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# The Effectiveness of Mortality Contingent Bonds under Adverse Selection: A Study of Selected Impact Factors

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# THE EFFECTIVENESS OF MORTALITY CONTINGENT BONDS UNDER ADVERSE SELECTION: A STUDY OF SELECTED IMPACT FACTORS

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### ABSTRACT

Recently, the increasing life expectancy witnessed in most industrialized countries has led to a greater demand by life insurance companies for possibilities to hedge the risk inherent in annuities. In particular, owing to a limited capacity of reinsurance companies, the need for alternatives to hedge against this risk has steadily increased. One of these alternatives is the transfer of longevity risk to the capital market by means of mortality contingent bonds or other capital market instruments. Previous analyses focus on aspects such as pricing, the impact of basis risk, or calibration of the hedge. The aim of this paper is to study the effectiveness of mortality contingent bonds for different selected characteristics of the bonds, including, e.g., its maturity, the investment strategy, or the policyholders' age under different assumptions concerning adverse selection. Toward this end, we use the model of adverse selection put forward by Gatzert and Wesker (2011) and model a survivor bond as proposed by Blake and Burrows (2011), thereby focusing on two default risk measures for analyzing the effectiveness of mortality contingent bonds. Our results show that, although the maturity of the bond should be sufficiently long for hedging to be efficient, the bond does not need to cover the complete maximum duration of the annuities.


## 1. INTRODUCTION

In many countries, the market for annuities and private pensions has increased considerably in recent years, partially owing to initiatives by governments to advocate private retirement saving.[1] At the same time, life expectancy in most industrialized countries has risen substantially, leading to higher than expected payouts for annuities for insurance companies. These two effects have led to an increasing demand by life insurance companies for possibilities to hedge the risk inherent in annuities. One possibility, which has lately received greater attention, is the possibility to transfer longevity risk to capital markets, for example by means of

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1 In Germany, for example, the so called "Riester"-annuity was introduced in 2001 to promote private saving for retirement to complement the social pension program, whereby the government supports these products through direct subsidies to the premium and tax savings (see, e.g. Kling, Russ, and Schmeiser (2006))

mortality contingent bonds (MCB) or standardized instruments such as q-forwards. Although twelve (re-) insurance companies and banks have collaborated to found the Life & Longevity Market Association (LLMA) with the aim to promote a liquid market for capital market instruments designed to hedge mortality and longevity related risks, thus far, there have been rather few successful transactions with respect to transferring longevity risk.[2] One of the obstacles for the creation of a successful and liquid market for mortality-linked securities is the occurrence of basis risk, which arises if the development of mortality in the population underlying the hedge is not perfectly correlated with the development of mortality within the insurer's portfolio. In hedging longevity risk, one important source of basis risk is adverse selection, which here refers to the fact that due to mortality heterogeneity and information asymmetries, mortality for annuitants is in general lower than for the population as a whole.[3] Since the payout of MCBs is often linked to the development of mortality for the entire population, adverse selection might substantially hamper the effectiveness of these instruments. The aim of this paper is therefore to study the impact of different selected characteristics of the MCB, including e.g., the coupon payment and the maturity of the bond, and of the insurance company, for example the riskiness of the asset portfolio, on the effectiveness of MCBs under adverse selection. We thereby focus on the risk situation of an insurance company selling a portfolio of annuities. Furthermore, following Gatzert and Wesker (2011) we analyze the impact of mortality information in underwriting by considering two assumptions differing in the ability of the insurance company to estimate and forecast adverse selection.

For analyzing the impact of adverse selection, annuitant mortality has to be estimated and forecasted. This can be achieved either by means of specifying a model for the relationship between annuitant and population mortality or by means of a two population model. With respect to modeling the relationship between population and annuitant mortality, Plat (2009) focuses on the relative difference between annuitant and population mortality, which is modeled through an age and time dependent portfolio-specific mortality factor. Ngai and Sherris (2011) use a similar approach and assume a portfolio specific mortality factor, which is constant over time and linear in age (this assumption is in line with Stevenson and Wilson (2008)). Brouhns, Denuit, and Vermunt (2002a) instead use a Brass-type relational model to capture the difference between the central death rates for annuitants and the central death rates for the population. A number of authors, including Li and Lee (2005), Jarner and Kryger (2009), Li and Hardy (2011), Cairns et al. (2011), and Dowd et al. (2011), propose two-population mortality models.

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2 Although transactions for hedging longevity risk have been scarce, there have been several examples with respect to hedging mortality risk (i.e., the risk of unexpected high mortality), for example by Swiss Re.

3 See Gatzert and Wesker (2011) for a more detailed discussion of adverse selection and its impact on the risk situation of an insurance company.

Concerning the effectiveness of MCBs (and other types of mortality-linked securities) under adverse selection and the impact of the resulting basis risk, extensive research has been conducted in recent years. The impact of basis risk on a survivor swap has been qualitatively studied by Sweeting (2007) in a utility-maximizing framework. He concludes that in his framework, basis risk is usually smaller than the risk premium that hedgers would be willing to pay and consequently basis risk should not be an obstacle to the creation of a market for longevity risk. In line with his results, Ngai and Sherris (2011), who use a static framework for quantifying basis risk, and Plat (2009), who studies a survivor swap, find that basis risk does not significantly affect hedging effectiveness. Cairns et al. (2011) aim to decompose the hedging effectiveness of a longevity swap by comparing a customized and a standardized longevity swap and find that the most important factors affecting hedging effectiveness are population basis risk, implemented through the differing mortality experience of Continuous Mortality Investigation (CMI) data and mortality for England & Wales, and recalibration risk. Coughlan et al. (2007) take another approach and use historical data to assess the hedging effectiveness of q-forwards for hedging insured lives when the q-forward is based on population mortality. They conclude that from a long-term perspective, the loss in efficiency is rather small. Furthermore, Coughlan et al. (2011) introduce a general framework for assessing basis risk and find in an illustrative example based on UK data that basis risk can be considerably reduced by applying their framework. In this context, Coughlan et al. (2011) define the concept of population basis risk, which refers to basis risk arising from a mismatch of demographic characteristics between the hedged and the underlying population. They contribute this to a mismatch of four factors, namely gender, age, country, and "subpopulation basis," the latter of which refers to hedging a subpopulation with mortality of the population as a whole. The effect of "country" population basis risk has been studied by Li and Hardy (2011), wherein the underlying mortality is based on mortality in the US and the hedged population is Canadian, and by Li and Luo (2011), who evaluate the impact of basing a mortality forward on the UK mortality for hedging mortality in Canada, France, and Scotland, respectively.[4] In this context, Zhou, Li, and Tan (2011) analyze the pricing of mortality-linked securities under population basis risk, which is implemented as subpopulation risk, i.e., the difference between the UK and Scottish mortality as well as the difference between the mortality experience of the UK and the mortality experience implied by the CMI data.

We extend the analysis in Gatzert and Wesker (2011) further and conduct a comprehensive analysis of the impact of different selected characteristics of the MCB, the investment strategy and the policyholders' age, on the hedge effectiveness of MCBs with respect to the risk situation of a life insurance company under different assumptions concerning adverse selection.

[4] Li and Luo (2011) present this study as an example of the effect of basis risk on their proposed calibration method for constructing efficient longevity hedges based on standardized mortality forwards.

We thereby consider the role of mortality information in underwriting by making different assumptions concerning the ability of the insurance company to estimate and price adverse selection as conducted in Gatzert and Wesker (2011). Adverse selection is modeled based on the extension of the Brass-type relational model devised by Brouhns, Denuit, and Vermunt (2002a) and used by Gatzert and Wesker (2011), which allows a difference in the level and trend of annuitant mortality as compared to population mortality. For risk management, we model a survivor bond as introduced by Blake and Burrows (2001), the payout of which is based on the mortality of the population as a whole. Basis risk therefore arises due to adverse selection effects, as the mortality of the population underlying the hedge differs from the mortality of the hedged population. For studying the effectiveness of the MCB in depth for different contract characteristics, the impact of the hedge effectiveness is analyzed under different assumptions about the size of the coupon payment and the maturity of the bond. Furthermore, we assess the impact of different characteristics of the insurance company and the insurance portfolio, that is, the riskiness of the asset base and the age of annuitants.

Our results show that the characteristics of the MCB can have a crucial impact on its effectiveness with respect to decreasing the risk level of an insurance company. In particular, one main result is that when considering the maturity of the bond, we find that the effectiveness of MCBs increases for longer maturities, but the greatest increases in the effectiveness of MCBs can be achieved for shorter durations. However, further increases in the maturities of MCBs of already relatively long durations yield almost negligible increases in effectiveness. This implies that MCBs can prove useful for hedging the risk inherent in annuities even if they do not cover the entire duration of the annuity. Furthermore, we find that the investment strategy has a considerable impact not only on the risk situation of an insurance company but also on the effectiveness of MCBs, whereby MCBs prove most effective for a rather conservative asset strategy. Lastly, the impact of adverse selection and the resulting basis risk on the hedge effectiveness of MCBs increases considerably for older annuitants.

The remainder of the paper is structured as follows. Section 2 introduces the methods for modeling and forecasting annuitant mortality as well as the model of the insurance company, the life insurance contracts considered, and the MCBs studied. Section 3 contains results of the numerical analyses and Section 4 presents the conclusion.

## 2. MODEL FRAMEWORK

In this section, we first present the model for forecasting annuitant mortality and subsequently introduce the model of the life insurance company and the MCB. Lastly, we present the risk measures used to assess the hedge effectiveness of MCBs. The model framework presented in this Section is based on the model introduced in Gatzert and Wesker (2011).

*Modeling and forecasting annuitant mortality*

For estimating and forecasting mortality of the population as a whole, we use the model proposed by Brouhns, Denuit, and Vermunt (BDV) (2002a), which is an extension of the Lee-Carter (1992) model and was also used in the analysis by Gatzert and Wesker (2011).[5] In this model the Poisson-distributed realized number of deaths at age $x$ and time $\tau$, $D_{x,\tau}$, is modeled as

$$D_{x,\tau} \sim Poisson\left(E_{x,\tau} \cdot \mu_x(\tau)\right) \text{ with } \mu_x(\tau) = e^{a_x + b_x \cdot k_\tau}, \quad (1)$$

where $a_x$ and $b_x$ are constant over time and represent the demographic part of the model, while $k_\tau$ is varying over time and constitutes the time series part of the model.[6] Furthermore, $E_{x,\tau} = \left(n_{x-1}(\tau-1) + n_x(\tau)\right)/2$ is the risk exposure at age $x$ and time $\tau$ from which the Poisson-distributed number of deaths $D_{x,\tau}$ arises. $n_x(\tau)$ hereby denotes the number of persons aged $x$ still alive at the end of year $\tau$ (see Brouhns, Denuit, and Vermunt (2002b)).[7]

For estimating the parameters of the BDV (2002a) model Maximum-Likelihood estimation is used whereby the maximization problem can be solved by using a uni-dimensional Newton method as proposed by Goodman (1979). Given the estimated parameters of $a_x$ and $b_x$, forecasts for the values of the time index $k_\tau$ are needed to predict future population mortality $\mu_x(\tau)$. For forecasting $k_\tau$, Lee and Carter (1992) propose to fit an ARIMA process of the form

$$k_\tau = \phi + \alpha_1 \cdot k_{\tau-1} + \alpha_2 \cdot k_{\tau-2} + ... + \alpha_p \cdot k_{\tau-p} + \delta_1 \cdot \varepsilon_{\tau-1} + \delta_2 \cdot \varepsilon_{\tau-2} + ... + \delta_q \cdot \varepsilon_{\tau-q} + \varepsilon_\tau$$

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5 An important advantage of the model used by Brouhns, Denuit, and Vermunt (2002a) is that the restrictive assumption of homoscedastic errors made in the Lee-Carter (1992) model is relaxed. Furthermore, the Poisson distribution is well suited for modeling the number of deaths (see Brillinger (1986)). The Lee-Carter (1992) model is one of the first models for stochastic mortality and is still widely used. In recent literature, however, alternative models have been proposed, which, depending on the respective country and population under consideration, might provide a better fit to the data. For the following analysis, the mortality model mentioned above thus might be substituted with another stochastic mortality model, whereas here the extension of the Lee-Carter (1992) model by Brouhns, Denuit, and Vermunt (2002a) is used as an example to highlight the characteristics influencing the effectiveness of MCBs.

6 For an interpretation of these parameters, see also Gatzert and Wesker (2011).

7 For simulation purposes, the above formula cannot be used, since $n_x(\tau)$ is not known. Following Brouhns, Denuit, and Vermunt (2002b), the formula $E_{x,\tau} = -n(\tau-1) \cdot q_x / \ln(p_x)$ is therefore used instead, whereby $q_x$ is the one year death probability of an $x$-year old and $p_x$ the respective survival probability.

on the estimated values of $k_\tau$, where the order $p$ and $q$ are chosen using methods from time series analysis, $\varepsilon_\tau$ is an error term with $E(\varepsilon_\tau)=0$ and constant variance, and $\phi$ is the drift term. Forecasts of $k_\tau$ are then obtained by replacing the coefficients $\phi$, $\alpha_i$, and $\delta_j$ with their respective estimates and setting $\varepsilon_\tau = 0$.

A general problem in insurance markets are asymmetric information and the resulting adverse selection effects. In the annuities market, adverse selection effects arise from mortality heterogeneity[8] and the inability of the insurance company to access this information and thus to distinguish between individuals with above or below average health. Furthermore, the individual health situation, which is at least partially known to the individual himself, usually influences insurance decisions (see Finkelstein and Poterba (2002)) such that individuals with above average health are more likely to buy annuities. Mortality heterogeneity and asymmetric information thus lead to adverse selection effects giving rise to differences in the level of mortality rates and in their development over time between annuitants and the general population (see, e.g., Brouhns, Denuit, and Vermunt (2002a), Gatzert and Wesker (2011)). Although the BDV (2002a) model can be used to predict mortality for the population as a whole, a separate model relating annuitant mortality to population mortality thus has to be specified to capture adverse selection effects.[9] We therefore use the extension of the brass-type relational model by Gatzert and Wesker (2011), which is based on the model proposed by Brouhns, Denuit, and Vermunt (2002a), given by

$$\ln\left(\mu_{x,\tau}^{ann}\right) = \alpha + \beta_1 \cdot \ln\left(\mu_{x,\tau}^{pop}\right) + \beta_2 \cdot \left(\ln\left(\mu_{x,\tau}^{pop}\right) \cdot \tau_{index}\right) + e_{x,\tau}. \tag{2}$$

In this model annuitant mortality (marked by the superscript "*ann*") is specified as a function of population mortality (marked by the superscript "*pop*") and time $\tau_{index}$, whereby population mortality can be forecasted using the BDV (2002a) model. As stated in Gatzert and Wesker (2011), $\beta_1$ reflects the improvement of annuitant mortality relative to the improvement of population mortality, whereas $\beta_2$ reflects the development of the speed of relative improvement, which is incorporated by means of the interaction term between mortality rates and a time index $\tau_{index}$ (see Gatzert and Wesker (2011)).

Based on the estimated coefficients $\alpha$, $\beta_1$, and $\beta_2$ and the forecasted population mortality $\mu_x^{pop}(\tau)$ given by the BDV (2002a) model, annuitant mortality $\mu_x^{ann}(\tau)$ can be predicted

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8 Mortality heterogeneity here refers to the fact that mortality rates are not identical for all individuals of the same age $x$ but differ depending, for example, on genetic predisposition or life style (see Gatzert and Wesker (2011)).

9 An alternative to this approach would be to use a two-population model for population and annuitant mortality.

whereby the normally distributed error term $e_{x,\tau}$, which is assumed to have zero mean and constant variance, is considered in forecasting to incorporate random deviations from the mean relationship between annuitant and population mortality (see Gatzert and Wesker (2011)). Given the force of mortality $\mu_x^{ann}(\tau)$, the Poisson-distributed number of deaths can be simulated using Equation (1).

When studying the impact of selected characteristics of the MCB and the insurance company on the effectiveness of MCBs under adverse selection, we furthermore concentrate on the role of mortality information in underwriting, which is of special importance due to the scarceness of data on annuitant mortality. Following Gatzert and Wesker (2011), we therefore consider two additional scenarios for adverse selection. On the one hand, we study a scenario that is referred to as "adverse selection misestimated," where the parameters of Equation (2) are misestimated, such that only the difference in the level and not in the trend between population and annuitant mortality is taken into account, i.e. $\beta_1 = 1$, $\beta_2 = 0$, and $\alpha \neq 0$. On the other hand, we consider a scenario wherein the insurance company has gained perfect information about annuitant mortality, for example, by way of experience rating, and is consequently able to estimate the parameters of Equation (2) correctly and to consider this information in pricing and reserving (referred to as "adverse selection perfectly estimated"). Overall, we thus study three different scenarios with respect to adverse selection as shown in Table 1 (see Gatzert and Wesker (2011)).[10]

Assuming a piecewise constant force of mortality $\mu_x(\tau)$, the death probability $q_x(\tau)$, which is the probability that an $x$-year old policyholder dies within the next year, can be calculated as

$$q_x(\tau) = 1 - \exp\left(-\mu_x(\tau)\right)$$

(see Brouhns, Denuit, and Vermunt (2002a), p. 376). Based on this,

$${}_n p_x = \prod_{i=0}^{n-1} p_{x+i}$$

is the probability that an $x$-year old male policyholder survives for the next $n$ years.

[10] In the following, the superscript "*ann*" refers to realized annuitant mortality, whereas the superscript "*A*" refers to the annuitant mortality assumed by the insurance company in pricing and reserving. These can differ due to estimation errors by the insurance company.

**Table 1**: Annuitant mortality under different assumptions concerning adverse selection (see Gatzert and Wesker (2011))

| | Coefficients of Equation (2) | Estimated relationship between annuitant and population mortality |
|---|---|---|
| no adverse selection | $\alpha = 0, \beta_1 = 1, \beta_2 = 0$ | $\mu_x^A(\tau) = \mu_x^{pop}(\tau)$ |
| adverse selection misestimated | $\alpha \neq 0, \beta_1 = 1, \beta_2 = 0$ | $\ln\left(\mu_{x,\tau}^A\right) = \alpha + \ln\left(\mu_{x,\tau}^{pop}\right) + e_{x,\tau}$ |
| adverse selection perfectly estimated | $\alpha \neq 0, \beta_1 \neq 1, \beta_2 \neq 0$ | $\ln\left(\mu_{x,\tau}^A\right) = \alpha + \beta_1 \cdot \ln\left(\mu_{x,\tau}^{pop}\right) + \beta_2 \cdot \left(\ln\left(\mu_{x,\tau}^{pop}\right) \cdot \tau_{index}\right) + e_{x,\tau}$ |

*Note: The superscript "A" refers to estimated annuitant mortality, whereas the superscript "pop" refers to population mortality*

*Modeling a life insurance company*

To gain insights into the effectiveness of MCBs under adverse selection with respect to the risk situation of a life insurance company, we model the life insurance company as a whole, considering assets and liabilities. Table 2 thus shows a simplified balance sheet of the life insurance company at time $t$ = 0.

**Table 2**: Balance sheet of the insurance company at time $t$ = 0 (see Gatzert and Wesker (2011))

| **Assets** | **Liabilities** |
|---|---|
| $S_{low}(0)$ | $E(0)$ |
| $S_{high}(0)$ | $L(0)$ |
| $M_{bond}(0)$ | |

We assume that the insurance company sells $n_A(0)$ immediate annuities paying a yearly annuity $a$ in arrears each year as long as the insured is alive in return for a single premium $SP$ paid in the beginning of the contract (i.e., in $t$ = 0). On the liability side, $L(t)$ denotes the value of liabilities for the sold annuity products at time $t$, whereas $E(t)$ is the value of equity of the insurance company at time $t$. $E(0)$ is thereby given by the initial contribution by shareholders, while $E(t)$ is the difference between assets and liabilities, i.e. it is determined residually. Thus, $E(t)$ can be expressed as

$$E(t) = \begin{cases} E(0) & \textit{for } t = 0 \\ A(t) - L(t) & \textit{for } t = 1, 2, ..., T-1 \end{cases}.$$

The insurance company hereby pays a fraction $r_e$ of the earnings each year, given that they are positive, as a dividend to shareholders (see Gatzert and Wesker (2011)).

The asset side of the modeled insurance company $A(t)$ consists of three elements. The value of the mortality contingent bond $M_{bond}(t)$ and the capital base available for investments in the capital market S($t$), whereby $S(t)$ is further divided into the market value of high and low risk assets $S_{high}(t)$ and $S_{low}(t)$ (see Gatzert and Wesker (2011)).

The initial capital base $S(0)$ is given by

$$S(0) = E(0) + n_A(0) \cdot SP_A - n_B \cdot \Pi_{x,M} ,$$

where $n_A(0)$ is the number of annuities sold, $n_B$ is the number of MCBs purchased and $\Pi_{x,M}$ is the premium for the MCB with maturity $M$ based on a population aged $x$ in the beginning of the contract. Subsequent values of $S(t)$ can be calculated by taking into account the cash-flows occurring each year, i.e.

$$S(t) = S_{high}(t) + S_{low}(t) - n_A(t) \cdot a + n_B \cdot X(t) - div(t), \tag{3}$$

where $n_A(t)$ is the number of annuitants still alive at the end of year $t$ and $X(t)$ is the coupon payment from one MCB (see Gatzert and Wesker (2011)). Concerning the investment strategy, we assume that the insurance company redistributes assets each year in such a way that a constant fraction $\alpha$ is invested in low risk assets, i.e. $S_{low}(0) = \alpha \cdot S(0)$ and $S_{high}(0) = (1-\alpha) \cdot S(0)$ (see Gatzert and Wesker (2011)). We assume that the market value of high and low risk assets, $S_i(t)$, *i* = *low, high*, follows a geometric Brownian motion with constant drift $\mu_i$ and volatility $\sigma_i$ for *i* = *low, high*.[11]

The total value of assets at time $t$ $A(t)$ is then given by the sum of the capital base $S(t)$ and the value of the MCB $M_{bond}(t)$ (see Gatzert and Wesker (2011)), i.e.

$$A(t) = S(t) + M_{bond}(t).$$

*Valuation of insurance liabilities*

Assuming independence between market and mortality risk (see, e.g., Carriere (1999, p. 340)) and following Gatzert and Wesker (2011), the insurance contracts can be evaluated using risk-neutral valuation. Consequently, the value of liabilities $L(t)$ is calculated as

[11] See Gatzert and Wesker (2011) for more details on the development of the geometric Brownian motion.

$$L(t) = n_A(t) \cdot \sum_{s=1}^{T-t} a \cdot {}_s p_{x+t}^A \cdot (1+r)^{-s} ,$$

with $T$ denoting the maximum duration of the annuity and $r$ denoting the risk-free rate. The superscript $A$ in the survival probabilities refers to the assumed annuitant mortality, which depends on the assumptions concerning adverse selection as shown in Table 1.[12] For calculating the annuity $a$ the actuarial equivalence principle is used, so that expected premium payments are equal to expected benefit payouts,[13] which can be expressed in the following manner:

$$\sum_{t=1}^{T} a \cdot {}_t p_x^A \cdot (1+r)^{-t} = SP . \tag{4}$$

*Modeling and valuation of a simple mortality contingent bond*

As an example of an MCB, following Gatzert and Wesker (2011) we model a so-called survivor bond as proposed by Blake and Burrows (2001), which is a coupon-based MCB.[14] The insurance company hereby receives an annual coupon payment $X(t)$ at the end of each year $t$ = 0, …, $M$-1, which is proportional to the number of survivors in a given reference population $n_{ref}(t)$. In return for these coupon payments, a premium $\Pi_{x,M}$ is paid in advance, i.e. in $t$ = 0, where $x$ denotes the age of the reference population of the MCB and $M$ is the duration of the bond. Furthermore, the insurance company can account for the value of the bond $M_{bond}(t)$ on the asset side of the balance sheet as shown in the previous Section.

Concerning the pricing of MCBs, an overview and comparison of different pricing methods is provided by, e.g., Bauer, Börger, and Russ (2010). Differences in pricing approaches result, for example, from different assumptions on the underlying processes for mortality and the application of different valuation approaches. In this paper, we follow Gatzert and Wesker (2011) and apply the pricing approach of the EIB/BNP Paribas bond to determine the premium of the MCB.[15] The coupon payments are hereby discounted using the risk free rate $r$ minus a certain risk premium $\lambda$ (see Cairns et al. (2005)), so that the premium for a bond with

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[12] In these formulas, the time subscript $\tau$ in the (age and time dependent) death and survival probabilities has been dropped for ease of illustration.

[13] In pricing, we do not consider the probability of default since we assume that the insurance benefits will continue to be paid out in case of a default (see also, e.g., Gatzert and Kling (2007, p. 553)), for example, because these are guaranteed by a guaranty fund.

[14] Blake, Cairns, and Dowd (2006), amongst others, offer a comprehensive overview for MCBs and other capital market instruments.

[15] While the EIB/BNP Paribas bond was withdrawn due to lack of interest, Blake et al. (2006) as well as Bauer, Börger, and Russ (2010), attribute this failure to weaknesses in design rather than mispricing.

duration $M$ based on a reference population aged $x$ at inception that pays out $X(t)$ in year $t$ can is given by

$$\Pi_{x,M} = \sum_{t=0}^{M-1} E\left(X(t)\right)\cdot(1+r-\lambda)^{-(t+1)},$$

with $n_B$ being the number of MCBs purchased at time 0. The annual payment $X(t)$ is then given by

$$X(t) = \frac{n_{ref}(t)}{n_{ref}(0)}\cdot C, \tag{5}$$

where $n_{ref}(0)$ is equal to an arbitrary number,[16] $n_{ref}(t)$ is the number of survivors in the reference population at time $t$, and $C$ is the initial coupon payment set in the contract (see Gatzert and Wesker (2011)). Mortality in the reference population is equal to population mortality and thus higher than annuitant mortality, so that $n_{ref}(t)$ is given by

$$n_{ref}(t) = n_{ref}(t-1) - d_{ref}(t),$$

where $d_{ref}(t)$ is the number of persons who died within year $t$ given by

$$d_{ref}(t) \sim Poisson\left(E_{x,t}^{ref}\cdot\mu_x^{pop}(t)\right) \text{ and } \mu_x^{pop}(t) = e^{a_x+b_x\cdot k_t},$$

where $E_{x,t}^{ref}$ is the exposure to risk of the reference population (see Gatzert and Wesker (2011)).[17] Hence, basis risk arises in the longevity hedge since annuitant mortality is equal to $\mu_x^{ann}(\tau)$ and therefore different from the mortality underlying the hedge.

Turning now to the calculation of the value of the bond at time $t$, $M_{bond}(t)$ is given by

$$M_{bond}(t) = n_B\cdot\sum_{j=t}^{M-1} E_t\left(X(j)\right)\cdot(1+r-\lambda)^{-(j-t+1)}, t=0,...,M-1.$$

i.e., the value of one MCB is determined as the expected present value of future cash flows given the information available at time $t$. To calculate $M_{bond}(t)$ this value is multiplied by the number of MCBs purchased at time $t$ = 0 ($n_B$) (see Gatzert and Wesker (2011)).

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16 As the coupon payment is expressed in relative terms, the value of $n_{ref}(0)$ does not affect the results.

17 Here, $E_{x,t}^{ref}$ is given by $E_{x,t}^{ref} = -\left(n_{ref}(t-1)\cdot q_x^{pop}\right)/\ln\left(p_x^{pop}\right)$ (see Brouhns, Denuit, and Vermunt (2002b)).

*Risk measurement*

Following Gatzert and Wesker (2011), we focus on the hedge effectiveness of MCBs with respect to the risk situation of a life insurance company, which we measure through two downside risk measures. Since we model the insurance company in a multi-period framework dynamic default during the contract term is taken into account, whereby $T_d = \inf\left\{t : A(t) < L(t)\right\}$ represents the time of default. For risk measurement, the probability of default (PD) is defined as

$$PD = P\left(T_d \leq T\right),$$

(see, e.g., Kling, Richter, and Russ (2007), Gerstner et al. (2008), Gatzert and Wesker (2011)), while the mean loss (ML) is given by

$$ML = E\left[\left(L\left(T_d\right) - A\left(T_d\right)\right)\cdot\left(1+r\right)^{-T_d}\cdot 1\left\{T_d \leq T\right\}\right],$$

(see Gatzert and Wesker (2011)), i.e., the mean loss is an LPM(1) at the time of default discounted to $t = 0$, whereby $1\left\{T_d \leq T\right\}$ denotes the indicator function.[18] Thus, while the probability of default takes only the frequency of shortfall into account, the mean loss also considers the extent of default.[19]

## 3. Numerical Analysis

Until otherwise stated, we will assume that the life insurance company sells $n_A(0) = 10{,}000$ annuities to $x = 65$ year old male policyholders in the year 2012. The maximum age attainable as implied by the BDV (2002a) model is 100 so that the maximum duration of an annuity $T$ is 35 years.[20] Assuming a risk-free interest rate of $r = 3\%$ and a single premium $SP = 10{,}000$, the fair annuity depends on the assumptions concerning adverse selection. Concerning the investment opportunities, we assume a drift (volatility) of $\mu_{low} = 6\%$ ($\sigma_{low} = 8\%$) for the low-risk assets and $\mu_{high} = 10\%$ ($\sigma_{high} = 24\%$) for the high-risk assets as well as a correlation of $\rho = 0.1$. The initial equity is set to $E(0) = 10$ Mio and the percentage of earnings distributed to shareholders is $r_e = 25\%$. Concerning the MCB, in the base case we assume that the maturity $M$ is equal to the maximum duration of the annuity $T$, i.e. $M = T$, such that the longevity hedge

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[18] The indicator function is thereby equal to one if the condition in the brackets is satisfied and zero otherwise.

[19] See Gatzert and Wesker (2011) for a more detailed interpretation of the mean loss.

[20] Assuming a maximum age of 100 might be considered too low. The scarcity of data especially at high ages nevertheless inhibits a reliable estimation of parameters at these ages. In the software accompanying the LifeMetrics index, for example, a maximum age of only 89 is recommended (see http://www.jpmorgan.com/pages/ jpmorgan/investbk/solutions/lifemetrics/software).

covers the complete duration of the annuity, $n_B = n_A(0)$, i.e. the insurance company purchases one MCB for each annuity it sells and an initial coupon payment $C = 100$. Following Gatzert and Wesker (2011), we set the loading $\lambda = 0$, since we do not assume any systematic mortality risk. These parameters were chosen for illustrative reasons and are subject to robustness tests.

To evaluate the effectiveness of MCBs under different assumptions concerning adverse selection, Monte-Carlo simulation is employed, whereby we simulate 100,000 paths for the asset portfolio and the realized mortality. To improve the comparability of results, the same sequence of random numbers was used for each simulation run.[21] As illustrated above, the value of the MCB at time $t$ depends on the information available at time $t$, so that valuation is conducted path-dependently for all 100,000 possible realizations of $n_{ref}(t)/n_{ref}(0)$ at each time $t$. The calculation of $M_{bond}(t)$ is thereby based on 1,000 simulation runs of future mortality, since computational intensity restricts a higher number of simulation runs; however, this is still enough to ensure robust results.[22]

*Estimation of annuitant mortality*

The estimation of population mortality is based on the central deaths rates for the UK from 1950 to 2009 available through the Human Mortality Database. The estimated parameters of the BDV (2002a) model are displayed in Figure 1 a) – c), whereby Figure 1 c) also displays the forecasted values of $k_\tau$, which are based on the estimated ARIMA process. Time series analysis indicates a Random Walk ($p = q = 0$) as sufficient to describe the dynamic of the mortality index. Subsequent residual analysis using Box-Ljung test as well as ACF and PACF analysis showed no significant residual autocorrelation. Thus, the forecasts shown below were calculated using an ARIMA (0,1,0) process with drift $\phi = -1.5403$ (standard error 0.3056).

To estimate adverse selection and thus annuitant mortality, data on the UK annuitant mortality from the CMI from 1947 to 2000 is used.[23] During this period, five mortality tables were published in the years 1947, 1968, 1980, 1992 and 2000, respectively. Applying the model for adverse selection on these data points, the results imply a coefficient $\beta_1 = 1.1618$ (0.0123) and a coefficient $\beta_2 = -0.0004$ (0.0002) (robust standard errors in parenthesis). This indicates a faster improvement of annuitant mortality as compared to the population as a whole; however,

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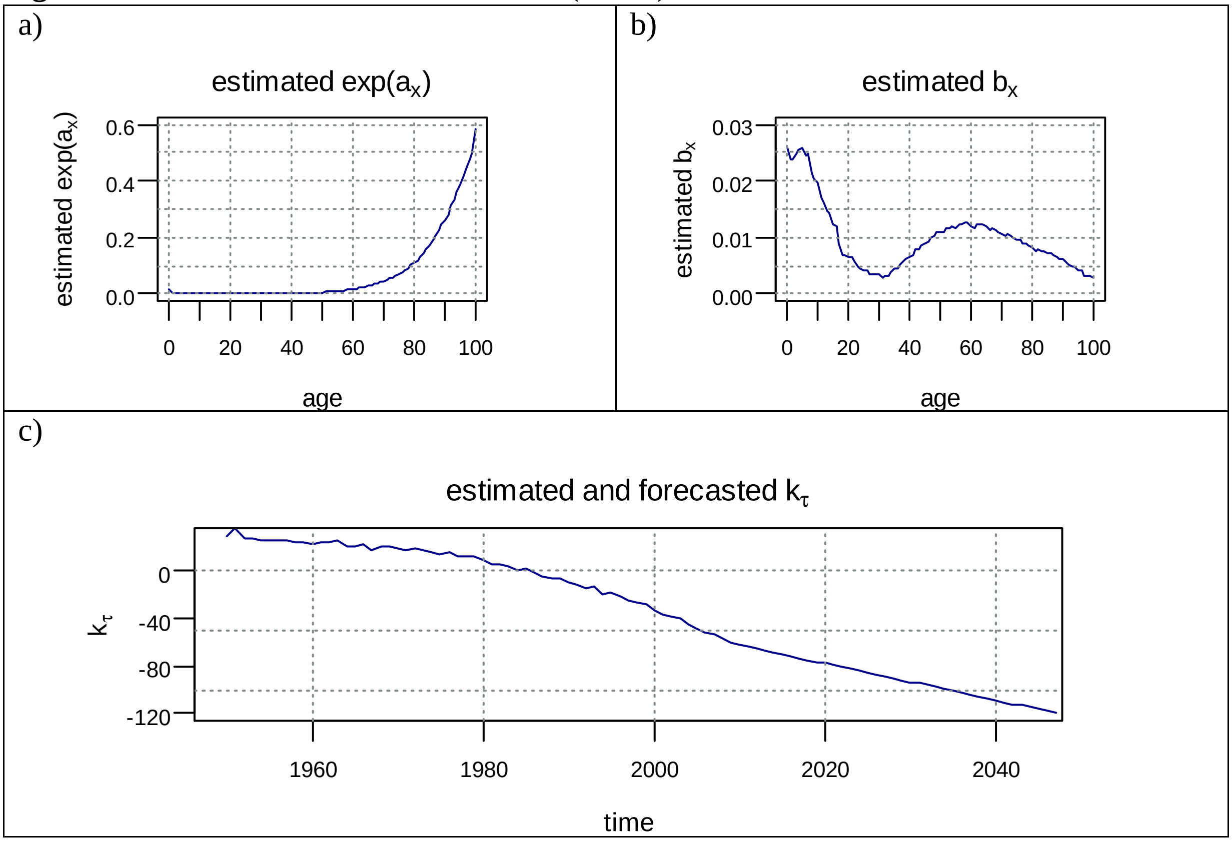
21 Concerning the robustness of results, the results are stable with respect to different sequences of random numbers.

22 The standard error of Monte-Carlo simulation for the value of the MCB at $t = 1$ $M_{bond}(1)$ is approximately 0.0322, whereas the expected value of $M_{bond}(1)$ is approximately 12 for an initial coupon payment of $C = 1$. The exact standard error depends on the path considered and the values for the standard error lie between 0.0291 and 0.0354.

23 This data is also used, for example, by Ngai and Sherris (2011) and Gatzert and Wesker (2011) to calibrate adverse selection.

this overimprovement decreases over time as shown by the negative coefficient of the interaction term between year $\tau_{index}$ and population mortality $\beta_2$.[24] The estimated intercept $\alpha$ is equal to -0.0275 (0.0198) and the estimated standard error of residuals $e_{x,t}$ is 0.1292, whereby the residuals are also considered for each year $t$ and age $x$ in forecasting.

**Figure 1**: Parameter estimates for the BDV (2002a) model




As shown in Table 1, three assumptions concerning adverse selection are made. We first assume no adverse selection, i.e. we assume annuitant and population mortality to be identical. Second, we differentiate between the ability of the insurance company to forecast and consequently consider adverse selection in pricing. In line with Gatzert and Wesker (2011), these analyses are intended to highlight the importance of mortality information in underwriting. Table 3 shows the estimated respectively assumed coefficients of Equation (2) as well as the implied remaining life expectancy of a 65-year-old male annuitant in the year 2012[25] and the fair annuity calculated according to Equation (4).

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[24] Note that $\tau_{index} = 1950 - \tau$, where 1950 is the first year for which mortality data is used and $\tau$ is the year under consideration.

[25] The remaining life expectancy of an $x$ year old in year $t$ $e_x(t)$ is given by $e_x(t) = \sum_{k \geq 0} \prod_{j=0}^{k} p_{x+j}(t+j)$ (see Brouhns, Denuit, and Vermunt (2002b)).

In the absence of adverse selection, population and annuitant mortality is identical resulting in a remaining life expectancy of 18.51 years for a 65-year-old annuitant and a fair annuity of $a$ = 748. When adverse selection is misestimated,[26] the estimated intercept $\alpha$ is equal to $\alpha$ = -0.2779, resulting in an assumed remaining life expectancy of 20.60 and a fair annuity of $a$ = 688, as shown in Table 3. Otherwise, when assuming that the insurance company is perfectly able to estimate adverse selection, the estimated parameters imply a remaining life expectancy for a 65-year-old annuitant of 21.58 and consequently a fair annuity $a$ = 663. As seen in Table 3, assumed and realized mortality differ for the case when adverse selection is misestimated. Assumed mortality is higher than realized mortality, which results in an expected life expectancy that is underestimated, such that on average, the insurance company has to pay out approximately one annuity more than expected.

**Table 3**: Estimated parameters, remaining life expectancy, and fair annuity under different assumptions concerning adverse selection

| | Coefficients of Equation (2) | Assumed annuitant mortality $q_x^A$ (life expectancy $e_{65}(2012)$) for pricing and reserving | Realized annuitant mortality $q_x^{ann}$ (life expectancy $e_{65}(2012)$) for risk measurement | Fair annuity $a$ |
|---|---|---|---|---|
| without adverse selection | $\alpha = 0$, $\beta_1 = 1$, ` $\beta_2 = 0$ | $q_x^A = q_x^{pop}$ (18.51) | $q_x^{ann} = q_x^{pop}$ (18.51) | 748 |
| adverse selection misestimated | $\alpha = -0.2779^{***}$, $\beta_1 = 1$, $\beta_2 = 0$ | $q_x^A = q_x^{ann^*,\alpha\neq 0,\beta_1=1,\beta_2=0}$ (20.60) | $q_x^{ann} = q_x^{ann}$ (21.58) | 688 |
| adverse selection perf. estimated | $\alpha = -0.0275$, $\beta_1 = 1.1618^{***}$, $\beta_2 = -0.0004^{**}$ | $q_x^A = q_x^{ann}$ (21.58) | $q_x^{ann} = q_x^{ann}$ (21.58) | 663 |

*Note: ***, **, and * denote values significant at the 1%, 5% and 10% levels, respectively.*

*Effectiveness of MCBs for different contract characteristics of the MCB*

In this section, we study the impact of different contract characteristics of the MCB on the effectiveness of MCBs with respect to the risk situation of the insurance company under different assumptions concerning adverse selection. We focus on the effect of the initial coupon payment $C$ and the maturity of the bond $M$. The impact of a differing maturity of the bond $M$ on the effectiveness of MCBs under different assumptions concerning adverse selection is shown in Figure 2. Under these assumptions, the risk situation of the insurance company remains constant if no MCB is purchased and is shown only for comparison. When comparing the red line with crosses and the blue line with triangles, one can see that the risk of an insur-

26 When aderverse selection is misestimated, only the difference in the level and not in the trend of annuitant mortality is taken into account as described in the model section, i.e. $\beta_1 = 1$, $\beta_2 = 0$, and $\alpha \neq 0$.

ance company can be reduced considerably through the use of MCBs but that the amount of risk reduction achievable depends substantially on the assumptions about adverse selection and the maturity of the MCB, *M*.

In general, when considering the risk level of an insurance company, as measured through the probability of default and the mean loss if no MCB is purchased for risk management, the risk level of the insurance company is higher when adverse selection is present (see Figure 2, Part b)) than under no adverse selection (Figure 2, Part a)), whereby a misestimation of adverse selection further increases the risk level of an insurance company as compared to the case when adverse selection is perfectly estimated (Figure 2, Part b, i) and ii), respectively).[27] Furthermore, the relative risk reduction achievable by means of MCBs decreases under adverse selection, whereby especially a mispriced adverse selection hampers the effectiveness of MCBs, as can be seen by the dashed line in Figure 2, which shows the relative risk reduction achievable through the use of MCBs.[28] For example, although the mean loss can be reduced by 53.8% when no adverse selection is assumed by purchasing a MCB with a maturity of $M$ = 20, the risk reduction that is achievable under adverse selection amounts to only 47.0% when adverse selection is misestimated and to 51.8% when adverse selection is perfectly estimated. These results are in line with those in Gatzert and Wesker (2011).

Concerning the impact of differing maturities $M$, the results show that, in general, an increase in the maturity of the MCB $M$ comes along with a decrease in the risk level of an insurance company and thus with an increase in the effectiveness of MCBs.[29] When considering the marginal increase in the effectiveness of the MCB, however, the results imply that the marginal increase in the effectiveness of MCBs is diminishing, i.e. the greatest increase in the effectiveness of MCBs can be achieved for the shorter durations studied ranging from about 5 to 20 years. For example, when increasing the maturity of the MCB from $M$ = 5 to $M$ = 15 years when no adverse selection is present, the probability of default can be reduced by 45.4% for $M$ = 15 as compared to 13.3% for $M$ = 5, which corresponds to a substantial reduction in the probability of default from 0.53% when purchasing MCBs with maturity $M$ = 5 to 0.33% when purchasing MCBs with maturity $M$ = 15. Otherwise, when increasing the maturity of the

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[27] For a more comprehensive analysis of the impact of adverse selection on the risk situation of an insurance company, see Gatzert and Wesker (2011).

[28] The relative difference is here defined as $\left(PD_{without\ MCB} - PD_{with\ MCB}\right) / PD_{without\ MCB}$ for the probability of default and $\left(ML_{without\ MCB} - ML_{with\ MCB}\right) / ML_{without\ MCB}$ for the mean loss.

[29] Please note that all analyses conducted in this paper are *ceteris paribus* analyses. Thus, they serve to identify the impact of the change in one influence factor keeping all other factors constant. It might be possible to enhance the effectiveness of MCBs by combining different factors studied in this paper. For example, the effectiveness of an MCB with a rather short duration might be improved through increasing the coupon $C$ or changing the investment strategy.

MCB from $M$ = 30 to $M$ = 35 years, the reduction in the probability of default corresponds to only 59.7% for $M$ = 30 as compared to 60.2% for $M$ = 35 (corresponding to a reduction in the probability of default from 0.25% for $M$ = 30 to 0.24% for $M$ = 35), whereby this increase is almost negligible. The results thus imply that in this model framework, under the stated assumptions, the effectiveness of MCBs is not substantially hampered if the duration of the MCB does not cover the maximum duration of the annuity, if the difference between the duration of the bond $M$ and the maximum duration of the annuity $T$ is not too large.[30] However, if the duration of the MCB is too short, the risk reduction achievable by means of MCBs is reduced substantially.

Lastly, considering the impact of adverse selection for different maturities $M$ of the MCB, the results imply that the loss in efficiency of MCBs through adverse selection and the resulting basis risk decreases for a longer maturity $M$ of the bond, which indicates that the impact of basis risk brought along by adverse selection can be reduced by increasing the duration of the hedge. For example, when considering the relative difference between the risk reduction achievable through an MCB with adverse selection misestimated and without adverse selection, the results show that this difference corresponds to approximately 15% for a maturity of $M$ = 5, whereas it amounts to only approximately 11% for $M$ = 35.[31] Overall, our results imply that a longer duration of the MCB leads, on the one hand, to an increase in the efficiency of MCBs in general, and on the other hand, decreases the impact of adverse selection and the resulting basis risk. When hedging the risk inherent in annuities, the maturity of the MCB should thus be sufficiently great for longevity hedging to be efficient.

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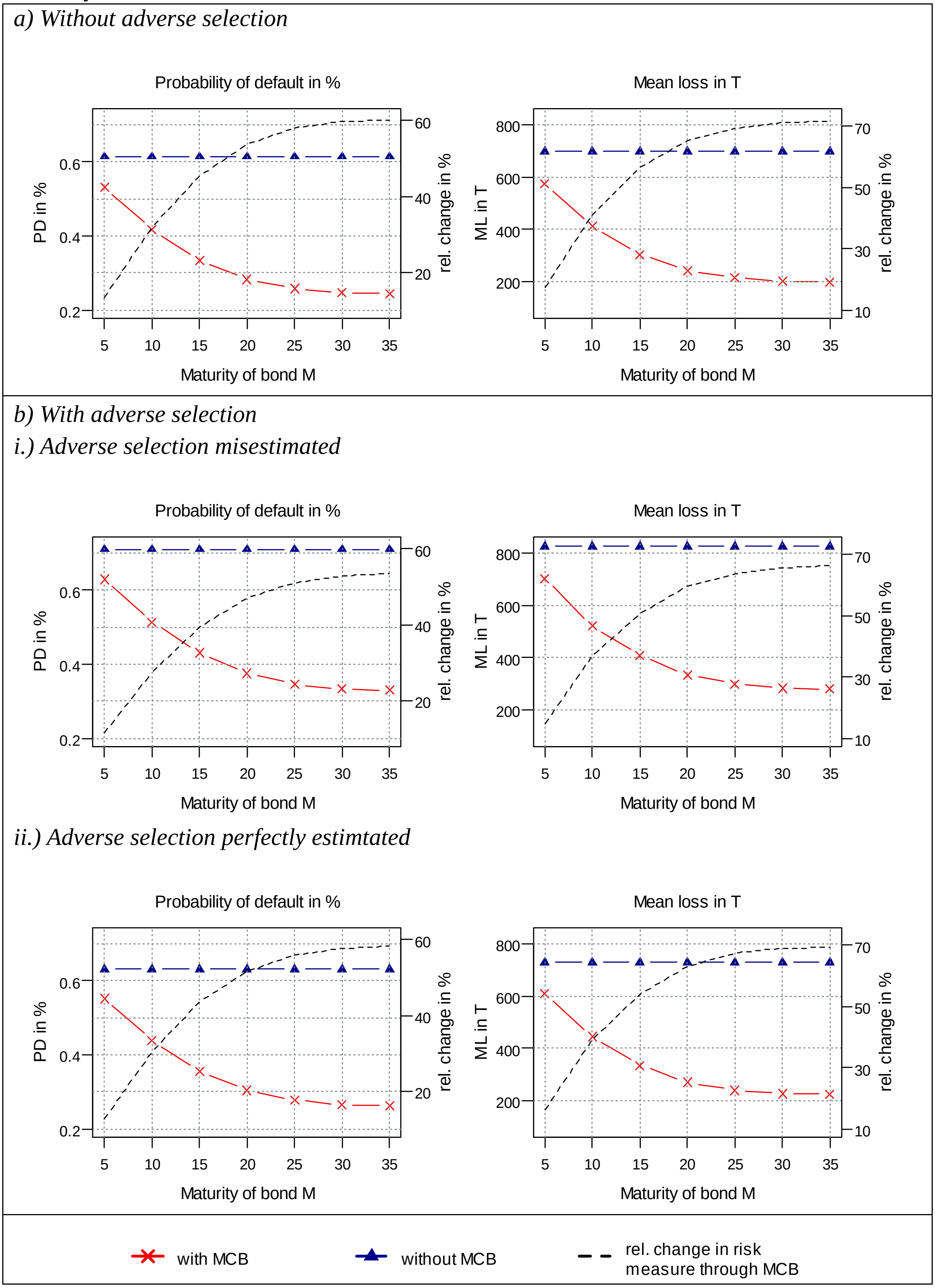
[30] These results thus imply that the duration of the BNP Paribas bond, which amounted to 25 years and therefore did not cover the maximum duration of the annuities, might not have resulted in a considerable loss in efficiency.

[31] This difference is defined as $\left(rel_{PD}^{no\ adverse\ selection} - rel_{PD}^{adverse\ selection\ misestimated}\right) \Big/ rel_{PD}^{no\ adverse\ selection}$ for the example of the probability of default with $rel_{PD} = \left(PD_{without\ MCB} - PD_{with\ MCB}\right) \Big/ PD_{without\ MCB}$ as the relative risk reduction achievable by means of MCBs defined as above.

**Figure 2**: The impact of risk management using mortality contingent bonds (MCBs) on an insurer's risk situation under different assumptions concerning adverse selection for different maturity *M* of the MCB

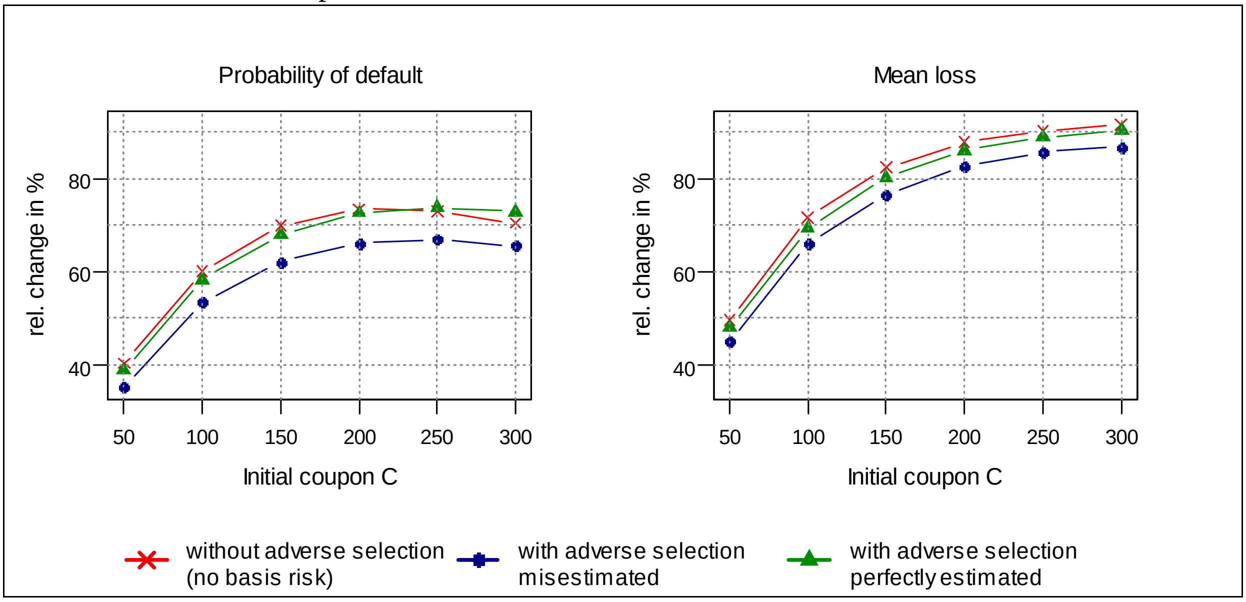


Turning now to the impact of the initial coupon $C$, Figure 3 shows the relative risk reduction achievable by means of MCBs under different assumptions concerning adverse selection for different values of the initial coupon $C$. While a higher value of the initial coupon $C$ comes along with higher payments to the insurance company during the contract term (see Equation 5), it also leads to a higher premium for the MCB which has to be paid in $t = 0$. Thus, there exists a trade-off between higher coupon payments and a higher premium. The results show that in this model framework under the stated assumptions generally in the considered range, a higher initial coupon $C$ *ceteris paribus* leads to a greater reduction in the risk level of an insurance company. However, for the probability of default, the maximum risk reduction is, under the stated assumptions for the considered discrete values, reached at approximately 200 to 250, depending on the assumption about adverse selection, whereas the risk reduction achievable through the use of MCBs decreases for higher values of $C$. This might be owing to the higher premium, which has to be paid in the beginning of the contract, associated with the higher coupon payment, which decreases the asset base and thus the amount of money available for investment in the capital market, thereby possibly leading to a higher default probability. For the mean loss, the results show that, in the considered range, a higher coupon payment $C$ leads to a higher risk reduction, but that, the extent of the efficiency increase decreases for higher coupon payments. For example, although an increase in the initial coupon $C$ from 50 to 100 leads to an increase in the relative risk reduction from 49.7% for $C = 50$ to 71.6% for $C = 100$ (which corresponds to a decrease in the mean loss from 351 for $C = 50$ to 198 for $C = 100$), an increase from $C = 250$ to $C = 300$ implies a relative risk reduction of 90.2% for $C = 250$ as compared to 91.5% for $C = 300$ (corresponding to a decrease in the mean loss from 68 for $C = 250$ to 59 for $C = 300$), whereby this increase is substantially smaller.

Although the direction of effects is identical for all three assumptions concerning adverse selection, the maximum relative risk reduction achievable for the probability of default through the use of MCBs is reached for lower coupon payments when no adverse selection is present than under perfectly estimated adverse selection. This implies that for relatively high coupon payments, the risk reduction achievable by means of MCBs is slightly higher under perfectly forecasted adverse selection as compared to no adverse selection. This effect might be due to very high coupon payments from the MCBs in the initial contract years, in which a high percentage of the reference population is still alive. In these years, the payouts for annuities are smaller when adverse selection is perfectly forecasted as compared to the case when no adverse selection is present, owing to the lower fair annuity $a$ (see Table 3). Under perfectly forecasted adverse selection, the insurance company might be able to build up sufficient reserves in the early policy-years to outbalance the lower coupon payments in later policy-years.

Concerning the case where adverse selection is misestimated, the risk reduction achievable through the use of MCBs is considerably lower than under perfectly estimated adverse selection as found in the previous analysis as well as in the analyses by Gatzert and Wesker (2011). When considering the impact of a higher coupon payment, the effects are almost identical to the case where no adverse selection was assumed. For example, the results show that for the probability of default the trade-off between the higher premium and the higher coupon payments, leads to a reduction in the effectiveness of MCBs for a relatively high initial coupon *C*. For the mean loss otherwise, the effectiveness of the MCB increases steadily in the considered range.

**Figure 3**: The impact of risk management using mortality contingent bonds (MCBs) on an insurer's risk situation under different assumptions concerning adverse selection for different values of the initial coupon *C*




Summing up, our results indicate that MCBs are very effective for rather low initial coupon payments *C*, while very high initial coupons might even reduce the hedge effectiveness of MCBs. Furthermore, for very high coupon payments, the impact of adverse selection and the resulting basis risk vanishes if adverse selection can be perfectly forecasted, which might be owing to the complex interaction between assets and liabilities and the time structure of cash flows, which differ for the varying assumptions concerning adverse selection.

*Effectiveness of MCBs under different characteristics of the insurance company and the insurance portfolio*

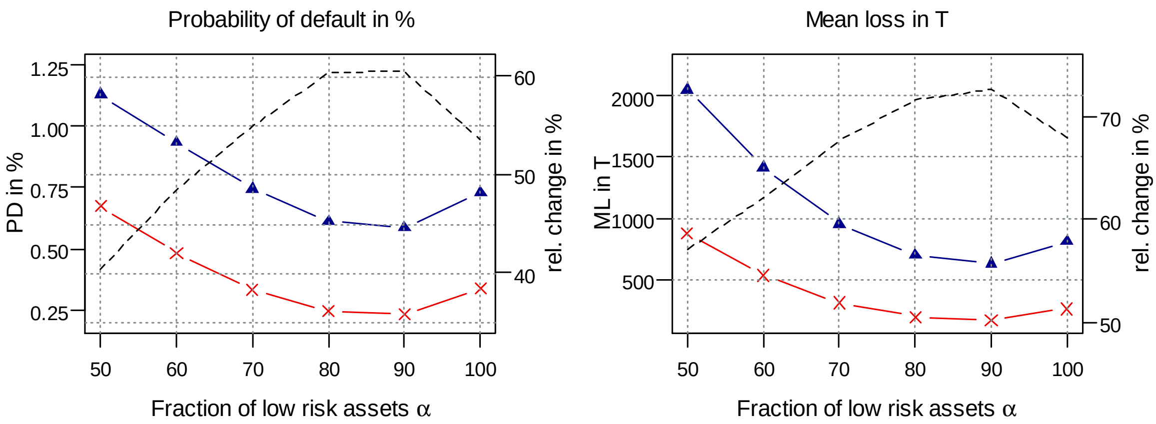
Beside the characteristics of the MCB itself, the characteristics of the insurance company, for example, its investment strategy or the characteristics of the hedged insurance portfolio, e.g., the policyholders' age, might influence the effectiveness of MCBs for reducing the risk level of an insurance company. Therefore, in this section, we analyze the impact of different asset allocations and the influence of the policyholders' age on the effectiveness of MCBs and the impact of different types of adverse selection on the effectiveness of MCBs.

Figure 4 shows the risk of an insurance company for varying investment strategies as reflected in different fractions α of low risk assets with and without purchasing an MCB based on different assumptions concerning adverse selection. The results show that the investment strategy has a substantial impact on the risk level of an insurance company, whereby a higher fraction of low risk assets α in general leads to lower risk, except for α = 100%, for which the risk level increases. This effect might be due to the effect of the expected return, as a more conservative asset strategy, i.e. a higher fraction of low risk assets α, while coming along with lower risk as reflected in lower volatility, also implies a lower expected return. For an investment in only low risk assets, the effect of the lower expected return appears to outweigh the impact of the lower volatility, which leads to an increase in both the probability of default and the mean loss, whereas for a lower fraction of low risk assets, the effect is reversed and a higher volatility implies a higher probability of default and mean loss despite a higher expected return. Under the stated assumptions for the considered discrete values, the risk level of the insurance company, as measured by both the probability of default and the mean loss, is thus minimal for approximately α = 90% or α = 80% for all assumptions concerning adverse selection.

Concerning the effectiveness of MCBs for different investment strategies, the results imply that the investment strategy has a substantial impact on the relative risk reduction achievable by means of MCBs as reflected in the black dashed line in Figure 4. When no adverse selection is present or when adverse selection can be perfectly forecasted, the highest risk reduction is achieved when the insurance company invests approximately α = 90% in low risk assets, that is, for the investment strategy coming along with the lowest risk level. Under imperfectly estimated adverse selection, the risk reduction achievable for both the probability of default and the mean loss is slightly higher for a fraction of low risk assets α = 80%. Otherwise, in the considered range for the chosen discrete values, the relative risk reduction reaches its minimum for α = 50%, at the highest risk level in the analyzed range. The results thus indicate that MCBs can prove especially efficient for a conservative investment strategy and consequently an insurance company with a low level of default risk, since the relative risk reduction achievable through the use of MCBs is highest for a rather conservative investment strategy.

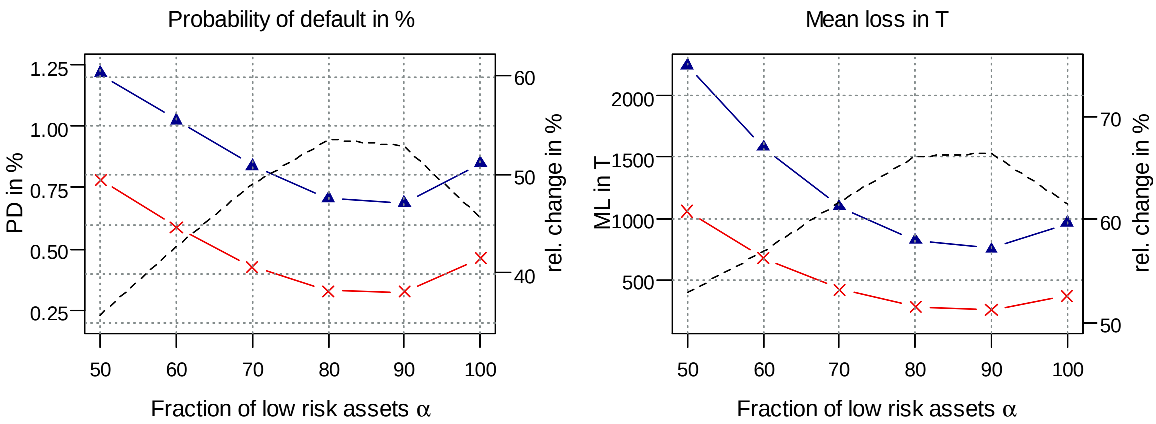
**Figure 4**: The impact of risk management using mortality contingent bonds (MCBs) on an insurer's risk situation under different assumptions concerning adverse selection for different asset allocation

*a) Without adverse selection*

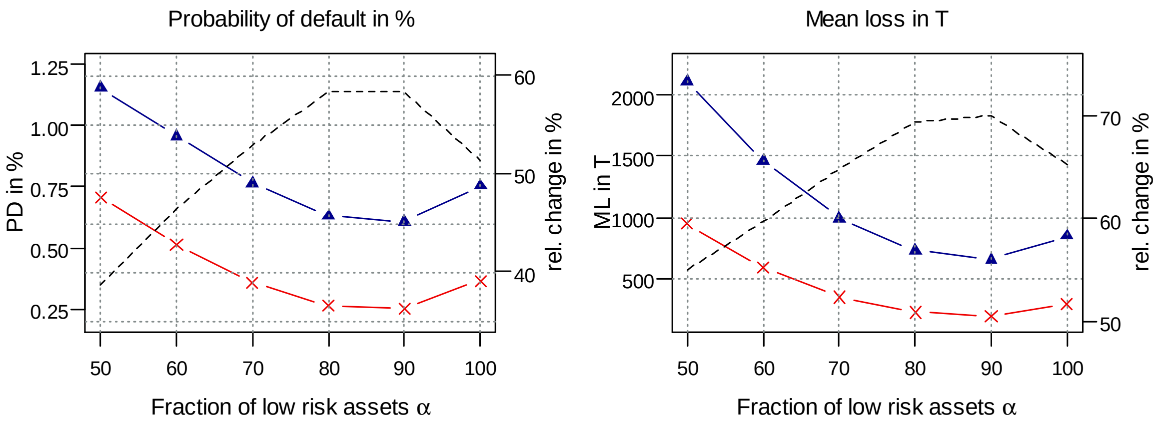



*b) With adverse selection*

*i.) Adverse selection misestimated*




*ii.) Adverse selection perfectly estimtated*




When studying the impact of adverse selection on the effectiveness of MCBs for different investment strategies, one can see that the development of the relative risk reduction is almost identical for all assumptions concerning adverse selection, i.e. a higher fraction of low risk assets $\alpha$ leads to a greater effectiveness of MCBs till about $\alpha$ = 80% or $\alpha$ = 90%, while increasing $\alpha$ further lowers the risk reduction achievable through MCBs.[32]

Lastly, the impact of the policyholders' age $x$ on the effectiveness of MCBs is studied. We vary the age of the annuitant from 60 to 80 years.[33] The effectiveness of MCBs for varying age $x$ under different assumptions concerning adverse selection is illustrated in Figure 5, which shows the relative reduction in the probability of default and the mean loss achievable through the purchase of MCBs. The results show that in general under the stated assumptions for the considered range, the effectiveness of MCBs decreases with increasing age of policyholders. Without adverse selection, for example, the probability of default can be decreased by approximately 65% for a portfolio of annuitants aged $x$ = 60, whereas the risk reduction effect amounts to only 39% for a portfolio of annuitants aged $x$ = 80. One reason for this effect might be the shorter duration of the MCB and consequently the fewer coupon payments for older annuitants.

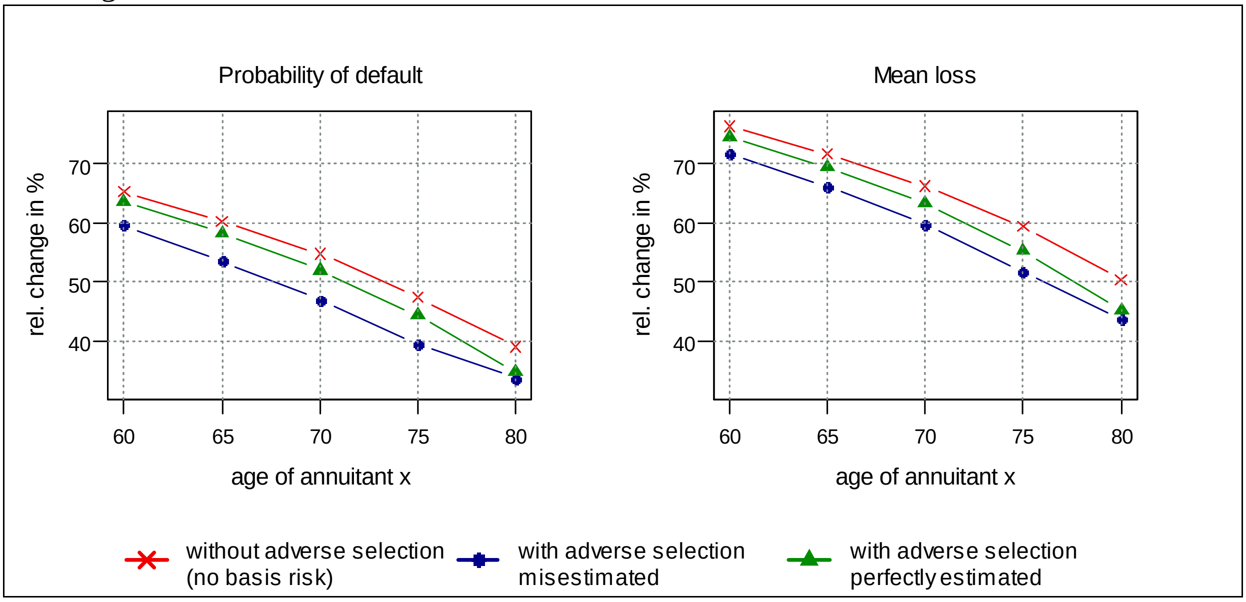
Concerning the impact of adverse selection and the resulting basis risk on the effectiveness of MCBs for varying ages of policyholders, the impact of adverse selection even if it is perfectly estimated increases for older annuitants as can be seen by the slightly greater difference between the red line with crosses and the green line with triangles in Figure 5. This reflects the risk reduction achievable through MCBs without adverse selection and with perfectly estimated adverse selection, respectively. For example, considering again the relative difference between the risk reduction achievable through an MCB with adverse selection perfectly estimated and without adverse selection, this difference corresponds to a probability of default of about 2.5% for $x$ = 60 year old annuitants, whereas it amounts to approximately 11% for $x$ = 80 year old annuitants. For the mean loss, the results are very similar.

[32] However, when considering the amount of risk reduction achievable through the use of MCBs, the effectiveness of MCBs varies substantially for the difference assumptions concerning adverse selection, e.g. for $\alpha$ = 90% the probability of default can be reduced by about 60.4% when no adverse selection is assumed, while the risk reduction achievable through the use of MCBs amounts to only 52.9% when adverse selection is misestimated. This difference is in line with the results found in the previous analyses as well as with the results by Gatzert and Wesker (2011).

[33] As stated previously, we assume $M$ = $T$. Thus, varying the age of policyholders also influences the maturity of the MCB $M$ as the maximum duration of the contract $T$, which is given by the difference between the inception of the contract and the maximum age as implied by the model (here: 100 years), changes. For example, for $x$ = 60 year old annuitants the maturity of the MCB $M$ (as well as the maximum duration of the annuity $T$) is equal to 40 years, while for $x$ = 80 the maturity of the MCB amounts to only 20 years.

When considering the impact of a misestimated adverse selection, the results show that the difference between the two studied assumptions "adverse selection misestimated" and "adverse selection perfectly estimated", decreases. This effect might be due to the shorter duration of both – the annuity as well as the MCB – such that the impact of the misestimated trend between annuitant and population mortality decreases.

**Figure 5**: The impact of risk management using mortality contingent bonds (MCBs) on an insurer's risk situation under different assumptions concerning adverse selection for annuitants aged *x*




Overall, the effectiveness of MCBs in general decreases for older annuitants. In addition, the impact of perfectly estimated adverse selection and the resulting basis risk increases considerably, so that especially for older annuitants and in the presence of adverse selection, the effectiveness of MCBs is considerably reduced.

## 4. SUMMARY

In this paper, we examine the impact of certain characteristics of the mortality contingent bond, namely the maturity and the initial coupon payment, insurance company (i.e., the investment strategy), and insurance portfolio (i.e., the age of annuitants) as well as the impact of adverse selection and the resulting basis risk on the hedge efficiency of an MCB under different assumptions concerning adverse selection. We therefore explicitly model adverse selection using the extension of the Brass-type relational model initially put forward by Brouhns, Denuit, and Vermunt (2002a) and used by Gatzert and Wesker (2011) to estimate the relationship between population and annuitant mortality based on the UK annuitant mortality data by CMI and population mortality for the UK from the Human Mortality Database. To estimate population mortality, the extension of the Lee-Carter (1992) model by Brouhns, Denuit, and Vermunt (2002a) is used. Following Gatzert and Wesker (2011), to highlight the importance of mortality information in underwriting, we study two different assumptions about adverse selection and the ability of the insurance company to estimate and forecast annuitant mortality. We first assume that the insurance company misestimates adverse selection, for example due to lack of data on annuitant mortality. We then assume that the insurance company has gained perfect information about annuitant mortality, so that this information can be considered in pricing and reserving. To analyze the effectiveness of MCBs, we model the insurance company holistically, considering assets and liabilities and focusing on two default risk measures, namely the probability of default and the mean loss.

One main result is that the maturity of the MCB is an important determinant of the risk reduction effect of the MCB, whereby generally a longer duration leads to a higher risk reduction. In our model, under the stated assumptions, the highest risk reduction effect is achieved when the duration of the bond is equal to the maximum duration of the annuity. However, the marginal increase in the effectiveness of the MCB is diminishing, i.e. for very long durations of the bond the increase in the risk reduction in response to an increasing duration of the bond is almost negligible. For example, for $M$ = 30 the probability of default can be reduced by 59.7%, while for $M$ = 35 the risk reduction achievable amounts to 60.2% for the case without adverse selection. On the other hand, a substantially reduced maturity leads to a considerable loss in efficiency. For example, although the mean loss can be reduced by 71.2% when purchasing a MCB with a duration of $M$ = 30 years, the risk reduction effect corresponds to only 41.1% for $M$ = 10 and to only 17.5% for $M$ = 5 years again for the case without adverse selection. Considering the effect of adverse selection, our results show that the impact of adverse selection and the resulting basis risk can be reduced by increasing the maturity of the MCB. For example, the relative difference between the risk reduction achievable through an MCB without adverse selection and with misestimated adverse selection decreases from approximately 15% for $M$ = 5 to approximately 11% for $M$ = 35. Our results thus indicate that for hedging to be efficient, the maturity of the MCB has to be sufficiently long, especially under adverse selection. In our model framework under the stated assumptions, however, the loss in efficiency is rather small if the hedge does not completely cover the maximum duration of the annuity. This indicates that a fixed maturity of the bond rather than a stochastic maturity that covers the complete duration of the annuity portfolio until the last annuitant dies, might result in a rather small loss in efficiency from the perspective of the insurance company.

Concerning the impact of the characteristics of the insurance company, we found that the investment strategy has a substantial impact on the effectiveness of MCBs, whereby MCBs prove most useful for a rather conservative investment strategy that comes along with a low level of default risk, whereas the impact of adverse selection and the resulting basis risk does not change considerably for a varying investment strategy. For the stated assumptions and the considered discrete values, for example, the maximum risk reduction of 72.6% is achieved for a fraction of low risk assets $\alpha$ = 90% for the mean loss and the case without adverse selection, whereas a riskier investment strategy with a fraction of low risk assets $\alpha$ = 50% yields a risk reduction effect of only 57.1%.

Lastly, we found that the age of annuitants substantially influences the impact of adverse selection and the resulting basis risk, whereby the older the annuitants in the portfolio, the greater the loss in efficiency due to adverse selection. For example, when hedging a portfolio for $x$ = 65 year old annuitants, the relative difference between the risk reduction effect without adverse selection and with perfectly estimated adverse selection amounts to approximately 3% for both the probability of default and the mean loss, whereas this difference increases to approximately 10% or 11% for the probability of default and the mean loss, respectively, for $x$ = 80.

Overall, our results indicate that when purchasing MCBs for risk management, attention has to be paid to the characteristics of the insurance company and the insurance portfolio in order to be able to forecast the risk reduction achievable through the purchase of MCBs correctly. The characteristics of the MCB also have to be considered carefully. Our results indicate that from the perspective of the insurance company, a fixed maturity might result in a rather small loss in efficiency, whereby the fixed maturity would be rather positive for potential investors in MCBs.

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