AN ANALYSIS OF PRICING AND BASIS RISK FOR INDUSTRY LOSS WARRANTIES

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ABSTRACT

In recent years, industry loss warranties (ILWs) have become increasingly popular in the reinsurance market. The defining feature of ILW contracts is their dependence on an industry loss index. The use of an index reduces moral hazard and generally results in lower prices compared to traditional, purely indemnity-based reinsurance contracts. However, use of the index also introduces basis risk since the industry loss and the reinsured company's loss are usually not fully correlated. The aim of this paper is to simultaneously examine basis risk and pricing of an indemnity-based industry loss warranty contract, which is done by comparing actuarial and financial pricing approaches for different measures of basis risk. Our numerical results show that modification of the contract parameters to reduce basis risk can either raise *or* lower prices, depending on the specific parameter choice. For instance, basis risk can be reduced by decreasing the industry loss trigger, which implies higher prices, or by increasing the reinsured company attachment, thus inducing lower prices.

1. INTRODUCTION

Industry loss warranties (ILWs) are innovative index-based reinsurance instruments that have become increasingly popular in recent years (Guy Carpenter, 2006). Payment on these contracts is triggered by an industry loss, where the contracted trigger amount varies by geographic region and type of catastrophic event. Two predominant ILW forms are binary and indemnity-based contracts: binary contracts pay a fixed amount if the industry loss is triggered; indemnity-based contracts take into consideration the reinsured company's loss (SwissRe, 2006). Compared to other forms of alternative risk capital, ILWs are easier to draw up, more flexible, and incur fewer frictional costs than, for example, catastrophe bonds. In comparison to traditional reinsurance contracts, the underwriting and claim processes are simple and moral hazard is reduced substantially by integrating an industry index. In general, this type of contract can be offered at a lower price than that charged for traditional indemni-

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ty-based reinsurance contracts.

On the other hand, the reinsured has to bear the basis risk induced by ILWs, which arises if the industry-wide loss and the reinsured company's loss are not fully correlated (see, e.g., Harrington and Niehaus, 1999; Doherty and Richter, 2002). This leads to a difference between the index-based payoff and the reinsured's actual loss. Thus, for reinsurers, appropriate premium pricing is crucial, whereas taking basis risk into consideration is vital for the reinsured. The aim of this paper is to consider both perspectives by analyzing and comparing several approaches for pricing ILWs and by studying the reinsured's basis risk.

To date, the relevant literature is primarily focused on other forms of alternative risk capital, such as cat bonds, for which pricing approaches, basis risk, and moral hazard have been analyzed (see, e.g., Doherty and Richter, 2002; Lee and Yu, 2002, 2007). Previous literature on ILWs has generally concerned itself with pricing binary contracts by calculating a risk load using the coefficient of variation (Ishaq, 2005) or with analyzing basis risk in the case of binary ILW contracts (Zeng, 2000). Beyond this, Zeng (2003) analyzes the tradeoff between basis risk and the cost of index-based instruments.

Cummins et al. (2004) conduct an empirical study of general index-based instruments for catastrophic losses. In particular, basis risk is analyzed by examining the hedging effectiveness of risk reduction using different risk measures. In addition, the relationship between hedging effectiveness and insurer characteristics is studied. Zeng (2005) applies an optimization method based on the genetic algorithm to measure the reinsurance efficiency of index-based contracts, thereby taking into account cost and benefit.

There are several ways that the basis risk associated with ILWs can be reduced. One can either change the underlying industry index or book of business (thus also changing the volatility), or adjust contract parameters, such as the industry loss trigger level, the company's attachment point, or the limit of coverage. However, these changes will also have an effect on the price, depending on the pricing concept applied. In this paper, we examine the influence of such modifications on the pricing and basis risk of an individual indemnity-based ILW contract. We compare different established actuarial and financial pricing concepts and different measures for basis risk.

Among the financial pricing approaches we consider are the contingent claims approach and the capital asset pricing model. The actuarial pricing principles include the expected value principle, the standard deviation principle, and the variance principle.[1] Furthermore, we also include actuarial investment-equivalent reinsurance pricing as developed by Kreps (1998), which is commonly used in reinsurance practice.

In contrast to Zeng (2003), we do not establish a benchmark contract or use optimization tools that depend on the risk management objective of the reinsured company. Instead, we analyze two types of basis risk. The first type is the risk that the reinsured suffers a substantial loss given that the industry-wide loss does *not* exceed the predetermined threshold. This risk is measured in two ways—(1) the probability of occurrence and (2) the expected payoff of the ILW contract that the reinsured does not receive because the industry-wide loss was not great enough to trigger coverage. The latter risk measure allows taking the extent of such a shortfall event into account.

The second type of basis risk that will be considered here includes the probability that the industry loss does not exceed the trigger, conditioned on the event that the company loss exceeds the attachment point and the expected loss, given by the difference between the expected payoff of a traditional indemnity-based reinsurance contract and the expected ILW payoff.

A numerical sensitivity analysis is conducted for changes in the reinsured company's attachment point, the industry loss trigger, and the correlation coefficient between the company loss and the industry loss. We also study the impact of the underlying processes' volatility. We show that reducing basis risk through a modification of contract parameters can lead to both higher *or* lower prices, depending on which parameter is adjusted. Moreover, price is strongly dependent on the type of pricing approach employed and can vary substantially.

The remainder of the paper is organized as follows. In Section 2, the model framework is discussed, including the ILW contract, a discussion and comparison of actuarial and financial pricing approaches, and the definition of basis risk measures. Section 3 contains numerical results based on a simulation study. Section 4 concludes.

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[1] An overview and discussion of different pricing approaches is presented in Embrechts (1998).

2. MODEL FRAMEWORK

2.1 The Industry Loss Warranty Contract

Industry loss warranty contracts can be designed in a variety of ways.[2] A binary contract pays out a fixed amount if the industry-wide loss exceeds a predefined threshold. Another common design is indemnity-based, i.e., the reinsured company's loss must exceed a certain amount and the industry loss must be larger than a preset trigger. However, the one feature that occurs in all ILWs is the presence of a trigger based on industry losses.

The ILW contract analyzed in this paper contains an aggregate excess of loss contract with a layer limit L. Furthermore, the contract incorporates a second trigger that is based on the industry loss distribution I_1 in $t = 1$ (see Zeng, 2000; Wharton Risk Center, 2007). Let S_1 denote the company's loss distribution in $t = 1$, A the attachment of the company loss, Y the industry loss trigger, and $1\{I_1 > Y\}$ the indicator function, which is equal to 1 if the industry loss in $t = 1$ is greater than the trigger and 0 otherwise. Hence, the payoff of the contract X_1 in $t = 1$ can be written as[3]

$$X_1 = \min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\{I_1 > Y\}. \tag{1}$$

The most frequently used reference indices for insured catastrophic events are those provided by the Property Claim Services (PCS) in the United States. Thus, the industry loss is usually determined by referencing a relevant PCS index. Outside of the United States, the insurance industry was lacking comparable indices in the past and had to rely on published loss figures provided by Swiss Re sigma or Munich Re's NatCat service. In Europe, the market participants have established the PERILS industry loss index service, an industry loss data provider similar in concept to the PCS, to overcome this problem (see e.g. SwissRe, 2009). In addition, commercial modeling vendors offer parametric indices which are based on proprietary industry exposure data. Burnecki, Kukla, and Weron (2000) show that, in general, a lognormal distribution provides a good fit to the analyzed PCS indices. We thus model both random variables I and S as geometric Brownian motions under the physical measure P, i.e.,

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[2] An overview of ILW contracts is provided in SwissRe (2006).

[3] This contract form is also called a double-trigger contract (see, e.g., Gründl and Schmeiser, 2002; SwissRe, 2009).

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_t^S, \tag{2}$$

$$dI_t = \mu_I I_t dt + \sigma_I I_t dW_t^I, \tag{3}$$

with empirical drift μ_S and μ_I, volatility σ_S and σ_I, and W^S, W^I denoting standard Brownian motions on a probability space $(\Omega, \mathcal{F}, P)$ with filtration $\mathcal{F}_t$. The two Brownian motions are correlated with $dW^S dW^I = \rho dt$. The solution of the stochastic differential equations is given by (see, e.g., Björk, 2004)

$$S_1 = S_0 \cdot \exp\left(\mu_S - 0.5 \cdot \sigma_S^2 + \sigma_S \cdot \left(W_t^S - W_{t-1}^S\right)\right),$$

$$I_1 = I_0 \cdot \exp\left(\mu_I - 0.5 \cdot \sigma_I^2 + \sigma_I \cdot \left(W_t^I - W_{t-1}^I\right)\right),$$

thus leading to a lognormal distribution for S_1 and I_1, since the increments ($W_t - W_{t-1}$) follow a standard normal distribution.

2.2 Pricing Methods

Most premium calculation methods derive insurance prices Π by determining a certainty equivalent *CE* for the uncertain contract loss payoff, thus making an investor indifferent between the stochastic loss payout and the deterministic certainty equivalent. In the premium calculation, the certainty equivalent is discounted with the risk-free interest rate. For the payoff of the contract at hand, this leads to

$$\Pi(X_1) = \exp\left(-r_f\right) \cdot CE(X_1),$$

with r_f denoting the continuous one-period risk-free rate of return. Hence, the price of the contract depends on the way the certainty equivalent is determined.[4] In the following sections, we discuss different actuarial and financial valuation methods for calculating the certainty equivalent.

[4] Embrechts (1998) points out parallels between the determination of the certainty equivalent in actuarial mathematics and in financial valuation.

2.2.1 Actuarial Pricing Approaches

In general, actuarial valuation methods rely on the individual decisionmaker's risk preferences, usually assuming risk aversion (see Cummins, 1990a, p. 125) and thus calculate a loading that is added to the expected loss of the contract in order to determine a certainty equivalent for the loss distribution. In the actuarial literature, the assumption of risk aversion is usually based on classical ruin theory, which states that a premium equal to the net risk premium leads to certain ruin in an infinite planning horizon, regardless of how much equity capital the insurer holds (see Bühlmann, 1996, pp. 141–144). Bühlmann (1985) also relates insurance premiums to ruin theoretical stability criteria, i.e., a certain probability of ruin, and thus deduces actuarial premium calculation principles with the implicit assumption of risk aversion. There are several different actuarial approaches for determining the loading, resulting in correspondingly different pricing principles.[5] Below, we consider four actuarial pricing principles.

Expected Value Principle

Under the expected value principle, the certainty equivalent is determined by loading the expected contract loss $E(X_1)$ with a percentage δ_E (> 0) of itself:

$$CE = E(X_1) + \delta_E \cdot E(X_1),$$

where E denotes the expectation operator under the physical measure P. This approach is not risk sensitive since it considers only the expected value and no quantity that would represent the risk inherent in the contract. However, it requires only the first moment of the contract's loss distribution and thus can be easily implemented.

Standard Deviation Principle

A widely used pricing principle in actuarial practice is the standard deviation principle, which determines the certainty equivalent by loading a portion δ_S (> 0) of the standard deviation $\sigma(X_1)$ on the expected contract loss, leading to:

$$CE = E(X_1) + \delta_S \cdot \sigma(X_1).$$

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[5] For an overview, see Goovaerts et al. (1984).

Variance Principle

The variance principle can be derived from utility theory using an exponential utility function or normally distributed wealth (see Pratt, 1964). It defines the certainty equivalent as the expected loss and a fraction δ_V (> 0) of the variance of the contract loss ($\sigma^2(X_1)$), depending on the risk aversion of the company issuing the contract:

$$CE = E(X_1) + \delta_V \cdot \sigma^2(X_1).$$

Investment-Equivalent Reinsurance Pricing

Kreps (1998) proposed the investment-equivalent reinsurance method for pricing individual contracts by specifying risk and return criteria. This concept assumes that the reinsurer allocates assets so as to be able to reimburse contract losses. The reinsurance company requires at least the same return and at most the same risk on these assets as if they were invested in some other equivalent financial instrument (target investment). Therefore, under this pricing concept, the risk load of a contract can be interpreted as opportunity cost.

Kreps (1998) considers two cases for the derivation of pricing formulas. First, costs are defined as the loss of investment income that results from investing in the risk-free instruments (to secure the losses) instead of in the risky target investment; this is called the “switch case.” In the second case, costs are determined through hedging by buying European put options on the underlying target investment with a strike price equal to the investment in risk-free securities. In the following, we will focus on the switch case, which generally results in higher prices than the option case and thus gives an upper bound for the premium.

In the switch case, the risk load *R* is determined using a benchmark target investment and conditions on the internal rate of return of the resulting cash flows (taking into account outflow of contract losses and inflow of the allocated assets’ return). This leads to two constraints for the contract: The loss safety constraint (left part of the maximum operator in Equation (4)) represents the return requirement for the assets allocated to the contract, where assets must be sufficient to cover losses to a certain safety level amount. The investment variance constraint (right part of the maximum operator in Equation (4)) requires that the variance of the internal rate of return is at most as large as the variance of the target investment. The maximum of these two requirements is then set equal to *R*, resulting in

$$R = \max\left(\frac{\left(E(y)-r_f{}^d\right)\left(q_\alpha^X - E(X_1)\right)}{1+E(y)}, \frac{\left(E(y)-r_f{}^d\right)\sigma(X_1)}{\sigma(y)}\right), \tag{4}$$

where y is the yield rate of the target investment, which is used as a benchmark in pricing the contract. $E(y)$ and $\sigma(y)$ are the expected value and the standard deviation of the yield rate, $r_f{}^d$ denotes the discrete equivalent of $r_f \left(= \ln\left(1+r_f^d\right)\right)$, and q_α^X is the specified safety level amount (in absolute terms), i.e., the α-quantile of the contract's loss distribution X_1. The resulting certainty equivalent is then given by

$$CE = E(X_1) + R. \tag{5}$$

The expected losses are loaded with the risk load derived from the investment criteria, thus again representing a risk-averse decisionmaker.

2.2.2 Financial Pricing Approaches

In contrast to actuarial pricing approaches, financial pricing concepts rely on the duplication of cash flows and are thus independent of individual preferences. Hence, in this model framework, one needs to assume that there are financial instruments that can be used to replicate the evaluated contract's payoff.

Capital Asset Pricing Model (CAPM)

In the CAPM, the certainty equivalent is defined as the expected value of the contract's payoff and a risk adjustment, such that[6]

$$CE = E(X_1) - \lambda \cdot Cov(X_1, r_m), \tag{6}$$

where r_m stands for the return of the market portfolio in $t = 1$ and λ denotes the market price of risk, given by

$$\lambda = \frac{E(r_m) - r_f{}^d}{\sigma^2(r_m)},$$

[6] For pricing insurance contracts in a CAPM framework, see, e.g., Fairley (1979), Hill (1979), D'Arcy and Doherty (1988), and Cummins (1990a).

where r_f^d again stands for the discretely compounded risk-free interest rate.

Contingent Claims Approach

Under the contingent claims approach,[7] insurance contract prices are determined by taking the expected value of the payoff with respect to the risk-neutral martingale measure *Q*, leading to a certainty equivalent of:

$$CE = E^Q\left(X_1\right).$$

By changing the probability measure to the risk-neutral measure *Q*, the drift of the stochastic processes in Equations (2) and (3) changes to the risk-free interest rate r_f and the company loss and the industry loss at time 1 are given by

$$S_1 = S_0 \cdot \exp\left(r_f - 0.5 \cdot \sigma_S^2 + \sigma_S \cdot \left(W_t^{Q,S} - W_{t-1}^{Q,S}\right)\right),$$

and

$$I_1 = I_0 \cdot \exp\left(r_f - 0.5 \cdot \sigma_I^2 + \sigma_I \cdot \left(W_t^{Q,I} - W_{t-1}^{Q,I}\right)\right),$$

respectively, where $W_t^{Q,S}$ and $W_t^{Q,I}$ are standard *Q*-Brownian motions. Hence, in case of geometric Brownian motions, the contract's cash flow is adjusted for systematic risk by changing the drift, while the standard deviation of the stochastic processes is not affected.

2.2.3 Comparison of Pricing Approaches

There are several important differences in the pricing approaches discussed in the previous sections. The actuarial methods in Section 2.2.1 evaluate individual contracts without considering diversification in the market or in the insurer's portfolio. Hence, only the contract's payoff is evaluated. In contrast, the financial methods in Section 2.2.2 assume that investors perfectly diversify unsystematic risk. Thus, only systematic risk is relevant for pricing insurance contracts. However, since the financial pricing approaches lead to present-value calcu-

[7] See, e.g., Doherty and Garven (1986), Cummins (1990b), and Gatzert and Schmeiser (2008).

lus, prices are additive for any portfolio of contracts. Therefore, the composition of the insurer's portfolio has no impact on pricing individual contracts.

One further difference in the pricing methods arises from the effect of the contract's volatility $\sigma(X_1)$. An increase in $\sigma(X_1)$ generally tends to induce higher premiums for all the pricing approaches under consideration—except for the expected value principle. In case of CAPM, this holds true only if there is systematic risk in the contract, i.e., if $\mathrm{Cov}(X_1,r_m) < 0$.

For all actuarial pricing concepts—except for investment-equivalent reinsurance pricing—the risk-free interest rate influences prices only as discount factor, but has no impact on calculating the certainty equivalent. In contrast, the risk-free interest rate does have an effect on the certainty equivalent determined with financial pricing methods. For investment-equivalent reinsurance pricing, lowering the risk-free interest rate leads to a higher risk load R, and thus to a higher certainty equivalent (see Equation (4)). Under the CAPM, the market price of risk λ increases when the risk-free rate is decreased. Hence, the effect of the covariance between the contract's payoff X_1 and the return of the market portfolio on the certainty equivalent is intensified if $\mathrm{Cov}(X_1,r_m) \neq 0$. Regarding the contingent claims approach, lowering r_f implies a lower probability of exceeding the triggers A and Y. Thus, the contract payoff is reduced, leading to a lower certainty equivalent.

In general, the distribution of X_1 is non-normal and typically heavily skewed, as observed in a simulation study. For all pricing methods, the payoff structure of X_1 is taken into account using the expected value (under the physical or the risk-neutral measure) or standard deviation of X_1. The only approach that can account for higher moments is investment-equivalent reinsurance pricing, which integrates the α-quantile of X_1 when calculating the certainty equivalent (see Equations (4) and (5)). Therefore, a higher α-quantile implies a higher certainty equivalent and thus higher premiums if the maximum function in Equation (4) is equal to the left expression in the maximum operator.

2.3 Basis Risk

From a buyer's perspective, the ILW contract should protect the company from losses that could endanger its survival. Thus, the situation where the insurance company suffers a severe loss while the industry has moderate losses represents a risk to the buyer since both triggering events must be fulfilled for the contract to pay out an indemnity. In general, this basis risk arises when using index triggers since company loss and industry loss are usually not fully correlated (see, e.g., Doherty and Richter, 2002). There are several ways of defining basis risk (see Zeng, 2003, p. 253).

We consider two types of basis risk. First, we define as Type I basis risk the case where the company's loss exceeds the attachment *A*, given that the industry loss does not exceed the trigger. Second, we consider as Type II basis risk the opposite case, i.e., the situation where industry loss is not triggered, given the insurance company has a severe loss.

Type I Basis Risk

To assess the Type I basis risk, we calculate the probability of occurrence under the real-world measure *P*:

$$P\left(S_1 > A \middle| I_1 < Y\right) = \frac{P\left(S_1 > A, I_1 < Y\right)}{P\left(I_1 < Y\right)}. \tag{7}$$

Furthermore, we measure basis risk by calculating the average loss amount the insurance company will not receive because the industry loss does not exceed the trigger:

$$E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \middle| I_1 < Y\right). \tag{8}$$

Hence, in contrast to Equation (7), Equation (8) takes the extent of company loss into account.

Type II Basis Risk

We further calculate the conditional probability that the industry loss does not exceed the trigger given that the company has a loss greater than the attachment, which can be considered as a critical level (see Zeng, 2000):

$$P\left(I_1 < Y \middle| S_1 > A\right) = \frac{P\left(I_1 < Y, S_1 > A\right)}{P\left(S_1 > A\right)}.$$

In addition, we consider the extent of missed indemnity payments for the buyer by examining the difference between a traditional reinsurance contract and the ILW. A traditional reinsurance contract X^{trad} with the same contract parameters as the ILW contract can be divided into two parts using the industry loss index:

$$\begin{aligned} E\left(X^{trad}\right) &= E\left(\min\left(\max\left(S_1 - A, 0\right), L\right)\right) \\ &= E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 > Y\right\}\right) + E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 < Y\right\}\right) \\ &= E\left(X_1\right) + E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 < Y\right\}\right), \end{aligned}$$

which illustrates the relationship between the traditional reinsurance contract and an ILW contract, i.e.,

$$E\left(X_1\right) = E\left(X^{trad}\right) - E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 < Y\right\}\right). \tag{9}$$

Thus, the ILW buyer can expect payment for only a part of the expected loss that could be claimed in full under a traditional reinsurance contract. The remainder, that is, the expected amount of payment not made, can then be considered as a measure of Type II basis risk, i.e.,

$$E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 < Y\right\}\right).$$

Equation (9) also illustrates that prices based on expected losses under the ILW reflect the reduced indemnity payments and thus generally result in a lower price for this type of contract compared to a traditional reinsurance contract.

3. Simulation Study

This section analyzes the sensitivity of basis risk and premiums derived under different pricing concepts with respect to changes in input parameters. The following examples will serve to identify key drivers for the basis risk. In addition, similarities and differences of the pricing approaches are examined.

Input Parameters for the Reference Contract

Based on data provided in Hartwig (2005), we set the expected value of the company loss at the end of the contract term, $E(S_1)$, to \$58 million, and the standard deviation of S_1 to $\sigma(S_1)$ = \$134 million. We follow Hilti, Saunders, and Lloyd-Hughes (2004) and assume that regional hurricane losses account for 50% of the historical U.S. industry total loss. Thus, for the industry loss, we set the expected value at the end of the contract term to $E(I_1)$ = \$1,450 million and the corresponding standard deviation to $\sigma(I_1)$ = \$3,550 million. The correlation coefficient between company loss S_1 and industry loss I_1 is set to $\rho(S_1, I_1) = 0.60$ and will be subject to sensitivity analysis. The empirical drift of S and I is assumed to be equal to an inflation rate of 2.50%, as measured by the U.S. Consumer Price Index for the year 2006 (see U.S. Department of Labor, 2007). These assumptions lead to an initial nominal value of the company's loss S_0 = \$56.57 million and an initial nominal value of the industry loss I_0 = \$1,414 million. Hence, the resulting standard deviations of the stochastic processes in Equations (2) and (3) are given by σ_S = 135.89% and σ_I = 139.47% (the derivation of these quantities is provided in the Appendix). The contract specifications include a layer limit of L = \$150 million, an attachment of the company's loss of A = \$150 million, and an industry loss trigger of Y = \$5,000 million.

Regarding the expected value principle, and according to Andreadakis and Waters (1980), the safety loading is set to δ_E = 30.00%, for the standard deviation principle, δ_S = 10.00% (see Wang, 2000), and the variance principle has a loading $\delta_V = 1.50 \cdot 10^{-7}$, a value that is close to data used by the Insurance Services Office in the United States (see Meyers and Kollar, 1999). For investment-equivalent reinsurance pricing, the risk load is determined by an expected target yield rate $E(y)$ = 5.30% and a standard deviation of $\sigma(y)$ = 8.40% (see Kreps, 1998).

To obtain results under the financial pricing concept CAPM, we use the input data in Gründl and Schmeiser (2002). The correlation coefficients between industry loss and return of the market portfolio and between company loss and return of the market portfolio are fixed at -0.20 and -0.10, respectively. Furthermore, the expected value and standard deviation of the return of the market portfolio are chosen as $E(r_m)$ = 8.00% and $\sigma(r_m)$ = 4.00%. Thus, the correlation coefficient between the market rate of return r_m and the contract payoff X_1 can be estimated from the simulated data and leads to a correlation coefficient $\rho(X_1, r_m)$ of approximately -0.05. The correlation coefficient between X_1 and r_m determines $Cov(X_1, r_m)$ in Equation (6), which is needed to calculate the certainty equivalent under the CAPM. Finally, the discrete rate of return r_f^d is given by 4.92%, a value based on 1-year U.S. Treasury constant maturity rates (see Federal Reserve, 2007), which corresponds to a continuously compounded rate of return r_f = 4.80%. The input data for the reference contract are summarized in Table 1.

Table 1: Input parameters for the reference contract

Contract parameters	$E(S_1)$	\$58 million
	$\sigma(S_1)$	\$134 million
	$E(I_1)$	\$1,450 million
	$\sigma(I_1)$	\$3,550 million
	μ_S, μ_I	2.50%
	$\rho(S_1, I_1)$	0.60
	L	\$150 million
	Y	\$5,000 million
	A	\$150 million
	r_f^d	4.92%
	r_f	4.80%
Expected value principle	δ_E	30.00%
Standard deviation principle	δ_S	10.00%
Variance principle	δ_V	1.5×10^{-7}
Investment-equivalent reinsurance pricing	$E(y)$	5.30%
	$\sigma(y)$	8.40%
	q_α^X	99%-quantile of X_1
CAPM	$\rho(r_m, I_1)$	-0.20
	$\rho(r_m, S_1)$	-0.10
	$E(r_m)$	8.00%
	$\sigma(r_m)$	4.00%

Numerical results are obtained using Monte Carlo simulation with 50,000 paths. To achieve better comparability, the same sequence of random numbers is used for all analyses. Normally distributed random variates are generated with the Box-Mueller scheme (see, e.g., Glasserman, 2004, p. 65).

In the following examples, we study the influence of the correlation coefficient $\rho(S_1, I_1)$, the attachment of the company's loss A, and the industry loss trigger Y, keeping everything else constant. We also look at the impact of industry loss volatility and company loss volatility, again leaving all else unchanged. Results are compared for the six pricing methods outlined above.

In addition to prices and basis risk, we also calculate the safety loading resulting from each pricing method. The safety loading is calculated as the difference between the certainty equivalent and the expected contract payoff and is given as a percentage of the expected contract payoff:

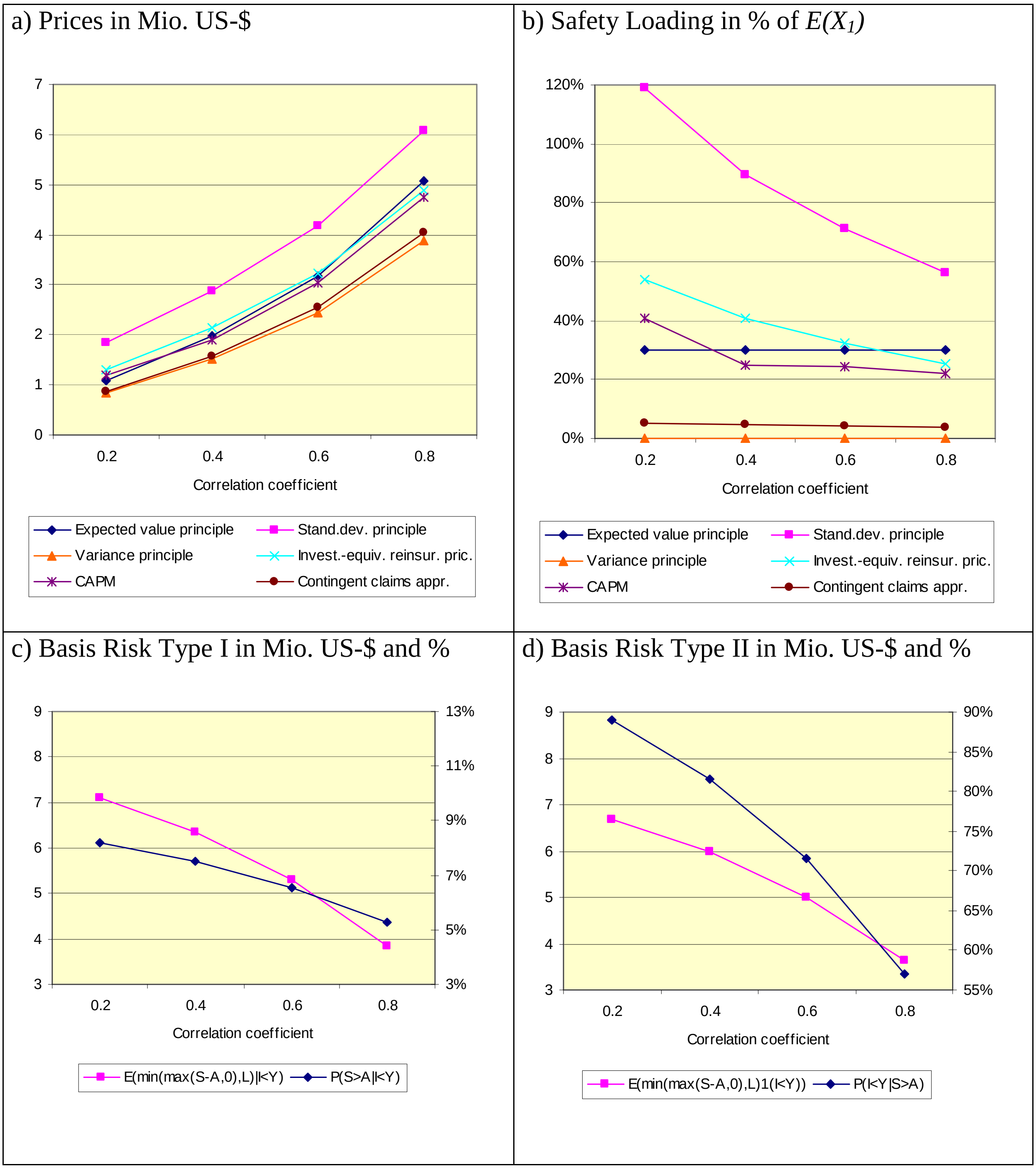
$$Safety\,Loading = \frac{CE - E\left(X_1\right)}{E\left(X_1\right)}. \tag{10}$$

Varying the Correlation Coefficient between Company and Industry Loss

We begin our numerical analysis by studying the impact of the correlation coefficient $\rho(S_1, I_1)$. Figure 1 shows results for prices, safety loadings, and basis risk when $\rho(S_1,I_1)$ = 0.2, 0.4, 0.6, and 0.8, where $\rho(S_1,I_1)$ = 0.6 corresponds to the reference contract.

Figure 1 demonstrates that all ILW prices increase with increasing coefficient of correlation (Part a)), which is due to the higher probability that both triggers A and Y are exceeded. However, at the same time, the safety loading on the expected contract loss decreases for most pricing concepts (Part b)), since the expected contract loss increases more than does the certainty equivalent (see Equation (10)). This is particularly obvious in the case of the standard deviation principle, which shows a decrease from 120% to 55%, and for investment-equivalent reinsurance pricing (from 54% to 25%). Exceptions are the expected value principle, which is constant at a rate of δ_E = 30%, and the variance principle with a nearly 0% loading.

Figure 1: Prices, safety loadings, and basis risk as functions of the correlation coefficient $\rho(S_1, I_1)$



Basis risk is independent of the type of valuation method used. Since the probability of a concurrent company loss and industry loss increases the higher the correlation coefficient, Part c) of Figure 1 exhibits a decreasing Type I basis risk (see Equations (7) and (8)). This observation also holds for the Type II basis risk. However, the analyzed contract setup results in a high level of basis risk. For a correlation of 0.8 between industry losses and company losses, the probability that the industry trigger is not exceeded given that the company has a severe loss is still 57%. By taking advantage of the industry index granularity, this basis risk can be reduced substantially. Cao and Thomas (1999) show that using ZIP-code level index values instead of state level index values increases the average correlation coefficient for a company comprising the index from 0.401 to 0.921. As PCS loss estimates are available on state level, the distribution of these index values on county level can be simulated (see e.g. Cummins, Lalonde, and Phillips, 2004). The PERILS index values will be available on CRESTA level, corresponding aggregated postcode zones.

In Figure 1, substantial discrepancies can be observed between prices derived by different pricing schemes in the considered example. In general, the price curves in Part a) can be grouped in essentially three categories. First, the prices calculated with the financial contingent claims approach and the actuarial variance principle are very close, which is also visible in Part b) of Figure 1. Because of the low value of the coefficient δ_V, the safety loading of the variance principle is only 0.002% in relation to the expected loss, which means that the price obtained is very close to the expected contract payoff $E(X_1)$. A similar reasoning holds for the contingent claims approach, which has a loading that decreases from 5% to 4% with increasing correlation coefficient. Here, the safety loading is due to changing the measure and thereby adjusting the drift of the underlying processes. Therefore, the market price of risk in this case is given by the difference between the empirical drift and the risk-free rate over the standard deviation of the respective process. With a continuous risk-free rate (4.80%) and an empirical drift rate (2.50%), the loading imposed by the price derived under the contingent claims approach remains moderate.

Second, the financial method CAPM and two actuarial methods—the expected value principle and investment-equivalent reinsurance pricing—have very similar results. Comparing the loading formulas for the CAPM and investment-equivalent reinsurance pricing, one can identify comparable elements since both concepts rely on an excess return above a risk-free investment, and a risk-sensitive quantity in the denominator, contributing to the similar curvature of these two concepts. The expected value principle is close to the other two concepts. However, the CAPM and investment-equivalent reinsurance pricing both include quantities that are sensitive to variation of contract losses, namely, the standard deviation of contract losses, in the denominator. This leads to decreasing safety loadings since the contract losses become increasingly volatile with increasing correlation $\rho(S_1,I_1)$. In contrast, the safety loading of the expected value principle is a constant percentage of the expected contract loss.

Third, the actuarial standard deviation principle results in the highest prices out of all the pricing concepts studied. Further analysis showed that all concepts—except the contingent claims approach—are very sensitive to changes in the risk-aversion parameters and safety loadings δ_S, δ_E, and δ_V.

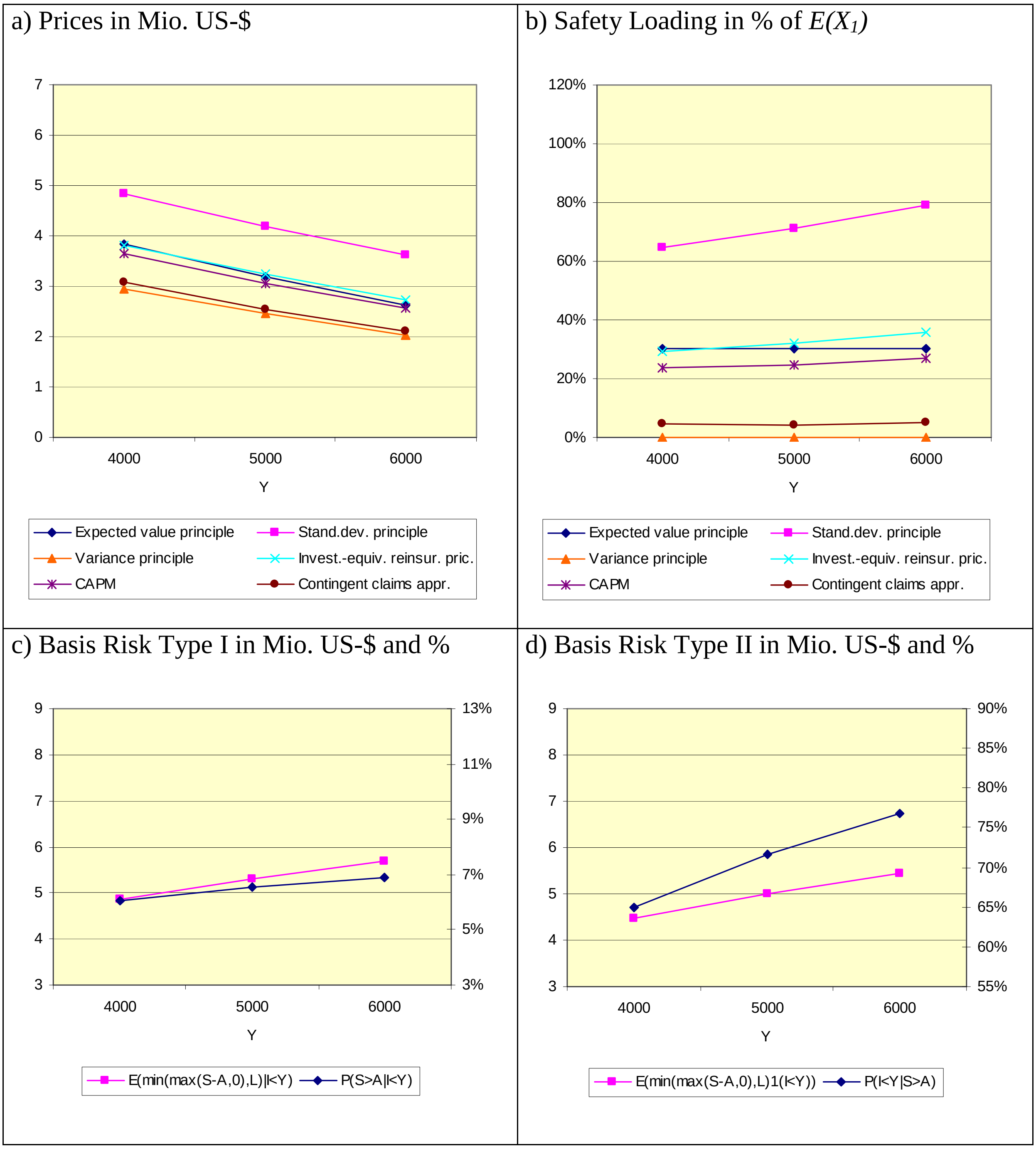
Pricing indications received from market participants exhibit increasing safety loadings with increasing industry triggers, e.g. based on modeling results from a catastrophe modeling firm, the risk load for a \$20bn industry loss warranty equals 70% of expected loss and can be as high as 5 times the expected loss for \$100bn contracts. As the assessment of expected loss can differ dramatically from one modelling firm to another, the absolute value of the loading fluctuates with the expected loss value. These multiples stand in contrast to a risk neutral assessment of industry loss warranties. On the other hand, the risk neutral prices represent a lower bound which could be achieved if the instruments were liquid, and traded constantly in significant volumes on an exchange. The current efforts to standardize these contracts and thus increase secondary market trading (see e.g. SwissRe, 2009) can thus lead to significant price reductions. Cummins and Weiss (2009) observe that the cyclicality of reinsurance prices can also be observed in the ILW market. This phenomenon may be weakened for exchange-traded products with sufficient capacity.

Varying Industry and Company Loss Trigger

In a next step, the effect of changes in the industry loss trigger is analyzed by changing the trigger level from Y = 5,000 to 4,000 and then to 6,000 (Figure 2). The impact of changes in the reinsured company loss trigger is studied by varying A from 150 to 100 and then to 200 (Figure 3).

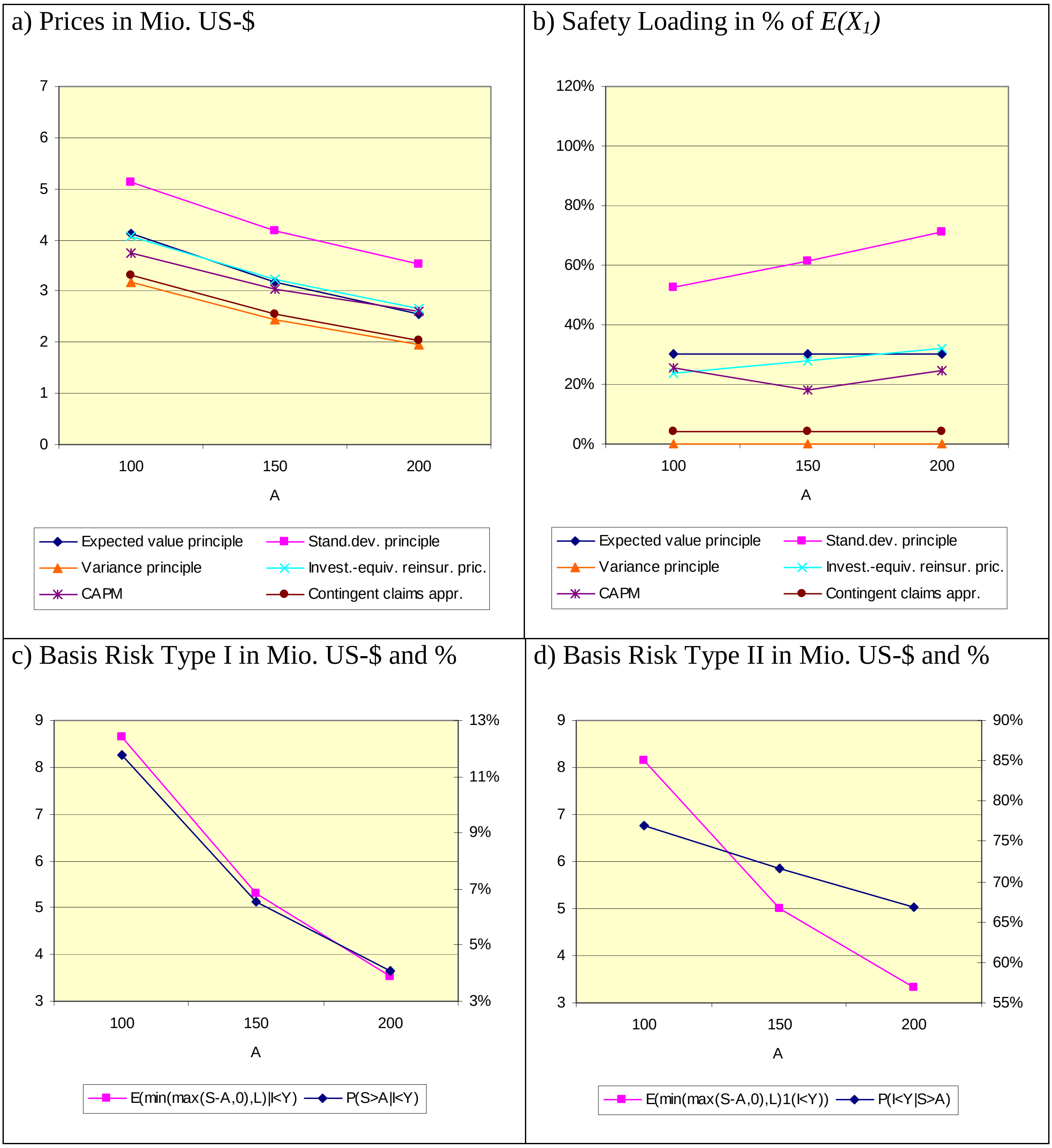
Figures 2 and 3 exhibit some similarities to the results displayed in Figure 1. As observed in the previous case, the standard deviation principle results in higher prices than all other pricing approaches for the given calibration. Furthermore, the contingent claims approach and the variance principle have similar results; CAPM, the expected value principle, and investment-equivalent reinsurance pricing all result in comparable prices.

Figure 2: Prices, safety loadings, and basis risk as functions of the industry loss trigger *Y*



In both Figures 2 and 3, increasing the trigger levels *A* and *Y* leads to lower ILW prices, whereas the loadings generally increase and are thus in contradiction to the decreasing price, as was also observed in Figure 1. One exception is the CAPM price curve, which first decreases and then increases. The effect on basis risk of varying *A* and *Y* is the complete opposite of the effect this variation has on price. With an increasing industry loss trigger *Y* (Figure 2), basis risk increases, whereas a higher company loss trigger *A* results in a lower basis risk (Figure 3). This effect is explained by the definition of basis risk used (see, e.g., Equations (7) and (8)). A higher industry loss trigger level Y results in a higher probability that the industry loss does not exceed Y and thus the conditional probability of $\{S > A\}$ given $\{I < Y\}$ decreases. For the same reason, basis risk decreases when the company loss trigger A is raised because, in this case, the probability of the event $\{S > A\}$—ceteris paribus—decreases. Hence, increasing the company loss trigger A can make it possible for the reinsurer to offer lower prices (due to a lower expected contract payoff) and also reduce the basis risk associated with the contract. Increasing the industry loss trigger Y also leads to decreasing prices; however, the basis risk increases.

Figure 3: Prices, safety loadings, and basis risk as functions of the company loss trigger A



Varying the Volatility of Company Loss and Industry Loss

The effects on pricing and basis risk of company loss volatility $\sigma(S_1)$ and industry loss volatility $\sigma(I_1)$ are illustrated in Figures 4 and 5. In both figures, volatility is reduced to -50% and increased up to +150% of the initial value, leaving everything else unchanged. Industry loss volatility is usually not influenced by a single company; modifications can be achieved only by changing the underlying industry index.

The three groupings of pricing methods are immediately obvious from Figures 4 and 5, as observed previously. Otherwise, however, the price and basis risk curves differ substantially from previous analyses. For instance, both Figures 4 and 5 show that ILW prices first increase and then decrease with increasing volatility of company loss and also with increasing volatility of industry loss. In contrast, in these scenarios, basis risk exhibits characteristics that are very different compared to the previous analyses and are also different for changes in $\sigma(S_1)$ and $\sigma(I_1)$.

In particular, increasing the volatility of company loss (Figure 4) first leads to an increase in basis risk and then, for even higher volatility, the risk decreases. In Part c) of Figure 4, basis risk measured by probability of occurrence is highest for the original volatility (+0%), and the highest expected loss caused by basis risk is at approximately +50%. Different results are found in Part d). Hence, the results are strongly dependent on the choice of risk measure.

In Figure 5, the basis risk curves in Parts c) and d) first decrease and then slightly increase as the volatility of industry loss is further increased, resulting in a convex form.

Hence, changes in volatility of industry loss or company loss have a similar impact on premiums, but considerably different basis risk profiles. For company loss volatility (Figure 4), basis risk follows a concave curve, implying that within the ranges considered, basis risk can only increase up to a maximum value when varying the volatility of the company's business (e.g., by underwriting certain business). For changes in the volatility of industry loss, however, basis risk is limited from below, in the examples considered and thus only lead to reductions in basis risk above a certain threshold.

Figure 4: Prices, safety loadings, and basis risk as functions of percentage changes in the company loss volatility $\sigma(S_1)$

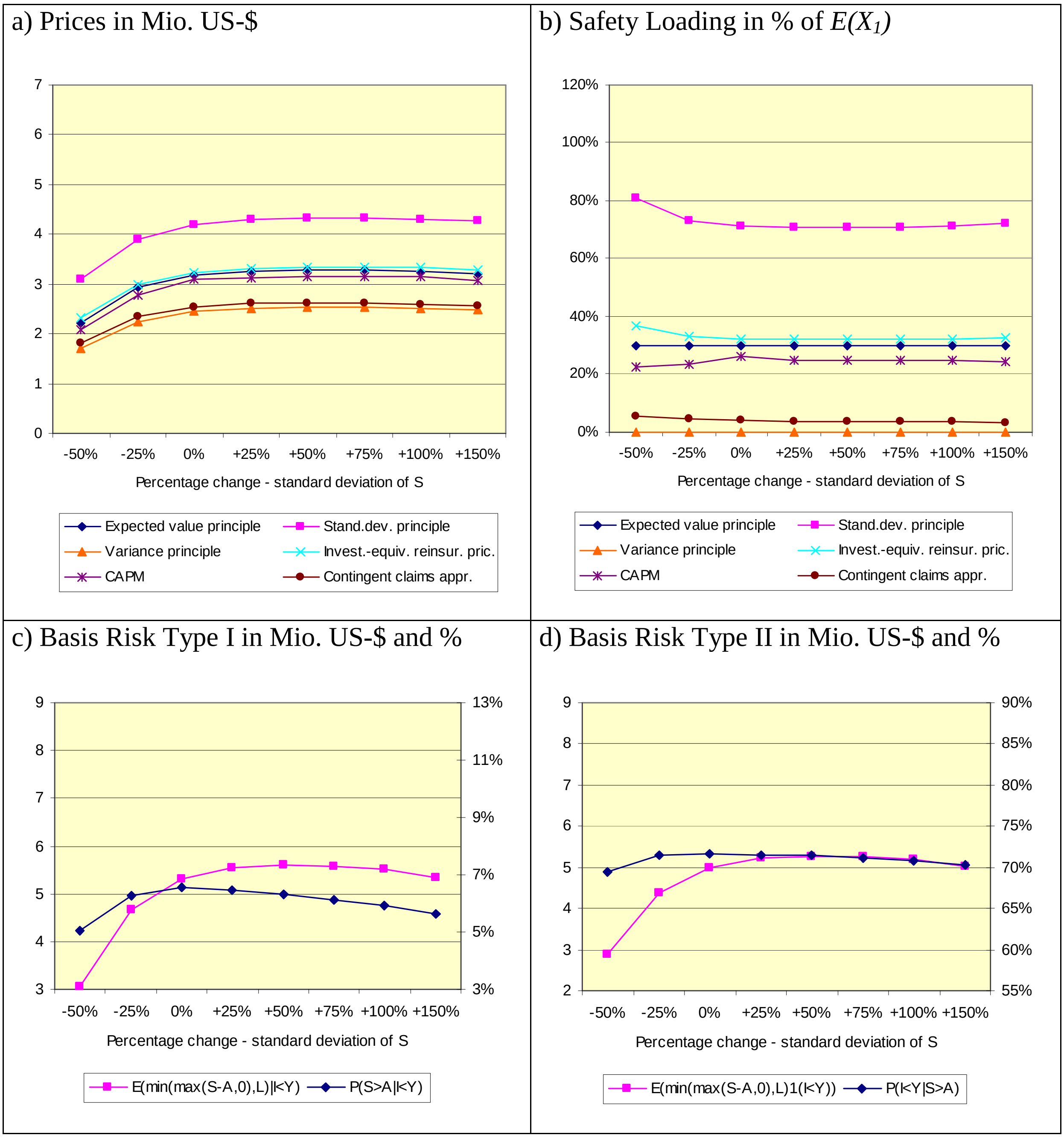
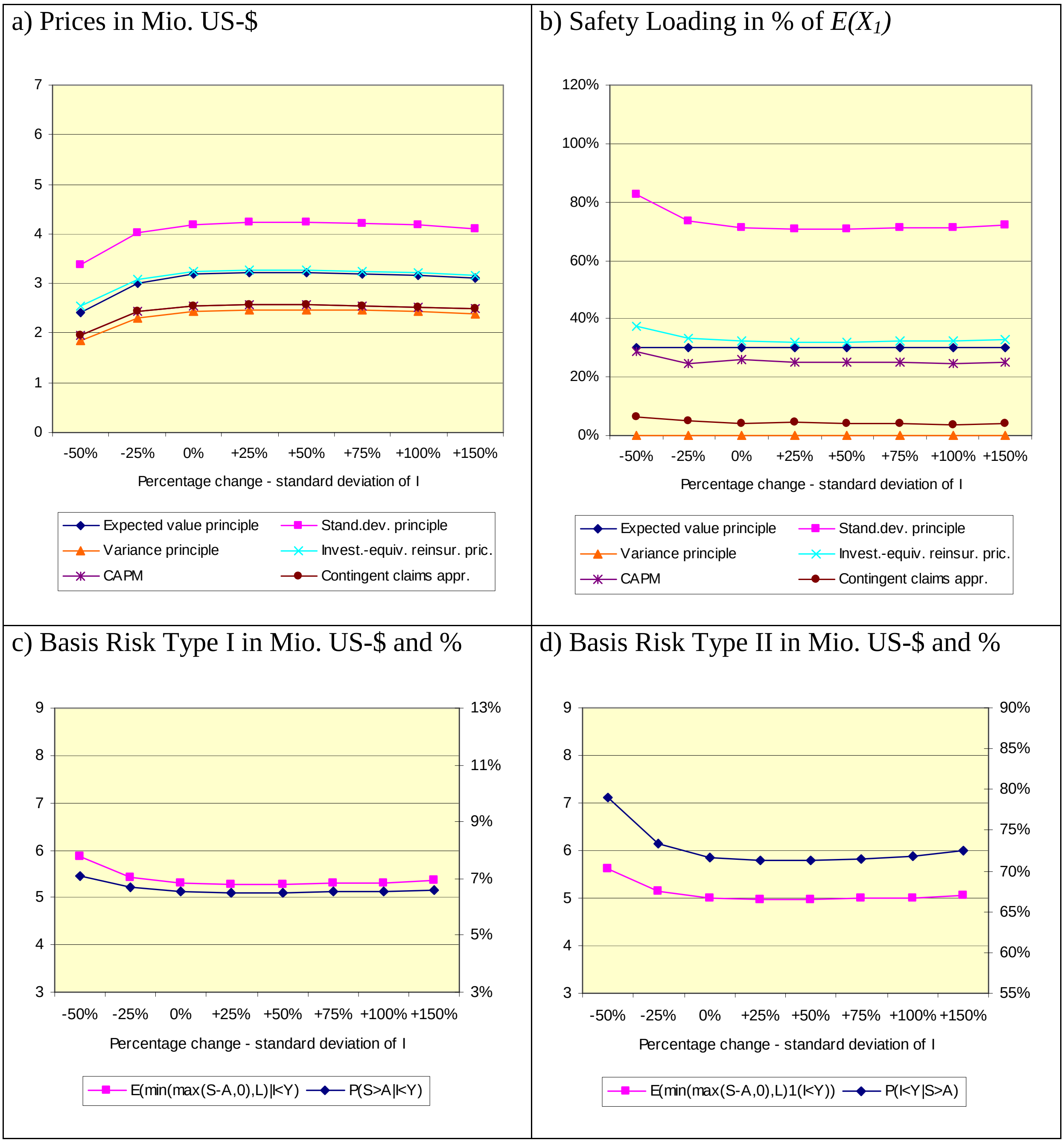


Figure 5: Prices, safety loadings, and basis risk as functions of percentage changes in the industry loss volatility $\sigma(I_1)$



4. Summary

This paper examines an indemnity-based industry loss warranty contract with regard to pricing and basis risk. Since the measures for basis risk are independent of the applied valuation method, calculated price curves do not necessarily reflect basis risk. To identify the different key drivers for premium and basis risk, we conducted a sensitivity analysis with respect to modifications in contract parameters for different measures of basis risk. We also compared several common actuarial and financial pricing approaches. Among the actuarial concepts are the expected value principle, the standard deviation principle, the variance principle, and investment-equivalent reinsurance pricing; among the financial concepts are the contingent claims approach and the capital asset pricing model.

Our numerical results reveal substantial discrepancies between the prices obtained using different pricing schemes. For the calibration setup, three groups of pricing methods could be identified that led to similar price levels. In general, these differences and similarities result from economic differences in the considered pricing approaches and from the choice of model parameters. We further provided the safety loadings implied by the different pricing schemes by calculating the difference between certainty equivalent and expected contract payoff, given in percent of the expected contract payoff. For all examples considered, the safety loading curves ran opposite to the pricing curves. For instance, raising the industry loss trigger led to strictly decreasing prices, but to higher (or equal) safety loadings for all analyzed pricing methods.

In the numerical analysis, an increase in the correlation coefficient between company and industry loss leads to higher prices and higher basis risk for all pricing concepts and risk measures. Furthermore, price and basis risk curves were concave with respect to increasing volatility of a reinsured book of business in the ranges considered. Increasing the volatility can thus induce lower prices for the ILW contract and, simultaneously, lower basis risk. The price curves for changes in industry loss volatility were concave as well. In contrast to the reinsured company's volatility, all basis risk curves were convex.

Regarding the trigger levels, we found that raising the industry loss trigger leads to an increase of basis risk, whereas increasing the reinsured company loss trigger leads to decreases in basis risk. However, in both cases, raising the trigger levels leads to lower prices.

This investigation of ILWs using different concepts of valuation and risk measurement provides insight into the structure and characteristics of this form of insurance product. The results demonstrate that a simultaneous analysis of both basis risk and pricing can be of substantial informational value to both insurers and insurance buyers.

APPENDIX

For the stochastic process as given in Equation (2), one obtains

$$S_1 = \exp\left(N\left(\ln\left(S_0\right) + \mu_S - 0.5\sigma_S^2, \sigma_S\right)\right) = \exp\left(N\left(a,b\right)\right),$$

where $N(a,b)$ denotes a normally distributed random variable with expected value $a = \ln\left(S_0\right) + \mu - 0.5\sigma^2$ and standard deviation $b = \sigma_S$, leading to a lognormal distribution for S_1. Given the expected value

$$E\left(S_1\right) = \exp\left(a + 0.5b^2\right)$$

and the standard deviation

$$\sigma\left(S_1\right) = \sqrt{\left(\exp\left(b^2\right) - 1\right) E\left(S_1\right)^2},$$

σ_S and S_0 can be obtained by transforming these equations. Using

$$a = \ln\left(E\left(S_1\right)\right) - 0.5b^2,$$

and

$$b = \sqrt{\ln\left(1 + \sigma^2\left(S_1\right) \big/ E\left(S_1\right)^2\right)},$$

the standard deviation of the stochastic process σ_S and the initial nominal value S_0 are given by

$$\sigma_S = \sqrt{\ln\left(1 + \sigma^2\left(S_1\right) \big/ E\left(S_1\right)^2\right)}$$

$$
\begin{aligned}
& a = \ln\left(E\left(S_1\right)\right) - 0.5b^2 \\
\Leftrightarrow \quad & \ln\left(S_0\right) + \mu - 0.5\sigma^2 = \ln\left(m\right) - 0.5b^2 \\
\Leftrightarrow \quad & S_0 = E\left(S_1\right)\exp\left(-\mu\right).
\end{aligned}
$$

A derivation of I_0 and σ_I for the industry loss distribution I_1 can be done analogously.

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