the net shareholder value as the relevant value as illustrated in Equation (3), which can be reformulated by replacing the values for assets A_0^V (see Equation (2)), policyholders' claims S_0 and the default put option DPO, as follows:

$$\begin{aligned}
SHV_0 &= E^Q\left(e^{-r_f \cdot T} \cdot E_T\right) - E_0 \\
&= e^{-r_f \cdot T} \cdot E^Q\left(A_T - S_T + \max\left(S_T - A_T, 0\right)\right) - E_0 \\
&= A_0^V - S_0 + DPO - E_0 \\
&\overset{(2)}{=} \left(A_0 + \sum_{i \in \{ILW, gap, re\}} E^Q\left(e^{-r_f \cdot T} \cdot X_T^i\right)\right) - S_0 + DPO - E_0 \\
&\overset{(1)}{=} E_0 + \left(S_0 - \left(1 + \tau_{FD}\right) \cdot DPO\right) \cdot \left(1 + \delta^{S_T}\right) \\
&\quad - \sum_{i \in \{ILW, re, gap, XL, gap-XL\}} \pi^i + \sum_{i \in \{ILW, re, gap, XL, gap-XL\}} E^Q\left(e^{-r_f \cdot T} \cdot X_T^i\right) - S_0 + DPO - E_0 \\
&\overset{(5)}{=} \left(S_0 - DPO\right) \cdot \delta^{S_T} - \tau_{FD} \cdot DPO \cdot \left(1 + \delta^{S_T}\right) - \sum_{i \in \{ILW, re, gap, XL, gap-XL\}} \delta^i \cdot \left(E^Q\left(e^{-r_f \cdot T} \cdot X_T^i\right)\right)
\end{aligned} \tag{6}$$

Hence, in order to increase the net shareholder value, the tradeoff between increasing the safety level (to reduce the DPO and thus the impact for costs of financial distress, and to increase the policyholders' willingness to pay reflected in the loading) by purchasing risk management instruments and the premium payments for risk management (which reduce the SHV_0) must be addressed.

The derivation of the optimal hedging strategy is highly endogenous due to the interrelation between the insurer's premium income at time $t = 0$ and the shortfall probability at time T. Purchased fractions α, β, λ, ω, and ϑ of the risk management instruments at time $t = 0$ influence assets and liabilities at time T and thus also the shortfall probability differs, which has an impact on the premium income at time $t = 0$. The premium income in turn at the same time determines the available capital for risk management and the investment opportunities at time $t = 0$. In our analysis, this problem is solved by a root-searching algorithm. In a real-world setting, the procedure can be interpreted that an insurer "chooses" a shortfall probability and given a certain amount of risk management activities, the shortfall probability is then

[13] For a detailed discussion on an insurer's rationale for hedging, see, e.g. Cummins and Song (2008).

communicated to policyholders through a rating agency (see also Gründl, Post and Schulze, 2006 for a similar argumentation).[14]

Hence, the higher the insurer's rating and thus safety level is, the higher are the product prices that are accepted by policyholders. Accordingly, the insurer's objective in this setting is to maximize the net shareholder value as displayed in Equation (6) through purchasing optimal fractions $\alpha, \beta, \lambda, \omega$, and ϑ of risk management instruments (see Table 1 for the hedging strategies under consideration)

$$SHV_0^{\max} = \max_{\substack{\alpha,\beta,\lambda,\omega,\vartheta \\ A^{XL}, A^{gap-XL}}} \left\{ \left(S_0 - DPO\right) \cdot \delta^{S_T} - \tau_{FD} \cdot DPO \cdot \left(1 + \delta^{S_T}\right) - \sum_{i \in \{ILW, re, gap, XL, gap-XL\}} \delta^i \cdot \left(E^Q \left(e^{-r_f \cdot T} \cdot X_T^i \right) \right) \right\} \tag{7}$$

subject to the constraint[15] c that the insurer's shortfall probability is not allowed to exceed a predefined level $\overline{SP_T}$, that the fractions $\alpha, \beta, \lambda, \omega$, and ϑ are in the range between zero and one, and that the premium payment is consistent with the shortfall probability implied by the risk management strategy,

$$c = \begin{pmatrix} SP_T \le \overline{SP_T} \\ 0 \le i \le 1 \\ \left(S_t - DPO\right) \cdot \left(1 + \delta^{S_T}\right) \end{pmatrix}, \; i = \alpha, \beta, \lambda, \omega, \vartheta.$$ [16]

Note that different types of risk measures could be included in regard to the policyholders' demand sensitivity and in the constaints of the optimization problem, which impacts the optimal risk management strategies. In this context, it is crucial which part of the insurer's surplus distribution is taken into account by the risk measure. The shortfall probability solely considers whether the surplus is below or above zero, while in case of the expected loss, for instance, the extent of the shortfall is also taken into account. When using the latter risk measure, the optimal attachment points would be lower in case of the excess of loss reinsurance

[14] In an empirical analysis for U.S. property-liability insurers, Cummins, Lin and Phillips (2006) detect an inverse relationship between insurance prices and insolvency risk, measured through A. M. Best's financial ratings.

[15] The number of constraints, which are involved in the optimization, can be optionally expanded. In general, the constraints can, e.g. constitute regulatory requirements an insurer might need to fulfill, such as a certain level of solvency or mandatory restrictions set by law.

[16] See Krvavych (2007) for further possibilities to formulate the shareholder value maximization problem. Krvavych (2007) shows that maximizing shareholder value under solvency constraints can be equivalent to maximize shareholder value using an isoelastic utility function.

and excess of loss gap insurance contracts, whereas fractions of proportional reinsurance and proportional gap insurance would be higher.

For the optimization problem displayed in Equation (7), differential evolution (DE) is applied. DE is a parallel stochastic direct search method, which belongs to the family of evolutionary algorithms introduced by Storn and Price (1997). To optimize an objective function, DE starts with a number *NP* of D-dimensional vectors, called the population, which are randomly chosen on a uniformly distributed probability space. After the initialization of the first generation *g*, DE creates trial vectors, representing the intermediary populations, through mutating and crossing over the initial vectors. To create the next generation, initial vectors are replaced through trial vectors if a trial vector leads to an equal or better result than an initial vector (see Storn and Price, 1997). After this selection, the procedure is repeated until a stopping condition, e.g. a maximum number of generations is reached or if the best value of the objective function could not be improved for a specified number of generations. DE offers advantages compared to traditional optimization methods (see Price, Storn and Lampinen, 2006) as, for instance, the simultaneous search for an optimal solution with multiple starting points reduces the probability to identify a false peak. Furthermore, DE does not rely on additional information such as derivatives to find the optimum. By virtue of these attributes, DE is capable to optimize non-smooth or non-linear functions.[17]

Basis risk

The crucial parameter for the effectiveness of hedging strategies including an index-linked instrument is basis risk. In the following analysis, basis risk is measured using the counter value of the hedging efficiency as employed by Zeng (2003), which is calculated based on the proportionate risk reduction of a predefined risk measure attainable through an index-based hedging strategy as compared to a benchmark for the index-hedge. In the following analysis, the shortfall probability is chosen as the relevant risk measure, which accounts for assets and liabilities. As a benchmark, an index-based hedge, assuming perfect dependence between the insurer's loss and the index, is defined so that the proportionate risk reduction RR^i for the hedging strategy i = *I, II, III, IV, V, perfect* (see Table 1) and the respective strategies under perfect dependence between index and insurer's loss (*perfect*), can be determined through

[17] In the numerical analysis, DE is implemented using the "DEoptim" package of the statistical software R (http://www.r-project.org). An explanation of the package is provided by Mullen et al. (2011).

$$RR^i = 1 - \frac{SP_T^i}{SP_T^{without}},$$

where SP_T^i denotes shortfall probabilities under the corresponding hedging strategy and $SP_T^{without}$ the shortfall probability if no hedging instrument is purchased. With these definitions, the counter value of the hedging efficiency CHE^i for the hedging strategies *i = I, II, III, IV, V* is given by (see Cummins, Lalonde and Phillips, 2004)

$$CHE^i = 1 - \frac{RR^i}{RR^{perfect}}.$$

3. Numerical Analysis

This section investigates the effectiveness of gap insurance to maximize net shareholder value under solvency constraints in the presence of basis risk, given that policyholders are sensitive with respect to an insurer's default. To quantify the impact of gap insurance, we compare results of net shareholder value, shortfall probability and basis risk for a hedging strategy consisting exclusively of an ILW ("ILW-hedge", HS^I), one combining an ILW and a proportional reinsurance contract ("ILW-reinsurance-hedge", HS^{II}), one with an ILW and a proportional gap insurance contract ("ILW-gap-hedge", HS^{III}), one with an ILW and an excess of loss reinsurance contract ("ILW-XL-hedge", HS^{VI}), and a hedging strategy consisting of an ILW and an excess of loss gap insurance contract ("ILW-gap-XL-hedge", HS^V) (see Table 1). The aim of this analysis is to study whether gap insurance can increase the maximum net shareholder value and/or improve an insurer's solvency situation. We thereby also investigate to what extent basis risk can be reduced through the usage of gap insurance. Sensitivity analyses are conducted concerning the degree and type of dependence between the insurer's loss and the index, different degrees of the insurer's loss volatility, the policyholder's risk sensitivity and the type of cost function with respect to the loading of indemnity-based risk management instruments (impacted by moral hazard) in order to identify key drivers for the effectiveness of gap insurance.

Input parameters

The input data for our reference contract are summarized in Table 2, where the expected value under the real-world probability measure *P*, the drift as well as the volatility of the company's loss are based on empirical data of a non-life insurer as presented in Eling, Gatzert and Schmeiser (2009). The corresponding values of the industry index are adopted from Gatzert, Schmeiser and Toplek (2011), referring to Hilti, Saunders and Lloyd-Hughes (2004). Moreover, the drift and the parameters for the volatility process of high-risk assets are based on data from the S&P 500. All other input parameters are chosen for illustration purposes and were subject to robustness tests to ensure the stability of our general findings.

Table 2: Input parameters for the reference contract

Available equity capital at time 0	E_t	\$58 million
Expected value of the company loss	$E(S_T)$	\$117 million
Expected value of the industry index	$E(I_T)$	\$1450 million
Drift and volatility of the company loss	μ_{S_1}, σ_{S_1}	0.025, 0.53
Drift and volatility of the industry index	μ_{I_1}, σ_{I_1}	0.025, 1.39
Drift of high-risk assets	$\mu_{A_{1,high}}$	0.0729
Long term variance of high-risk assets	θ	0.0482
Mean-reversion of high-risk assets	κ	2.50
Variance's volatility of high-risk assets	σ_V	0.3131
Market price of risk for the volatility process	λ_V	0.00
Correlation among high-risk assets processes	$\rho_{A_{t,high},V_t}$	-0.4235
Risk-free interest rate	r_f	2%
Investment in high-risk investment	γ	25%
Policyholder's risk sensitivity	q	5
Kendall's tau for company and index losses	$\rho_\tau(S_1, I_1)$	0.70
Premium loading for an insurer without default risk	$\delta^{\max,S_1}$	40%
Premium loading ILW	δ^{ILW}	5%
Premium minimum loading for indemnity-based contracts	$\delta^{\min}$	5%
Parameters for the cost function of indemnity- based risk management instruments δ_{ib}^i	k, l, v	1,1,0.50
Maximum shortfall probability	$\overline{SP_T}$	5%
Layer limit for ILW	L^{ILW}	\$200 million
Industry loss trigger	Y	\$2,000 million
Number of generations within differential evolution	g	200
Population size per generation within differential evolution	NP	150

Numerical results are based on Monte Carlo simulation with 250,000 sample paths. To further improve the simulation's stability, latin hypercube sampling and control variates as a variance reduction techniques are applied (see Glasserman, 2010).[18] Recent literature illustrates the relevance of an adequate treatment of dependence structures between processes when modeling insurance risks (see, e.g. Eling and Toplek, 2009; Gatzert and Kellner, 2011). We assume assets to be independent from liabilities in order to specifically focus on the dependence structure between the industry loss index and the insurance company's losses and as these dependencies are the main drivers in regard to the effectiveness of index-linked hedging strategies. To obtain a holistic picture of the impact of the dependence structure, we first vary the degree of dependence using Kendall's rank correlation ρ_τ, since this is invariant against (non-linear) transformations (see, e.g. McNeil, Frey and Embrechts, 2005), and additionally examine the impact of the type of dependence using the Clayton (lower tail dependencies), the Gauss (linear dependencies), and the Gumbel copula (upper tail dependencies), using the Gauss copula as the reference case.

The effectiveness of gap insurance under different degrees of dependence

Table 3 exhibits results for the five hedging strategies under consideration (see Table 1) for different degrees of dependence between the insurer's loss and the index. For all strategies, the optimal hedging shares $(\alpha, \beta, \lambda, \omega, \vartheta)$ are derived by means of differential evolution and are displayed in Table 3 along with the implied shortfall probability SP_T^i, the value of payments to policyholders at time 0 V_0, the corresponding loading δ^{S_1}, the total premium π^{S_T} that results from V_0 multiplied with the loading based on the policyholders' sensitivity towards shortfall risk. Furthermore, the loading for the indemnity-based instruments δ^i, $i = re, XL, gap, gap - XL,$ basis risk CHE^i and the maximum net shareholder value $SHV_0^{\max}$ are presented.

Table 3 shows that a fraction of the indemnity-based hedging instrument is always purchased in combination with the index-linked ILW contract in the present setting. The results further indicate that the inclusion of indemnity-based hedging instruments leads to a considerable reduction in the shortfall probability, while, at the same time, the maximum net shareholder value increases despite the higher costs for risk management. This behavior may be ascribed

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[18] Due to the application of differential evolution, the number of sample paths appeared to be crucial for the simulation's stability. Thus, we chose a sufficiently high number of sample paths and further implemented variance reduction techniques to achieve low sample standard deviations (e.g., for the reference contract, the sample standard deviation of the net shareholder value amounts to 0.0829) and ensured that the results remain stable for different sets of random numbers.

to the reduction of costs for financial distress and an increased total premium income for the insurer that comes along with a reduction in the shortfall probability, which leads to a higher value of V_0 and, in addition, to a higher policyholder's willingness to pay. When comparing the indemnity-based hedging strategies $\left(HE^{II} - HE^{V}\right)$, it can be seen that the contracts with an excess of loss structure (*XL* and *gap-XL*) dominate the contracts with a proportional structure (*re* and *gap*) in that they generally yield a higher maximum net shareholder value, lower values of basis risk and a lower shortfall probability in the examples considered. This observation is mainly due to the fact that in contrast to *XL*-types of contracts, proportional contracts provide insurance coverage for the full range of losses, which on the one hand increases the expected payoff for these contracts, but on the other hand raises their price. Contrariwise, excess of loss contracts focus on the critical range of high losses and are thus better suited to insure critical loss levels at a lower price. This explanation is supported by the relatively high optimal attachment points $\left(A^{XL}, A^{gap-XL}\right)$, which are located between the 75% and the 85% quantile of the insurer's losses. Furthermore, this leads to lower loadings for the contracts exhibiting an excess of loss structures, since the rate of insurance is lower as compared to proportional contract structures, which in turn reduces the need to control for moral hazard (see Equation (4)).[19]

On the basis of this advantage, the fractions for the excess of loss contracts are higher than for the proportional contracts, while the fractions for the ILW are lower in case of the ILW-XL-hedge (HS^{VI}) and the ILW-gap-XL-hedge (HS^{V}). Moreover, a comparison of the ILW-reinsurance-hedge (HS^{II}) with the ILW-gap-hedge (HS^{III}) and the ILW-XL-hedge (HS^{VI}) with the ILW-gap-XL-hedge (HS^{V}) shows that the contracts with the gap insurance structure (*gap* and *gap-XL*) appear more effective than the traditional reinsurance contracts (*re* and *XL*) with respect to increasing the maximum net shareholder value and reducing shortfall probability and basis risk. The gap insurance contracts are advantageous as their payment structure directly takes into account a non-payment of the index-linked instrument. It thus provides more coverage in case of a non-payment from the index-linked instrument and less coverage when it is less needed (i.e. if the index-linked instrument provides a payment).

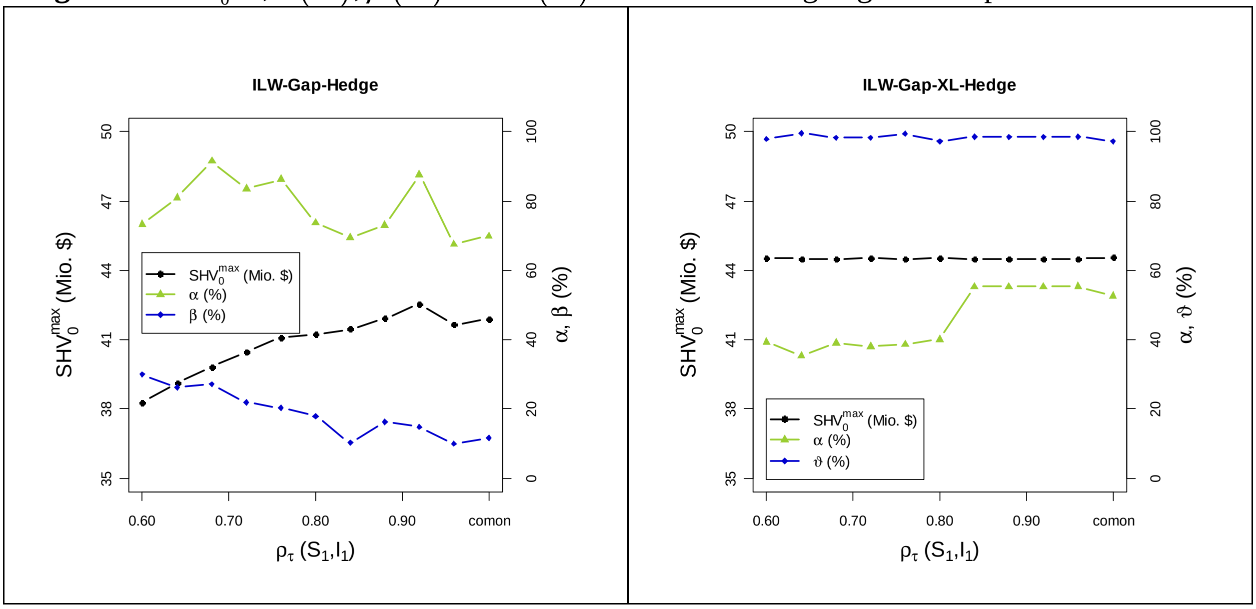
[19] Increasing the minimum loading of the excess of loss-type contracts above 5% to, e.g. 7.5%, leads to higher (lower) fractions of the index-linked (indemnity-based) instrument, but does otherwise not change the general results.

Table 3: Maximization of shareholder value in the case of all hedging strategies illustrated in Table 1 ($HS^I - HS^V$) for different degrees of dependence

	optimal α	*optimal* $\beta,\lambda,\omega,\vartheta$	*loading indemnity* δ^i	*optimal* A^{XL}, A^{gap-XL}	*Shortfall probability* SP_T^i	V_0	*loading* δ^{S_T}	*premium* π^{S_T}	*basis risk* CHE^i	*Shareholder Value* $SHV_0^{\max}$ (Mio. $)
$\boldsymbol{\rho_\tau(S_1,I_1)=0.65}$										
ILW	66.74%	0.00%			**4.39%**	114.33	31.41%	150.25	**15.71%**	**33.97**
ILW and r*e*	59.96%	20.98%	10.49%		**1.84%**	115.66	36.48%	157.85	**5.69%**	**38.23**
ILW and *gap*	82.06%	24.60%	9.41%		**1.51%**	115.83	37.18%	158.90	**4.83%**	**39.29**
ILW and *XL*	47.57%	95.56%	5.55%	149.26	**0.18%**	116.28	39.84%	162.61	**0.08%**	**44.10**
ILW and gap-*XL*	35.52%	99.79%	5.00%	169.00	**0.18%**	116.28	39.84%	162.60	**0.07%**	**44.57**
$\boldsymbol{\rho_\tau(S_1,I_1)=0.70}$										
ILW	69.52%	0.00%			**3.42%**	114.79	33.36%	153.09	**11.22%**	**36.31**
ILW and r*e*	68.05%	19.03%	9.51%		**1.49%**	115.81	37.17%	158.85	**4.51%**	**39.39**
ILW and *gap*	89.39%	22.88%	8.54%		**1.22%**	115.94	37.71%	159.66	**3.70%**	**40.18**
ILW and *XL*	29.57%	98.38%	5.00%	172.11	**0.24%**	116.24	39.71%	162.40	**0.16%**	**44.33**
ILW and gap-*XL*	38.98%	97.14%	5.00%	173.54	**0.19%**	116.28	39.80%	162.56	**0.14%**	**44.52**
$\boldsymbol{\rho_\tau(S_1,I_1)=0.75}$										
ILW	75.84%	0.00%			**2.62%**	115.19	34.97%	155.47	**8.08%**	**38.22**
ILW and r*e*	73.13%	15.26%	7.63%		**1.29%**	115.85	37.58%	159.39	**3.42%**	**40.48**
ILW and *gap*	87.73%	20.37%	7.60%		**1.01%**	115.97	38.16%	160.23	**2.42%**	**41.04**
ILW and *XL*	45.35%	92.05%	5.00%	157.81	**0.21%**	116.26	39.75%	162.47	**0.17%**	**44.15**
ILW and gap-*XL*	38.56%	99.41%	5.00%	173.98	**0.18%**	116.29	39.79%	162.55	**0.13%**	**44.53**

To study the impact of different degrees of dependence between the insurer's losses and the index $\rho_\tau(S_1, I_1)$ in more depth, Figure 1 illustrates the results for the proportional gap insurance (left graph) and the excess of loss gap insurance contract (right graph). It can be seen that the amount of proportional gap insurance (β) decreases for an increasing degree of dependence, since the ILW itself becomes more effective in enhancing the net shareholder value and in reducing the shortfall probability such that less proportional gap insurance is needed to hedge against basis risk. The results in Table 3 indicate that the same holds true for proportional reinsurance contracts.

Figure 2: $SHV_0^{\max}, \alpha(\%), \beta(\%)$ and $\vartheta(\%)$ for an increasing degree of dependence[20]



In case of an excess of loss gap insurance contract, in contrast, the optimal fraction (ϑ) remains relatively stable due to the lower fraction α of the ILW contract, which implies that the optimal hedging strategies are less sensitive with respect to changes in the degree of dependence between the insurer's loss and the industry index (as the indemnity-based contract dominates the hedging strategy). In summary, the results for the five hedging strategies under different degrees of dependence between the insurer's loss and the industry index show that solely purchasing an index-linked instrument for hedging purposes might not be sufficient from a shareholder's as well as a risk management's point of view. Combining an index-linked and an indemnity-based instrument generally allows higher net shareholder values and increases the hedging effectiveness, which depends on the respective gap insurance structure.

[20] Note that fluctuations in optimal risk management fractions (apart from the simulation) can be explained by the strong interrelations between expenses for risk management and an improvement in the solvency situation, which comes along with an increase in premium income and maximum net shareholder value.

The impact of the different types of dependence

Besides the degree of dependence, also the type of dependence can play a considerable role for the effectiveness of index-linked hedging strategies. In particular, even if the degree of dependence remains unchanged (in our reference case $\rho_\tau(S_1, I_1) = 0.70$), the hedging effectiveness can differ when varying the type of dependence. This is laid out in Table 4, which displays results for all hedging strategies using lower (Clayton copula) and upper (Gumbel copula) tail dependencies between the insurance company's losses and the industry loss index. When comparing these results with the reference case in Table 3, it can be seen that for all hedging strategies, the highest values for the maximum net shareholder value and the lowest values for basis risk and shortfall probability are achieved in the case of upper tail dependence. This observation can be attributed to the increase in the effectiveness of the ILW under upper tail dependencies as basis risk is reduced. In case of each hedging strategy except the ILW-gap-hedge (HE^{III}), this leads to an increase in the fraction invested in the ILW when upper tail dependence is given. At the same time, the fractions for indemnity-based instruments decrease in case of the ILW-reinsurance hedge (HE^{II}) and the ILW-gap-hedge (HE^{III}), as the need for these hedging instruments is reduced by virtue of the more effective ILW. Moreover, the general results from our reference case remain the same in that the hedging strategies including gap insurance like contract structures are most effective.

The impact of policyholders' risk sensitivity

Table 4 illustrates the impact of the policyholder's willingness to pay, which in the present setting depends on the insurer's risk situation and is based on empirical results in, e.g., Wakker, Thaler and Tversky (1997), who find that policyholders demand premium reductions for an insurer exhibiting default risk. As there is a lack of empirical data regarding reasonable assumptions about the policyholder's risk aversion parameter q, we examine its impact by means of a sensitivity analysis. For an increasing policyholders' risk sensitivity, fractions of risk management instruments in tendency increase, while the maximum net shareholder value generally decreases for all hedging strategies. At the same time, the difference between the $SHV_0^{\max}$ of the traditional indemnity-based (ILW-reinsurance-hedge and ILW-XL-hedge) and the gap insurance hedging strategies (ILW-gap-hedge and ILW-gap-XL-hedge) increases. The same holds true for the hedging strategies exhibiting a proportional contract design (ILW-reinsurance-hedge and the ILW-gap-hedge) and the hedging strategies exhibiting an excess of loss design (ILW-XL-hedge and the ILW-gap-XL-hedge).

Table 4: Maximization of shareholder value in the case of hedging strategies illustrated in Table 1 ($HS^I - HS^V$) for different types of dependence between the insurance company's losses and the industry loss index and different degrees of policyholders' risk sensitivity

	optimal α	*optimal* $\beta, \lambda, \omega, \vartheta$	*loading indemnity* δ^i	*optimal* A^{XL}, A^{gap-XL}	*Shortfall probability* SP_T^i	V_0	*loading* δ^{S_T}	*premium* π^{S_T}	*basis risk* CHE^i	*Shareholder Value* $SHV_0^{\max}$ (Mio. $)
Clayton copula										
*ILW**										
ILW and re	37.76%	26.66%	13.33%		**3.43%**	114.53	33.33%	152.71	**11.06%**	**33.04**
ILW and *gap*	75.80%	35.38%	13.87%		**2.44%**	115.19	35.29%	155.84	**9.63%**	**34.79**
ILW and *XL*	1.48%	96.42%	5.00%	166.57	**0.54%**	115.84	39.05%	161.05	**0.67%**	**43.57**
ILW and gap-*XL*	20.48%	86.18%	5.00%	167.39	**0.49%**	115.93	39.16%	161.34	**0.44%**	**43.77**
Gumbel copula										
ILW	76.47%	0.00%			**2.27%**	115.51	35.65%	156.69	**6.32%**	**39.11**
ILW and re	71.46%	13.65%	6.83%		**1.24%**	116.02	37.60%	159.65	**2.74%**	**40.81**
ILW and *gap*	78.07%	16.85%	6.46%		**1.10%**	116.05	37.87%	160.00	**1.89%**	**41.14**
ILW and *XL*	49.64%	91.28%	5.00%	163.77	**0.17%**	116.48	39.83%	162.87	**0.02%**	**44.35**
ILW and gap-*XL*	45.04%	94.67%	5.00%	143.88	**0.16%**	116.48	39.86%	162.91	**0.01%**	**44.54**
$\boldsymbol{q = 3}$										
ILW	73.17%	0.00%			**3.06%**	114.98	36.45%	156.89	**10.17%**	**39.85**
ILW and re	64.34%	14.23%	7.11%		**1.75%**	115.62	38.00%	159.56	**4.82%**	**41.11**
ILW and *gap*	80.78%	20.22%	7.73%		**1.33%**	115.85	38.48%	160.42	**3.60%**	**41.38**
ILW and *XL*	16.98%	94.34%	5.00%	165.97	**0.33%**	116.15	39.70%	162.26	**0.09%**	**44.40**
ILW and gap-*XL*	43.94%	94.90%	5.00%	156.19	**0.18%**	116.29	39.89%	162.67	**0.11%**	**44.54**
$\boldsymbol{q = 7}$										
ILW	68.05%	0.00%			**4.05%**	114.55	28.92%	147.68	**13.75%**	**31.23**
ILW and re	58.87%	20.94%	10.47%		**1.49%**	115.76	36.10%	157.56	**3.73%**	**37.87**
ILW and *gap*	88.84%	26.37%	9.86%		**1.07%**	116.01	37.21%	159.17	**3.03%**	**39.15**
ILW and *XL*	49.24%	94.76%	5.29%	151.55	**0.18%**	116.29	39.78%	162.55	**0.10%**	**44.07**
ILW and gap-*XL*	48.57%	96.12%	5.00%	166.10	**0.15%**	116.31	39.84%	162.65	**0.07%**	**44.48**

*In case of the ILW hedge only (HS^I) the shortfall probability condition $SP_T \leq \overline{SP_T}$ cannot be satisfied under lower tail dependencies.

This indicates that more effective hedging strategies such as the excess of loss and/or gap insurance contract design allow higher expenses for risk management, since the enhancement in maximum net shareholder value (arising through the reduction of shortfall probability by purchasing additional fractions of risk management instruments and the associated higher willingness to pay by policyholders) outweighs the reduction in the net shareholder value due to higher expenses for risk management and a lower default put option value. Contrariwise, hedging strategies that are comparatively less effective (proportional and/or traditional contract design) do not allow higher expenses for risk management instruments as the shortfall probability cannot be reduced by an amount that would result in a sufficient increase in total premium income to increase the maximum net shareholder value.

The impact of premium loadings and moral hazard

In contrast to the industry loss warranty contract, each of the remaining hedging instruments is indemnity-based and thus includes the insurer's loss in its payoff structure. This leads to moral hazard and, thus, to higher costs as compared to the index-linked risk transfer instrument (see Equation (4)). Table 5 illustrates results for all hedging strategies under different cost functions for the indemnity-based instruments. In the reference case, k is assumed to be 1.00, which leads to a linear cost function. To analyze the impact of a concave or a convex cost function, k is additionally set to 0.50 and 2.00. Table 5 shows that while the general results remain the same, it can be seen that the shape of the cost function has a great impact on the purchased amount of risk management. A convex cost function $(k = 2.00)$ reduces the costs as compared to the linear or the concave cost function, especially for lower fractions of $E\left(X_T^i\right)/E\left(S_T\right)$. This allows purchasing more proportional reinsurance and proportional gap insurance, as these contracts are generally purchased at lower fractions of β and λ (which become much less expensive as in the case of a linear or concave cost function). A similar behavior can be observed in case of the excess of loss and the excess of loss gap insurance contract, but to a lesser extent. Table 5 also illustrates that even under the concave cost function, fractions of indemnity-based instruments are still purchased, as the associated higher expenses are outweighed by an increase in the premium income (through the improvement in the solvency situation) and a decrease of costs of financial distress. The lower costs (going from a concave to a linear and a convex cost function) also generally imply a higher shareholder value and a lower shortfall risk.

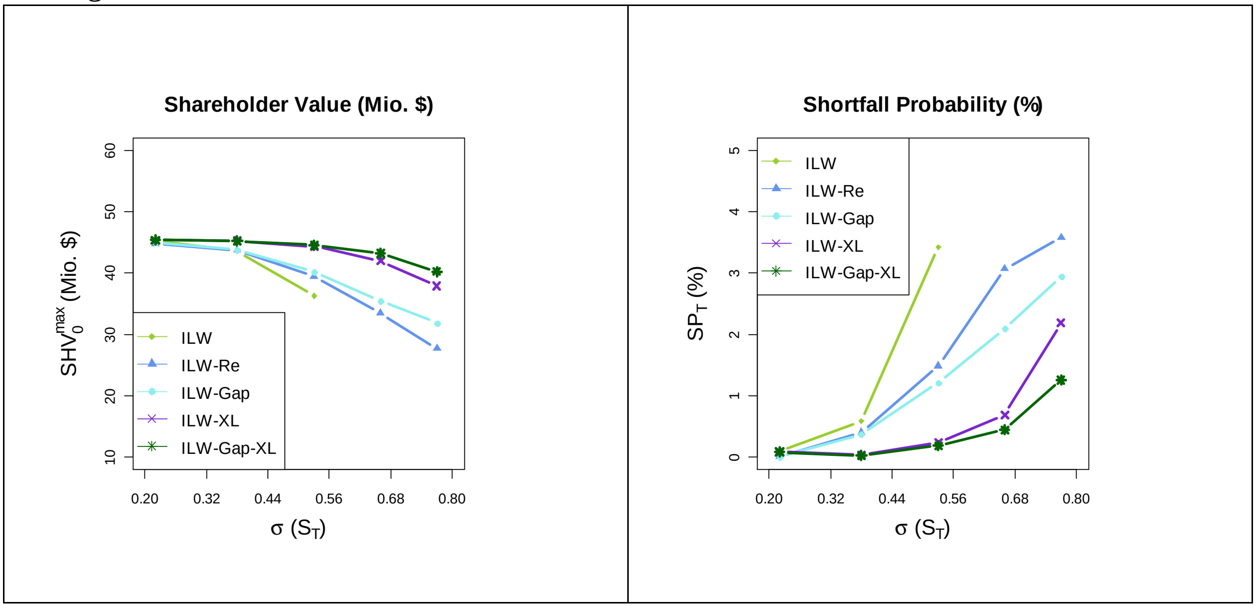
Table 5: Maximization of shareholder value in the case of hedging strategies illustrated in Table 1 ($HS^I - HS^V$) for different types of loading cost functions (k) reflecting moral hazard in case of the indemnity-based risk management instruments

	optimal α	*optimal* $\beta, \lambda, \omega, \vartheta$	*loading indemnity* δ^i	*optimal* A^{XL}, A^{gap-XL}	*Shortfall probability* SP_T^i	V_0	*loading* δ^{S_T}	*premium* π^{S_T}	*basis risk* CHE^i	*Shareholder Value* $SHV_0^{\max}$ (Mio. $)
$k = 0.50$ (concave)										
ILW	69.52%	0.00%	-		**3.42%**	114.79	33.36%	153.09	**11.22%**	**36.31**
ILW and r*e*	67.48%	9.89%	15.73%		**2.38%**	115.37	35.44%	156.25	**7.42%**	**37.36**
ILW and *gap*	89.18%	16.42%	17.51%		**1.79%**	115.72	36.59%	158.06	**6.16%**	**37.94**
ILW and *XL*	18.54%	81.52%	12.55%	174.07	**0.46%**	116.06	39.28%	161.64	**0.21%**	**43.40**
ILW and gap-*XL*	51.51%	94.55%	8.24%	164.71	**0.15%**	116.31	39.79%	162.59	**0.07%**	**44.30**
$k = 1.00$ (linear)										
ILW	69.52%	0.00%	-		**3.42%**	114.79	33.36%	153.09	**11.22%**	**36.31**
ILW and r*e*	68.05%	19.03%	9.51%		**1.49%**	115.81	37.17%	158.85	**4.51%**	**39.39**
ILW and *gap*	89.39%	22.88%	8.54%		**1.22%**	115.94	37.71%	159.66	**3.70%**	**40.18**
ILW and *XL*	29.57%	98.38%	5.00%	172.11	**0.24%**	116.24	39.71%	162.40	**0.16%**	**44.33**
ILW and gap-*XL*	38.98%	97.14%	5.00%	173.54	**0.19%**	116.28	39.80%	162.56	**0.14%**	**44.52**
$k = 2.00$ (convex)										
ILW	69.52%	0.00%	-		**3.42%**	114.79	33.36%	153.09	**11.22%**	**36.31**
ILW and r*e*	66.65%	33.22%	5.52%		**0.63%**	116.20	38.91%	161.41	**1.72%**	**41.80**
ILW and *gap*	69.74%	37.89%	5.00%		**0.62%**	116.15	38.94%	161.38	**0.67%**	**42.09**
ILW and *XL*	28.79%	98.55%	5.00%	176.24	**0.24%**	116.24	39.71%	162.39	**0.20%**	**44.35**
ILW and gap-*XL*	39.30%	97.10%	5.00%	173.72	**0.19%**	116.28	39.80%	162.56	**0.14%**	**44.52**

The impact of the loss volatility

Besides the choice of purchased risk management instruments, the insurer can try to reduce its risk through diversifying the insurance portfolio and thus lowering the loss volatility. Thus, in Figure 3 we further study the maximum net shareholder value (left graph) and the shortfall probability (right graph) under different degrees of loss volatility for each hedging strategy. It can be seen that in general, a well-diversified underwriting portfolio with a low degree of loss volatility reduces the insolvency risk and increases the maximum net shareholder value under an optimal hedging strategy. We further find that – consistent with the previous analyses – the "ILW-gap-XL" strategy implies the highest shareholder value and the lowest shortfall risk. However, one can also observe that the hedging strategies with excess of loss and/or gap insurance contract design are most effective in the considered examples when the underwritten risks (i.e. the insurance portfolio) exhibit a higher loss volatility. This is reflected in the increasing discrepancy between shareholder value and shortfall probability among the hedging strategies including proportional and excess of loss contract structures as well as among traditional reinsurance and gap insurance alike contracts.

Figure 3: $SHV_0^{\max}$ and SP_T for varying loss volatilities $\sigma(S_T)$ under the optimal hedging strategies



Summing up, our results reveal that the combination of index-linked and indemnity-based instruments can increase the effectiveness of risk management strategies. Furthermore, gap insurance-like contract structures are advantageous in comparison to traditional reinsurance contracts as their contract design directly takes into account if a payment from the index-lined instruments takes place. Our results are in line with those of Yow and Sherris (2006) and

Krvavych and Sherris (2006) in that in general, a tradeoff between reducing financial distress costs and higher expenses for risk management exists. Moreover, besides frictional costs, policyholders' reaction to the insurer's solvency situation play an important role for the purchase of risk management and thus for the insurance company's shareholder value. In addition, the findings found in this setting confirm the results of Doherty and Richter (2002) in a mean-variance framework, in that it is advisable to supplement the purchase of an index-linked instrument by gap insurance. Further analyses showed that when replacing the constraint of a maximum shortfall probability by minimum solvency capital requirements (calculated in line with the planned European risk-based regulatory framework Solvency II, using the Value at Risk as the relevant risk measure), the general results in Table 3 remain robust, even though the absolute values differ. Thus, our results show that gap insurance-like structures can be optimal for risk management in the presence of regulatory restrictions and distress if these instruments are accepted as a risk transfer instrument.[21]

4. CONCLUSION

This paper examined the effectiveness of index-based hedging strategies when gap insurance is purchased, which specifically reinsures the gap in a hedging strategy's payoff structure arising due to basis risk. Toward this end, we compared five hedging strategies, one that solely consists of an index-linked instrument (an industry loss warranty contract ILW), one that combines an index-linked instrument and proportional gap insurance, and one that combines an index-linked contract and proportional reinsurance, one that consists of an ILW and an excess of loss reinsurance contract, and a hedging strategy consisting of an ILW and an excess of loss gap insurance contract. The strategies were then analyzed with respect to their effectiveness for maximizing the net shareholder value in the presence of policyholders' risk aversion towards insolvency risk and reducing basis risk, which has not been done so far. Optimal hedging strategies that maximized the net shareholder value were numerically derived by means of differential evolution; basis risk was measured by means of the hedging efficiency with respect to lowering the shortfall probability, thus taking into account assets and liabilities.

One main result was that gap insurance can increase the effectiveness of index-based hedging strategies in multiple ways. The purchase of gap insurance in addition to the index-linked instrument led to a higher value of the net shareholder value and, at the same time, to a lower

[21] In general, alternative risk transfer instruments with a contract design that is similar to reinsurance contracts are accepted as risk transfer instruments (see SwissRe, 2009).

shortfall probability and a reduction of basis risk, where particularly excess of loss-type structures proved to be most effective in the considered setting. To analyze the impact of basis risk on the purchase of gap insurance in more depth, we conducted a sensitivity analysis for different degrees of dependence between the insurer's loss and the industry index and found that higher fractions of gap insurance are purchased for lower degrees of dependence to compensate for the higher basis risk associated with the index-linked contract. In that case, results regarding the net shareholder value and the shortfall probability could be considerably improved by purchasing gap insurance.

We further examined if the results persist if the more complex gap insurance transaction is substituted through a traditional reinsurance contracts (proportional or excess of loss), which cannot specifically reinsure the gap in the hedging strategy's payoff emerging by virtue of basis risk, but that can lower the probability for a non-payment of a hedging strategy. Even though this hedging strategy did not lead to the same increase in the hedging effectiveness as the corresponding gap insurance, the net shareholder value could be substantially increased and shortfall probability could be reduced compared to the hedging strategy solely containing the index-linked instrument. The same holds true in regard to basis risk.

To analyze if further parameters besides basis risk determine the effectiveness of index-based hedging strategies, we conducted sensitivity analyses concerning the price difference between the index-linked instrument and gap insurance (resulting from moral hazard) as well as reinsurance and the degree of policyholders' risk aversion. Our findings showed that the price at which each hedging instrument is available plays an important role. Furthermore, the optimal amount of gap insurance (in addition to the index-linked contract) increased for higher policyholders' risk aversion. This implied higher costs for risk management but also a higher premium income due to the reduction in shortfall risk. Overall, this tradeoff led to a lower net shareholder value for higher policyholder risk aversion, but also to a reduction in basis risk.

In summary, our findings demonstrate that a hedging strategy that is based on an index-linked instrument should be replenished through an indemnity-based instrument to counterbalance the negative impact of basis risk. An optimal use of gap insurance or even traditional reinsurance in addition to an index-linked contract can in fact considerably improve the effectiveness of the hedging strategy compared to the case of a hedge with an index-linked instrument only. In particular, by means of additional gap insurance, the maximum net shareholder value can be increased while, at the same time, the insurer's shortfall risk and basis risk can be reduced.

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