



FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

RECHTS- UND WIRTSCHAFTS-
WISSENSCHAFTLICHE FAKULTÄT

Mortality Risk and its Effect on Shortfall and Risk Management in Life Insurance

Nadine Gatzert, Hannah Wesker

Working Paper

Chair for Insurance Economics
Friedrich-Alexander-University (FAU) of Erlangen-Nuremberg

Version: February 2012

MORTALITY RISK AND ITS EFFECT ON SHORTFALL AND RISK MANAGEMENT IN LIFE INSURANCE

Nadine Gatzert, Hannah Wesker*

ABSTRACT

Mortality risk is a key risk factor for life insurance companies and can have a crucial impact on its risk situation. In general, mortality risk can be divided into different sub-categories, among them unsystematic risk, adverse selection, and systematic risk. In addition, basis risk may arise in case of hedging, e.g., longevity risk. The aim of this paper is to holistically analyze the impact of these different types of mortality risk on the risk situation and the risk management of a life insurer. Toward this end, we extend previous models of adverse selection, empirically calibrate mortality rates, and study the interaction among the mortality risk components in the case of an insurer holding a portfolio of annuities and term life insurance contracts. For risk management, we examine natural hedging and mortality contingent bonds. Our results show that particularly adverse selection and basis risk can have crucial impact not only on the effectiveness of mortality contingent bonds, but also on the insurer's risk level, especially when a portfolio consists of several types of products.

Keywords: Longevity risk, mortality contingent bonds, natural hedging, life insurance, risk management

JEL Classification: G22, G23, G32, J11

1. INTRODUCTION

Recently, there has been a growing interest in mortality risk and its management in the scientific literature as well as in practice, especially due to the demographic development in most industrialized countries. The increasing life expectancy poses serious problems to life insurance companies selling annuities and to pension funds. These problems are especially severe because of a scarcity of possibilities to hedge against this risk. Due to the limited capacity of reinsurance, several alternative instruments for managing demographic risk, e.g., by transferring mortality risk to the capital market or the use of natural hedging, have been discussed in the scientific literature and by practitioners. However, mortality heterogeneity as well as information asymmetries between the insurance company and the insured about these different mortality experiences of individuals can lead to adverse selection. In particular, annuitants generally have a systematically

* Nadine Gatzert and Hannah Wesker are at the Friedrich-Alexander-University (FAU) of Erlangen-Nuremberg, Chair for Insurance Economics, Lange Gasse 20, 90403 Nuremberg, Germany, nadine.gatzert@wiso.uni-erlangen.de, hannah.wesker@wiso.uni-erlangen.de.

lower mortality than the population as a whole.¹ Mortality heterogeneity and information asymmetries can thus severely limit the usefulness of these risk management tools. Therefore, the aim of this paper is to study the interaction among different types of mortality risk – unsystematic mortality risk, basis risk, adverse selection, and systematic mortality risk – with respect to the risk situation of an insurance company. Furthermore, we analyze the impact of mortality risk components on the effectiveness of two risk management tools: 1) a natural hedging strategy, using the opposed reaction towards changes in mortality of term life insurance and annuities for eliminating the impact of systematic mortality risk, and 2) a mortality contingent bond (MCB) for transferring mortality risk to the capital market.

In the literature, mortality risk is generally divided into different subcategories: 1) unsystematic mortality risk that the individual time of death is a random variable with a certain probability distribution (see Biffis, Denuit, and Devolder (2009)), 2) systematic mortality risk, which is the risk of unexpected changes in the underlying population mortality, e.g. due to common factors impacting the mortality of the population as a whole, which causes dependencies between lives and is thus not diversifiable through enlarging the portfolio (see Wills and Sherris (2010)), and 3) adverse selection, which refers to the fact that the probability distribution differs in the level and trend over age for different populations of insured, for example, for life insurance holders and annuitants² (see, e.g., Brouhns, Denuit, and Vermunt (2002a)). Furthermore, adverse selection, which is due to the mortality heterogeneity of individuals and information asymmetries between the insurance company and the insured, is one important source of basis risk when hedging longevity risk through mortality contingent bonds or other capital market instruments (see Sweeting (2007)). Basis risk arises if the population mortality underlying the hedge and the hedged portfolio mortality do not coincide. Thus, the differences in the mortality of the population and the mortality of the insured annuitants caused by adverse selection imply basis risk in longevity hedges. In this analysis, we solely consider the basis risk in longevity hedges³ and model all types of mortality risk explicitly in order to analyze their impact on a life insurer's risk situation.

Adverse selection (and basis risk) is modeled differently in the literature. Plat (2009) proposes to model the difference in mortality rates for annuitants and the population through an age and time

¹ See Finkelstein and Poterba (2002), Cohen and Siegelmann (2010).

² In general, adverse selection refers to information asymmetry and hidden characteristics. In this paper, we follow Brouhns, Denuit, and Vermunt (2002a) and refer to adverse selection as the observation that, due to mortality heterogeneity and asymmetric information, annuitants experience a lower mortality than the average population and therefore have a higher life expectancy. Other papers (e.g., Coughlan et al. (2009)) refer to this as basis risk. In the following analysis, we consider two cases in order to highlight the importance of mortality information in underwriting, one where the insurer is not fully informed about the mortality of its annuitants, and one case where adverse selection can be fully addressed.

³ Other potential sources of basis risk in longevity hedges are stated, e.g. by Sweeting (2007) or Coughlan et al. (2007) and include age mismatch or geographic differences.

dependent portfolio-specific mortality factor, which reflects the relative difference between annuitant mortality and population mortality. Ngai and Sherris (2011) also use a portfolio specific mortality factor and, following Stevenson and Wilson (2008), assume a linear and constant effect of age as the only impact factor. Brouhns, Denuit, and Vermunt (2002a) choose a different approach and model annuitant mortality through a Brass-type relational model for the central death rates. Concerning the effectiveness of mortality contingent bonds (or other instruments for transferring mortality risk to capital markets) under basis risk resulting from adverse selection, other certain aspects have already been discussed in the literature. Sweeting (2007) discusses the influence of basis risk when using a survivor swap qualitatively in a utility-maximizing framework and concludes that basis risk is comparatively small and thus will not hinder the occurrence of hedging transactions. In terms of the effectiveness of q-forwards⁴ based on the population mortality for hedging insured lives, Coughlan et al. (2007) use historical data and conclude that the loss in efficiency is small from a long-term perspective. Ngai and Sherris (2011) quantify the impact of basis risk in longevity bonds and q-forwards in a static framework and find that basis risk does not significantly affect the hedging effectiveness. Coughlan et al. (2010) introduce a general framework for assessing basis risk in longevity hedges and conclude that it can be reduced considerably by applying their framework for calibrating the hedge. A more general concept in this context, the so-called population basis risk, describes the risk of basing the payout of the risk management instrument on a different population⁵ and is discussed by Li and Hardy (2009) and Coughlan et al. (2007). Thus, to date, results in the literature suggest that basis risk in longevity hedges overall has a minor impact on the effectiveness of the hedge.

The second risk management instrument, natural hedging, has also been studied in the literature. Cox and Lin (2007) as well as Bayraktar and Young (2007) examine the impact of natural hedging on pricing. Gründl, Post, and Schulze (2006) and Hanewald, Post, and Gründl (2011) compare the effects of different risk management strategies on shareholder value, concluding that natural hedging is the preferred risk management tool, but only under certain circumstances. Wang et al. (2010) apply the concept of duration to mortality and derive an optimal liability mix, which is characterized by a portfolio-mortality-duration of zero, while Wetzel and Zwiesler (2008) show that the mortality variance, i.e. the variance due to fluctuations in mortality, can be reduced by more than 99% through portfolio composition. Gatzert and Wesker (2010) consider the insurer as a whole and show how to immunize a given risk level by simultaneously considering the investment and insurance portfolio.

⁴ A q-forward is a standardized mortality contingent swap, based on the LifeMetrics index by J.P. Morgan. The LifeMetrics index is distinguished by gender and age for the population of U.S., England and Wales, the Netherlands and Germany (for more information and current index data see <http://www.jpmorgan.com/pages/jpmorgan/investbk/solutions/lifemetrics>).

⁵ Potential sources of population mismatch include differences in geographic location, age, social status etc.

Despite a fair amount of research on mortality risk, the impact of all three mortality risk components (separately and combined) and basis risk resulting from adverse selection on the risk level of a life insurance company and on the effectiveness of different risk management strategies with respect to reaching a desired risk level as well as hedging against unexpected changes in mortality has not been systematically studied. Hence, in this paper, mortality risk is modeled comprehensively to gain deeper insight into the interaction among the different types of risk, incorporating unsystematic mortality risk, adverse selection, systematic mortality risk, and basis risk with respect to the risk management instruments. Based on this model, the impact of mortality risk on the risk level of a two-product life insurance company and for hedging longevity risk is analyzed. Population mortality is forecasted using the extension of the Lee-Carter (1992) model proposed by Brouhns, Denuit, and Vermunt (2002a).⁶ Adverse selection is modeled based on an extension of the Brass-type relational model by Brouhns, Denuit, and Vermunt (2002a) and estimated based on data from the Continuous Mortality Investigation (CMI).

Furthermore, in contrast to previous literature, we specifically study the impact of information asymmetries concerning mortality heterogeneity and the resulting adverse selection on an insurer's risk situation and the effectiveness of risk management. If the insurance company cannot observe the insured's individual mortality or if there is insufficient data on average annuitant mortality, adverse selection may lead to a misestimation of mortality experience for annuitants and thus to a difference between actual mortality and expected mortality, which is used in, e.g. pricing and reserving. Therefore, to examine the impact of mortality information, we first look at adverse selection under information asymmetry, implying a misestimation of annuitant mortality experience. Second, we examine the impact of adverse selection when the insurance company has gained perfect information about the mortality experience within the annuitant portfolio, e.g. by way of experience rating.

This consideration of adverse selection extends the work of Ngai and Sherris (2011) and Coughlan et al. (2007) and is intended to offer additional central insight regarding the effect of adverse selection and basis risk. In particular, our results show that adverse selection and the resulting basis risk in longevity hedges can in fact have a particularly strong impact on both an insurer's risk level and on the effectiveness of MCBs in reducing the level of risk, if the true mortality experience is partly hidden from the insurer. Thus, this effect should be taken into account when determining the amount of risk management needed to achieve a certain desired risk level. This is also true when determining the optimal MCB volume and portfolio composition to reduce the impact of systematic mortality risk. In this context, another contribution to previous literature,

⁶ This mortality model is taken as an example and can as well be replaced by other stochastic mortality models that provide a good fit depending on the concrete application (and the respective country).

including Coughlan et al. (2007), Ngai and Sherris (2011), and Sweeting (2007), is the consideration of systematic mortality risk in addition to basis risk as well as the analysis of natural hedging. In addition, the explicit inclusion of adverse selection and the model of systematic mortality risk in the analysis of natural hedging extend previous studies such as Gründl, Post, and Schulze (2006) and Gatzert and Wesker (2010), where focus is not laid on adverse selection.

The remainder of the paper is structured as follows. Section 2 introduces methods for modeling and forecasting population mortality. Furthermore, the model of the insurer and the MCB are presented. Section 3 contains results of the numerical analyses and Section 4 concludes.

2. MODEL FRAMEWORK

2.1 Modeling and forecasting mortality risk

Modeling unsystematic mortality risk

One of the most frequently used models for mortality is the Lee-Carter (1992) model, which consists of a demographic and a time series part. In this framework, the central death rate or force of mortality $\mu_x(\tau)$ is modeled through

$$\ln[\mu_x(\tau)] = a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau} \Leftrightarrow \mu_x(\tau) = e^{a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau}},$$

where a_x and b_x are time constant parameters indicating the general shape of mortality over age and the sensitivity of the mortality rate at age x to changes in k_τ , respectively, where k_τ is a time-varying index and shows the general development of mortality over time, and $\varepsilon_{x,\tau}$ is an error term with mean 0 and constant variance. Lee and Carter (1992) propose to fit an appropriate ARIMA process on the estimated time series of k_τ ,

$$k_\tau = \underbrace{\phi + \alpha_1 \cdot k_{\tau-1} + \alpha_2 \cdot k_{\tau-2} + \dots + \alpha_p \cdot k_{\tau-p}}_{\hat{k}_\tau} + \underbrace{\delta_1 \cdot \varepsilon_{\tau-1} + \delta_2 \cdot \varepsilon_{\tau-2} + \dots + \delta_q \cdot \varepsilon_{\tau-q}}_{\varepsilon_\tau} + \varepsilon_\tau = \hat{k}_\tau + \varepsilon_\tau$$

using Box-Jenkins time series analysis techniques with $\varepsilon_\tau \sim N(0, \sigma^2)$, where σ^2 is assumed to be constant over time. A more recent variation of the Lee-Carter (1992) model is the extension by Brouhns, Denuit, and Vermunt (BDV) (2002a), whose proposed modification results in slightly more attractive theoretical properties. They model the realized number of deaths at age x and time τ , $D_{x,\tau}$, as

$$D_{x,\tau} \sim \text{Poisson}(E_{x,\tau} \cdot \mu_x(\tau)) \text{ with } \mu_x(\tau) = e^{a_x + b_x \cdot \hat{k}_\tau},$$

where \hat{k}_τ is the forecasted realization of the time index used in BDV (2002a) for simulating random numbers of death, thus reflecting the unsystematic mortality risk and $E_{x,\tau}$ is the risk exposure at age x and time τ , defined as $E_{x,\tau} = (n_{x-1}(\tau-1) + n_x(\tau)) / 2$, where $n_x(\tau)$ is the number of persons (i.e., the population size) still alive at age x and the end of year τ .⁷ The advantages of the BDV (2002a) model are that the restrictive assumption of homoscedastic errors made in the Lee-Carter (1992) model is given up and that the resulting Poisson distribution is well suited for a counting variable such as the number of deaths.⁸

Modeling adverse selection and basis risk

Mortality heterogeneity refers to the fact that mortality rates are not identical for all individuals of the same age x but differ depending on, e.g., genetic predisposition or behavior. Individuals are usually able to gain some information about their individual mortality, for example through family history or their general health situation, which may influence their insurance decisions (see Finkelstein and Poterba (2002)). For instance, a person estimating its own mortality to be below average will be more likely to purchase an annuity than a person with below average mortality. Insurance companies generally do not have access to these information and thus cannot directly distinguish between individuals with above or below average health. These circumstances give rise to information asymmetries and thus the problem of adverse selection, as both the level of mortality rates as well as their development over time differ between annuitants and the general population. At the same time, adverse selection also implies basis risk when hedging against longevity risk due to the difference between the mortality rates of the reference population used as an underlying for the hedge and the annuitants' mortality rates. Hence, basis risk arises because the underlying and the hedged population are not perfectly dependent and can thus reduce the hedging effectiveness.⁹ In the following, adverse selection is modeled through an extension of the brass-type relational model used by, among others, Brouhns, Denuit, and Vermunt (2002a),

$$\ln(\mu_{x,\tau}^{ann}) = \alpha + \beta_1 \cdot \ln(\mu_{x,\tau}^{pop}) + \beta_2 \cdot (\ln(\mu_{x,\tau}^{pop}) \cdot \tau_{index}) + e_{x,\tau}, \quad (1)$$

which relates the mortality of annuitants (denoted by superscript “ann”) to that of a reference population (denoted by superscript “pop”). In this context, the parameter β_1 can be interpreted as the speed of improvement of annuitant mortality as compared to population mortality, where val-

⁷ For simulation purposes, $E_{x,t}^{ref} = -n_{ref}(t-1) \cdot q_x^{pop} / \ln(p_x^{pop})$ is used instead (see BDV (2002b)).

⁸ Depending on the respective country and application, other mortality models may be more appropriate to adequately forecast mortality rates of the population (see, e.g., Cairns et al. (2009)).

⁹ Here, adverse selection and basis risk are modeled identically through a differing mortality experience for annuitants and the population as a whole. However, the two terms refer to different aspects. While adverse selection refers to the difference in mortality experiences arising from mortality heterogeneity, basis risk refers to the consequences of this difference when hedging longevity risk through capital markets.

ues greater than one indicate a greater improvement in mortality rates for the group of annuitants. In contrast to previous literature, we additionally include an interaction term between mortality rates and a time index τ_{index} in order to incorporate time dependency in the speed of relative improvement. We expect a negative coefficient β_2 , indicating that the over-improvement in mortality rates of annuitants decreases over time. This regression model can then be used to obtain forecasts of annuitant mortality based on the estimated and forecasted population mortality. The normally distributed error term $e_{x,\tau}$ has zero mean and a constant variance over age and time and is taken into account in forecasting. As in the case of unsystematic risk, the realized number of deaths for annuitants is modeled by applying the Poisson distribution for a given exposure and the forecasted force of mortality $\mu_{x,\tau}^{ann}$.

In the context of adverse selection, we additionally focus on the role of mortality information in underwriting and its impact on the risk situation and risk management of an insurance company. Therefore, we first assume that the insurer cannot perfectly account for adverse selection, for example because of a lack of data on annuitant mortality and private information of the insured concerning his individual health situation, and that the parameters of Equation (1) are misestimated, such that $\beta_1 = 1$, $\beta_2 = 0$, and $\alpha \neq 0$ (referred to as “adverse selection misestimated”). Hence, since the actual relationship between annuitant and population mortality (equal to the mortality of term life insurance policyholders) is misestimated, the different development of mortality rates for annuitants and life insurance policyholders cannot be fully taken into account when calculating premiums and benefits. However, the insurance company may be able to gain information about the average mortality within the annuitant portfolio under adverse selection, e.g. by way of experience rating. Thus, second, we assume that the insurer is able to perfectly estimate and thus account for adverse selection effects and consequently to take this information into account when determining benefits and premiums of annuitants.¹⁰ This setting is referred to as “adverse selection perfectly estimated”.

Modeling systematic mortality risk

Systematic mortality risk is the risk that cannot be diversified through enlarging the insurance portfolio, i.e. it is the risk of unexpected deviations from the expected mortality rates applying to all individuals, which can result, e.g., from a common factor unexpectedly impacting mortality at all ages (see, e.g., Wills and Sherris (2010)). This can in general be attributed either to unexpected environmental or social influences, impacting mortality positively or negatively,¹¹ or to

¹⁰ Thus, adverse selection in the sense of hidden information is in fact eliminated.

¹¹ Additionally, certain other macroeconomic variables might have an influence on mortality (see, e.g., Hanewald (2011)).

wrong expectations about future mortality due to estimation errors.¹² Unexpected common factors that influence lives in a similar way induce dependencies and thus destroy diversification benefits of large pool sizes. In the literature, systematic mortality risk is modeled and accounted for in different ways. Hanewald, Piggot, and Sherris (2011) and Wills and Sherris (2010) characterize systematic (longevity) risk as uncertain changes in mortality applying to all individuals, which leads to dependencies between lives due to common improvement in mortality rates across individuals. Wang et al. (2010) describe systematic risk as a constant shock to the force of mortality, thus accounting for unexpected changes in mortality rates, similarly to Milevsky and Promislow (2003) and Gründl, Post, and Schulze (2006). Furthermore, Cox and Lin (2007) point out that while mortality risk may not be hedgeable in financial markets, it may be reduced or eliminated by insurers by means of, e.g., natural hedging, reinsurance, asset-liability management, or mortality swaps.

In the following, systematic mortality risk is modeled through different realizations of the time trend k_τ , where now the error term is taken into account, having $k_\tau^{syst} = \hat{k}_\tau + \varepsilon_\tau$, which we refer to as the “neutral scenario” as the mean life expectancy does not change. The factor ε_τ impacts mortality at all ages in year τ ¹³ and thus causes dependencies between lives, which cannot be diversified through enlarging the portfolio. To study systematic mortality risk in more detail, we conduct scenario analyses by distinguishing between a longevity scenario, in which mortality is unexpected low, and a scenario with unexpected high mortality (“mortality scenario”) using the absolute value of ε_τ , respectively, thus having

$$k_\tau^{syst, longevity} = \hat{k}_\tau - |\varepsilon_\tau| \quad \text{and} \quad k_\tau^{syst, mortality} = \hat{k}_\tau + |\varepsilon_\tau|.$$

Summary of modeled mortality risk

Based on the mortality model presented here, the probability that a male policyholder aged x in calendar year τ dies within the next year, given he has survived until age x , is calculated by $q_x(\tau) = 1 - \exp(-\mu_x(\tau))$ (see BDV (2002a), p. 376), and ${}_n p_x = \prod_{i=0}^{n-1} p_{x+i}$ is the probability that an x -year old male policyholder survives for the next n years. Based on the previous modeling of mortality rates, four different cases can be distinguished depending on the inclusion of unsystematic risk, adverse selection, and systematic mortality risk, laid out in Table 1.

¹² An example of a potential source of estimation error is the choice of the appropriate sample period, since k_τ is rather sensitive towards the specified period.

¹³ Due to the assumed ARIMA process for k_τ , subsequent years are also impacted by the realization of ε_τ .

Table 1: Overview of force of mortality depending on included mortality risk component

<i>Without adverse selection</i>	$\mu_x^{pop, unsyst}$	Mortality rates of the reference population: ➤ Only unsystematic risk
	$\mu_x^{pop, syst}$	Mortality rates of the reference population with systematic mortality risk (time trend k^{syst}) ➤ Unsystematic risk + systematic risk
<i>With adverse selection</i>	$\mu_x^{ann, unsyst}$	Mortality rates of annuitants ➤ Unsystematic risk + adverse selection
	$\mu_x^{ann, syst}$	Mortality rates of annuitants with systematic mortality risk (time trend k^{syst}) ➤ Unsystematic risk + adverse selection + systematic risk

2.2 Modeling and valuation of life insurance liabilities

Modeling a life insurance company

Table 2 depicts a simplified balance sheet of the life insurance company at time $t = 0$. The insurance company sells immediate annuities paying a yearly annuity a in arrear each year as long as the insured is alive, and a term life insurance paying a constant death benefit DB . The annuity is financed through a single premium SP_A^i paid in $t = 0$, which depends on assumptions regarding the mortality setting (see Table 1). In particular, $i = unsyst$ refers to the case where only unsystematic risk is modeled, and $i = syst$ is the setting that takes into account systematic risk. The term life insurance contract is financed through constant annual premiums P_L^i . We thereby assume that the insurance company sells $f_L \cdot n = n_L(0)$ term life and $(1 - f_L) \cdot n = n_A(0)$ annuity contracts, where f_L denotes the fraction of term life insurance and n the constant number of insurance contracts sold.

Table 2: Balance sheet of the insurance company at time $t = 0$ for $i = unsyst, syst$

Assets	Liabilities
$S_{low}^i(0)$	$M_A^i(0)$
$S_{high}^i(0)$	$M_L^i(0)$
$M_{bond}^i(0)$	$E(0)$

On the liability side, $M_A^i(0)$, or, more general, $M_A^i(t)$, denotes the value of annuities at time t and $M_L^i(0)$ represents the value of term life insurance liabilities at time $t = 0$ for $i = unsyst, syst$.

To increase the comparability in the numerical analysis, the total volume per contract¹⁴ of both contract types in $t = 0$ is identical. $E(0)$ denotes the initial equity contributed by shareholders in $t = 0$, and $E^i(t)$ is residually determined as the difference between assets and liabilities. In return for their investment, shareholders receive a constant fraction r_e of the positive earnings each year as a dividend, given by $div^i(t) = r_e \cdot \max(E^i(t) - E^i(t-1); 0)$, $i = unsyst, syst$. Furthermore, $M_{bond}^i(t)$ is the value of the mortality contingent bond (MCB) at time t as detailed in the next subsection, $S_{low}^i(0)$ stands for the market value of low-risk assets at time $t = 0$, and $S_{high}^i(0)$ represent the high-risk assets. If the insurance company purchases a mortality contingent bond, a premium $\Pi_{x,T}^i$ has to be paid in $t = 0$, implying that the initial capital at time 0 available for investment in the capital market $S^i(0)$ is given by

$$S^i(0) = E(0) + n_A(0) \cdot SP_A^i + n_L(0) \cdot P_L^i - \Pi_{x,T}^i, \quad i = unsyst, syst.$$

The total value of assets $A^i(t)$ in the balance sheet at time t in turn increases by the market value of the MCB, such that

$$A^i(0) = S^i(0) + M_{bond}^i(0), \quad i = unsyst, syst.$$

The market value of assets $S_j^i(t)$, $j = low, high$, is assumed to follow a geometric Brownian motion with μ_j the being drift and σ_j denoting the volatility. Let W_{low}^P and W_{high}^P denote two Brownian motions with correlation ρ under the real-world measure P on the probability space (Ω, \mathcal{F}, P) , where \mathcal{F} is the filtration generated by the Brownian motion. Hence, $S_j^i(t)$ can be expressed as (see Björk (2004))

$$S_j^i(t) = S_j^i(s) \cdot \exp\left(\left[\mu_j - \frac{1}{2}\sigma_j^2\right] \cdot (t-s) + \sigma_j \cdot (W_{j,t}^P - W_{j,s}^P)\right), \quad j = low, high, i = unsyst, syst.$$

Thus, the value of the capital investment $S^i(t)$ develops as

$$S^i(t) = S_{high}^i(t) + S_{low}^i(t) + n_L^i(t) \cdot P_L^i - n_A^i(t) \cdot a - d_L^i(t) \cdot DB + X^i(t) - div^i(t), \quad i = unsyst, syst.$$

As before, $n_A^i(t)$ is the number of annuitants still alive at the end of year t , $n_L^i(t)$ the number of term life insurance policyholders alive at the end of year t , $d_L^i(t)$ represents the number of deaths of life insurance policyholders during year t , and $X^i(t)$ denotes the coupon payment for the MCB in year t and $S_{low}^i(t-s) = \alpha \cdot S^i(t-s)$ and $S_{high}^i(t-s) = (1-\alpha) \cdot S^i(t-s)$. The total market value of assets is then given by

¹⁴ Volume here refers to the present value of expected benefit payments in $t = 0$.

$$A^i(t) = S^i(t) + M_{bond}^i(t), \quad i = unsyst, syst.$$

Valuation of insurance liabilities

We assume independence of market and mortality risk and evaluate the insurance contracts using risk-neutral valuation. The resulting market values for annuities and term life insurance at time t on the liability side of the balance sheet for annuities (paid in arrears at the end of each year) and life insurance, respectively are given by

$$M_A^i(t) = n_A^i(t) \cdot \sum_{s=1}^{T_A-t} a \cdot p_{x+t}^A \cdot (1+r)^{-s}, \quad i = unsyst, syst,$$

$$M_L^i(t) = n_L^i(t) \cdot \left(\sum_{s=0}^{T_L-t-1} DB \cdot p_{x+t}^L \cdot q_{x+t+s}^L \cdot (1+r)^{-(s+1)} - P_L^i \cdot p_{x+t}^L \cdot (1+r)^{-s} \right), \quad i = unsyst, syst,^{15}$$

with $n_A^i(t)$ being the number of annuity contracts and $n_L^i(t)$ being the number of life insurance contracts in t . T_L and T_A denote the maximum duration of the respective contract type and r is the risk-free rate. The superscripts A and L of the mortality rates used in the valuation of insurance contracts refer to the respective mortality rates for annuitants or life insurance holders, which depend on the mortality assumptions (unsystematic risk, unsystematic risk + adverse selection, see Table 1), but do not include the systematic mortality risk. The contract parameters a , DB and P_L^i , $i = unsyst, syst$, are calculated by using the actuarial equivalence principle, i.e., expected premiums must be equal to expected benefits. We refer to these premiums as SP_A^{fair} and P_L^{fair} .¹⁶ In the presence of systematic mortality risk, the insurance company additionally demands a risk premium $(1+\delta)$, which we assume to be equal for both products, resulting in the premiums P_L^{syst} and SP_L^{syst} . Furthermore, the contract volume of annuities and life insurance contracts, defined as the present value of the expected benefit payouts, is fixed to V for both contract types. Thus, the death benefit DB and the annual premium P_L^i as well as a are calculated by

$$V = \sum_{t=0}^{T_L-1} DB \cdot p_x^L \cdot q_{x+t}^L \cdot (1+r)^{-(t+1)} = \sum_{t=0}^{T_L-1} P_L^{fair} \cdot p_x^L \cdot (1+r)^{-t}, \quad (2)$$

$$V = \sum_{t=0}^{T_A-1} a \cdot p_{(t+1)}^A \cdot (1+r)^{-(t+1)} = SP_A^{fair}. \quad (3)$$

¹⁵ The time subscript τ in the death and survival probabilities has been dropped from the formulas for ease of illustration; however, all death and survival probabilities remain dependent on age and time.

¹⁶ The probability of default is not taken into account in pricing since we assume that the insurance benefits will continue to be paid out in case of a default; this is in line with the current situation in many countries, where benefits are guaranteed by a guaranty fund (see e.g., Gatzert and Kling (2007)).

Thus, in the setting without systematic risk, $P_L^{unsys} = P_L^{fair}$ and $SP_A^{unsys} = SP_A^{fair}$, while in the presence of systematic risk, $P_L^{sys} = P_L^{fair} \cdot (1 + \delta)$ and $SP_A^{sys} = SP_A^{fair} \cdot (1 + \delta)$, ensuring that the volume of benefits for both contract types is the same. Overall, the value of liabilities at time t , $L^i(t)$, is thus given by

$$L^i(t) = M_L^i(t) + M_A^i(t), \quad i = unsys, sys.$$

2.3 Risk management and risk measurement

Modeling and valuation of a simple mortality contingent bond

For risk management, a simple coupon-based mortality contingent bond (MCB), called a survivor bond, is used as proposed by Blake and Burrows (2001), which is considered to provide an efficient hedge against longevity risk. We assume that the variable coupon payment $X^i(t)$, for $i = unsys, sys$, at the end of each year $t = 0, \dots, T-1$ is proportional to the percentage of the reference population still alive at time t . At time $t = 0$, the insurance company pays a premium $\Pi_{x,T}^i$, for $i = unsys, sys$, where x denotes the age of the reference population of the MCB and T is the duration of the bond.

There is an extensive literature on the pricing of MCBs, assuming different processes for the underlying mortality and different valuation approaches (e.g., Lin and Cox (2005, 2008), Barbarin (2008). Blake et al. (2006), Dawson et al. (2011). However, there is still uncertainty concerning suitable methods for pricing a given instrument. Additionally, since there does not yet exist a liquid market for longevity risk, it is not easily possible to determine the risk premium for longevity risk. Therefore, in the following, the pricing approach applied for the EIB/BNP Paribas is used to determine the premium of the mortality contingent bond,¹⁷ which was derived using the projected survival rates by the U.K. Government Actuary Department, whereby coupon payments were discounted using the LIBOR rate minus a certain risk premium λ (see Cairns et al. (2005)). Hence, the price of a bond with duration T that pays out $X^i(t)$ in year t can be calculated by

$$\Pi_{x,T}^i = \sum_{t=0}^{T-1} E\left(X^{unsys}(t)\right) \cdot (1 + r - \lambda)^{-(t+1)}, \quad i = unsys, sys,$$

where r is the risk-free interest rate and λ the risk premium for systematic mortality risk, if $i = sys$ and $\lambda = 0$ if $i = unsys$. The actual cash-flow $X^i(t)$ at time t depends on the mortality of the reference population, i.e., on the forecasted force of mortality $\mu_x^{pop,i}$, and thus also on whether

¹⁷ This approach seems justifiable despite the non-success of the bond, since e.g. Blake et al. (2006) state that the failure was likely due to weaknesses in design rather than due to mispricing.

systematic risk is taken into account in the analysis or not ($i = unsyst, syst$). As before, in pricing and valuation, systematic risk is not taken into consideration in the mortality projection as it represents an unexpected change to mortality. However, systematic mortality risk is addressed by introducing the risk premium λ . Moreover, basis risk is involved in the hedge as the behavior of the underlying of the bond (with $\mu_x^{pop,i}$) is not identical to the development of the hedged position, i.e. the portfolio of annuities (with $\mu_x^{ann,i}$, see Table 1). Let $n_{ref}^i(t)$ denote the number of persons in the reference group still alive at the end of year t , which can be recursively calculated as $n_{ref}^i(t) = (n_{ref}^i(t-1) - d_{ref}^i(t))$, where $d_{ref}^i(t)$ is the number of persons who died within year t . This is calculated using

$$d_{ref}^i(t) \sim \text{Poisson}(E_{x,t}^{ref,i} \cdot \mu_x^{pop,i}(t)) \text{ and } \mu_x^{pop,i}(t) = e^{a_x + b_x \cdot k_t^i}, \quad i = unsyst, syst,$$

$n_{ref}(0)$ equal to an arbitrary number¹⁸ and C being the initial coupon payment agreed upon at inception of the contract. The exposure to risk of the reference population $E_{x,t}^{ref,i}$ is given by $E_{x,t}^{ref,i} = -(n_{ref}^i(t-1) \cdot q_x^{pop,i}) / \ln(p_x^{pop,i})$ (see BDV (2002b)). Then, the annual payoff $X^i(t)$ is equal to

$$X^i(t) = \frac{n_{ref}^i(t)}{n_{ref}^i(0)} \cdot C, \quad i = unsyst, syst.$$

Thus, the actual coupon payment taken into account in risk measurement by means of actual cash flows is $X^{syst}(t)$, whereas for valuation, $X^{unsyst}(t)$ is used. The value of the MCB at time t , $M_{bond}^i(t)$, $i = unsyst, syst$, is an asset for the insurance company and given by the expected value of future cash-flows discounted to time t and given the information at time t , multiplied by the number of MCBs purchased at time 0 (n_B),

$$M_{bond}^i(t) = n_B \cdot \sum_{j=t}^{T-1} E_t(X^{unsyst}(j)) \cdot (1+r-\lambda)^{-(j-t+1)}, \quad t=0, \dots, T-1, \quad \lambda=0 \text{ if } i = unsyst.$$

Risk management using natural hedging

We further examine the effect of natural hedging, which uses the opposed reaction towards changes in mortality rates of term life insurances and annuities to immunize a life insurer against systematic mortality risk. In the literature, natural hedging has been used for minimizing (e.g. Wetzel and Zwiesler (2008)) or immunizing (e.g. Wang et al. (2010)) the risk of an insurance company in response to unexpected (systematic) changes in mortality. In the following analysis,

¹⁸ $n_{ref}(0)$ is merely a scaling parameter that does not impact the result, since the coupon payment is expressed in relative terms.

we take an immunization approach as in Wang et al. (2010), but follow Gatzert and Wesker (2010) by considering the insurance company as a whole instead of only focusing on the liability side. In the case without adverse selection and taking the probability of default PD as the relevant risk measure, for example, the optimal portfolio composition f_L^* is defined as

$$g(f_L) = \Delta PD(f_L; \mu_x^{pop}; \mu_x^{pop, syst}) = PD(f_L; \mu_x^{pop}) - PD(f_L; \mu_x^{pop, syst}) = PD^{unsyst}(f_L) - PD^{syst}(f_L) \stackrel{!}{=} 0,$$

implying that the probability of default is the same with and without systematic risk. However, since the risk immunizing portfolio composition does not constitute the risk minimizing portfolio, the investment strategy, reinsurance, or MCBs can be used to achieve a desired risk level, which is at the same time immunized against changes in mortality as illustrated in the numerical section.

Risk measurement

To analyze the impact of mortality risk on the insurer's risk situation, we consider two downside risk measures, namely the probability of default (PD^i) and the mean loss (ML^i), which essentially correspond to the Lower Partial Moments (LPM) of order zero and one, adapted to account for the long duration of contracts and to take into account potential default during the contract period. The probability of default is defined as

$$PD^i = P(T_d^i \leq T), \quad i = unsyst, syst,$$

where $T_d^i = \inf \{t : A^i(t) < L^i(t)\}$ is the time of default. Thus, the PD only measures the frequency of default. The second risk measure, the mean loss, is defined as an LPM of order one at the time of default, discounted to $t = 0$, i.e.

$$ML^i = E \left[\left(L^i(T_d) - A^i(T_d) \right) \cdot (1+r)^{-T_d^i} \cdot 1\{T_d^i \leq T\} \right], \quad i = unsyst, syst$$

where $1\{T_d^i \leq T\}$ denotes the indicator function, which is equal to one if the condition in the brackets is satisfied. Thus, this risk measure is the discounted unconditional expected loss in case of default, which takes into account the extent of the default and thus reflects the amount by which assets are not sufficient to cover liabilities. It can thus be interpreted as the average amount of money necessary for funding a case of default during the contract term.

3. NUMERICAL ANALYSIS

This section presents results of the numerical analysis. We first discuss the relevant input parameters and the estimation of mortality rates. Second, all three types of mortality risk – unsystematic risk, adverse selection, and systematic risk – and their interaction are analyzed with respect to an insurer’s risk situation with special focus on adverse selection. Third, the impact of mortality risk on the effectiveness of risk management instruments is illustrated, thereby considering mortality contingent bonds and natural hedging.

3.1 Definition of input parameters and estimation of the mortality model

In the numerical analysis, we assume that the life insurance contracts are sold to $x = 35$ year old male policyholders for a duration of $T = 35$ years. The contract volume V is set to 10,000 for each of the two contract types.¹⁹ Using Equation (2), this assumption results in a fair death benefit of $DB = 164,547$. Concerning the premium calculation, data regarding the size of the loading for only systematic mortality risk is not available; hence, we follow Gründl, Post, and Schulze (2006) by assuming a loading of $\delta = 1\%$ and conduct sensitivity analyses. Thus, under unsystematic mortality risk, the constant annual premium $P_L^{unsys} = 465$ is used, and in the presence of systematic mortality risk, $P_L^{sys} = 469$. The age of the annuitants at inception of the contract is $x = 65$ and the maximum age attainable as implied by the BDV (2002a) model is 100, which corresponds to a maximum duration of $T = 35$ years.²⁰ Setting the single premium for the annuitant under unsystematic and systematic mortality risk equal to $SP_A^{unsys} = 10,000$ and $SP_A^{sys} = 10,100$, using Equation (3), the fair annuity a depends on whether or not adverse selection is taken into account in pricing and reserving. The calendar year of contract inception is set to 2012. Initial equity E_0 is set to 20 Mio and the dividend payment is $r_e = 25\%$. Regarding the assets, a constant risk-free rate r of 3% is assumed and the drift and volatility of high (low) risk assets are fixed at 10% (5%) and 20% (8%), with a correlation of 0.1 and a fraction $\alpha = 80\%$ invested in low-risk assets, where sensitivity analyses were conducted for robustness. To ensure comparability among portfolios, in addition to fixing the volume of each contract, the total number of contracts sold, $n(0)$, is fixed and equal to 10,000. All analyses are based on Monte-Carlo simulation with 100,000 paths for the asset portfolio and the same sequence of random numbers was used for each simulation run. As the value of the MCB at time t depends on the information available at time t , valuation is con-

¹⁹ These parameters were subject to robustness checks. Results are similar when considering a shorter duration of contracts (e.g. $T = 30$ or $T = 25$), younger term life insurance policyholders (e.g. $x = 30$), or older annuitants (e.g. $x = 70$, $x = 75$).

²⁰ This might be considered too low, however due to the scarcity of data especially at high ages, a reliable estimation of the parameters above this age is not possible. For example, J. P. Morgan recommends a maximum age of only 89 in the accompanying software for its LifeMetrics index (see <http://www.jpmmorgan.com/pages/jpmorgan/investbk/solutions/lifemetrics/software>).

ducted path-dependently for all 100,000 possible realizations of $n_{ref}^i(t)/n_{ref}(0)$, $i = unsyst, syst$ at each time t . Due to computational intensity, $M_{bond}^i(t)$ is thereby approximated based on 1,000 simulation runs of future mortality, still ensuring robust results.²¹

Estimating and projecting future mortality and adverse selection in the U.K.

Regarding the empirical estimation of mortality rates, the U.K. is chosen as an example of a typical industrialized country due to the availability of mortality rates for annuitants deduced from actuarial mortality tables that are derived from actual insurance data. Hence, the data basis for the estimation of mortality for both groups of insured (annuitant data and population data used for term life policyholders) is the number of deaths and exposure to risk for U.K. from 1950 to 2009 available through the Human Mortality Database and the U.K. annuitant mortality from the Continuous Mortality Investigation (CMI) from 1947 to 2000 as reflected in the five mortality tables for the years 1947, 1968, 1980, 1992 and 2000.²² The estimated demographic parameters of the BDV (2002a) model are consistent with the results stated in the original article by Lee and Carter (1992) and the estimated and forecasted mortality trend k_τ is obtained by applying Box-Jenkins time series analysis techniques, which indicated an ARIMA (0,1,0) model²³ with drift equal to $\phi = -1.5403$ (standard error 0.3056); the standard error of ε_τ is estimated as 2.3474.

Systematic mortality risk is modeled by simulating random realizations of ε_τ for each year. As illustrated in Figure 1, this common factor impacts mortality at all ages and thus leads to dependencies in the number of deaths at each date τ . Figure 1 exhibits the correlation between the random number of deaths $D_{x,\tau}$ in the year $\tau = 2020$ for different ages x .²⁴ Without systematic mortality risk, as shown in Part a) of Figure 1, the number of deaths $D_{x,\tau}$ for different ages x for a given year τ are generally uncorrelated. Thus, the benefits of risk pooling apply. However, under systematic mortality risk (Figure 1 b)), the common factor causes correlations between the number of deaths for different ages and the correlation coefficient increases with policyholders' age.

²¹ The standard error of Monte-Carlo simulation for the value of the mortality contingent bond at $t = 1$ $M_{bond}^{unsyst}(1)$ is about 0.0322 (the expected value is approximately 12; the exact standard error depends on the path considered and values for the standard error lie between 0.0291 and 0.0354. These values are calculated for an initial coupon $C = 1$), while for $t = 10$ it is about 0.0263 (between 0.0239 and 0.0287).

²² The corresponding graphs exhibiting the demographic parameters $\exp(a_x)$, b_x and k_τ can be found in the appendix; for the estimation procedure using a uni-dimensional Newton-method, we refer to BDV (2002a). Other annuitant data with more data points is not publicly available; however, the data still implies significant parameter estimates when calibrating the time series to estimate the effect of adverse selection.

²³ The Schwarz as well as the Akaike information criterion indicated a more complex model for the ARIMA time series. However, subsequent residual analysis using Box-Ljung test as well as ACF and PACF analysis showed no significant residual autocorrelation.

²⁴ All calculations of the correlations are based on $k_\tau^{syst} = \hat{k}_\tau + \varepsilon_\tau$, i.e. the "neutral" scenario. Results for the longevity and the mortality scenario are qualitatively similar, except that correlations are slightly smaller. The year 2020 was used as an example. The results are based on 100,000 simulation runs and a population of 10,000 policyholders for each age x .

Figure 1: Correlations between the random number of deaths at age x and age y for life insurance policyholders and annuitants, respectively, in the year $\tau = 2020$

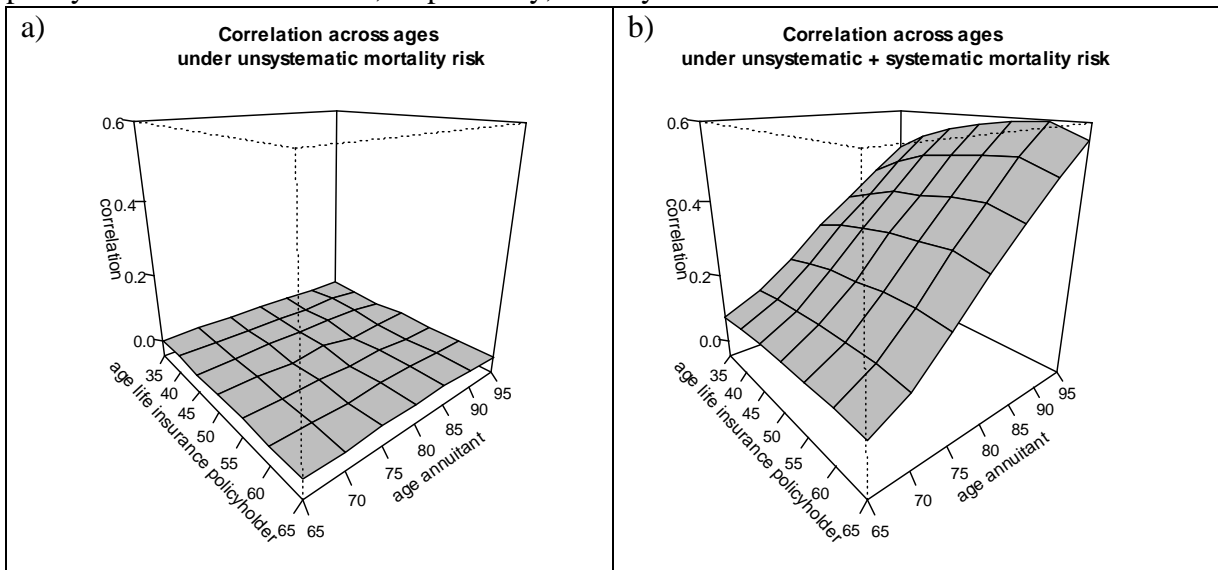
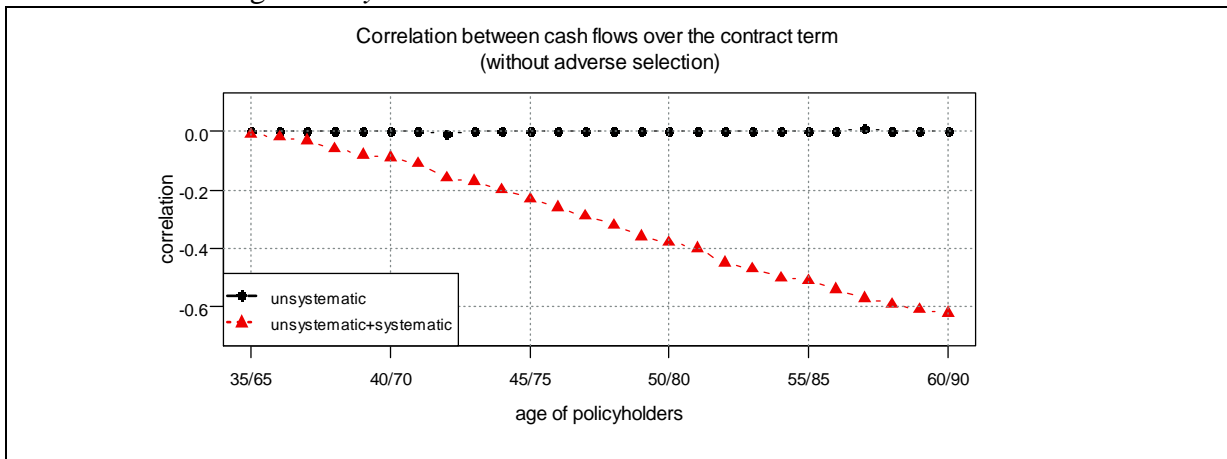


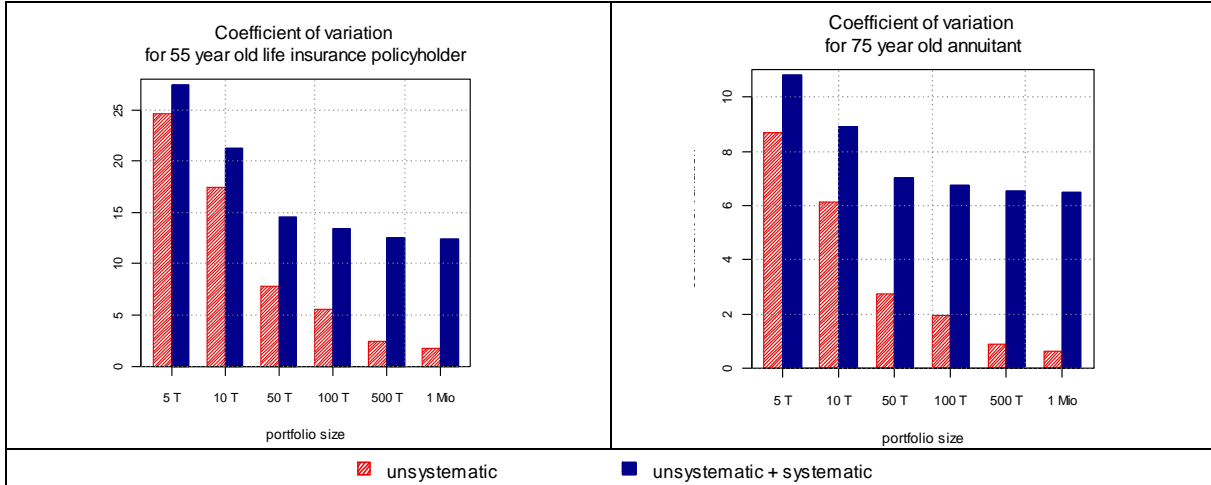
Figure 2: Correlation between the cash flows for death benefits and annuities for the cohort of 1977 and 1947 at age x and y over the contract term



These results can also be confirmed when considering the correlation between the cash flows for annuities and death benefits for the cohort of, e.g., 1947 (annuitants) and 1977 (life insurance policyholders) over the contract term, i.e. for the year 2012 to 2047, as shown in Figure 2. Without systematic mortality risk, the correlation at each point in time t is zero, while under systematic mortality risk, negative correlations between cash flows of annuities and death benefits can be observed, which increase over the contract term. These correlations between the number of deaths for different ages caused by systematic mortality risk in general destroy diversification benefits. This can further be seen in Figure 3, where the coefficient of variation $\sqrt{\text{var}(X)} / E(X)$ is displayed, which provides a relative measure of risk by relating the standard deviation to the expected value. Here, X denotes the random number of deaths for the cohort of 1947 (annuitants) or 1977 (life insurance policyholders) for different ages (i.e., different points in time) with and without systematic mortality risk. The results show that under unsystematic mor-

tality risk, the coefficient of variation decreases for an increasing portfolio size, i.e. the benefits of risk pooling and the law of large numbers apply. However, under systematic mortality risk, diversification benefits are limited and the risk reduction achievable through enlarging the portfolio is considerably reduced.

Figure 3: Measuring diversification – Coefficient of variation for different ages and different portfolio sizes under unsystematic and systematic mortality risk



Regarding adverse selection, the estimation according to Equation (1) indicates that annuitant mortality rates improve more rapidly than the population mortality rates, but that this greater improvement decreases over time. The estimated intercept α is equal to -0.0275 (0.0198), the parameter for the relationship between annuitant and population mortality (β_1) is 1.1618 (0.0123), and the interaction term between year τ_{index} and population mortality is slightly negative with $\beta_2 = -0.0004$ (0.0002) (robust standard errors in parenthesis).²⁵ The estimated standard error of residuals $e_{x,t}$ is 0.1292 , which are also taken into account for each year t and age x in forecasting. In case the insurer is not be able to perfectly account for adverse selection (“adverse selection misestimated”), the parameters of Equation (1) are misestimated, such that $\beta_1 = 1$, $\beta_2 = 0$, and $\alpha = -0.2779$. Concerning the interaction of adverse selection and systematic risk, further analysis shows that the correlation between lives implied by systematic mortality risk is reduced through adverse selection, which is due to the difference between the mortality experience of the population and annuitants induced by mortality heterogeneity and adverse selection.

²⁵ The estimate for β_1 and β_2 are significantly different from zero (on the 1% and 5% significance level, respectively), while the intercept is not. The inclusion of β_2 additionally leads to a slight improvement in R^2 by 0.1 percentage point. Note that $\tau_{index} = 1950 - \tau$, where 1950 is the first year for τ which mortality data is used and τ is the year under consideration. The R^2 indicates that more than 98% of the variance of annuitant mortality can be explained through the model, which is to be expected, since impact factors for mortality rates of annuitants and the population should generally be similar.

3.2 The impact of mortality risk on an insurer's risk situation

Mortality risk in pricing and risk measurement

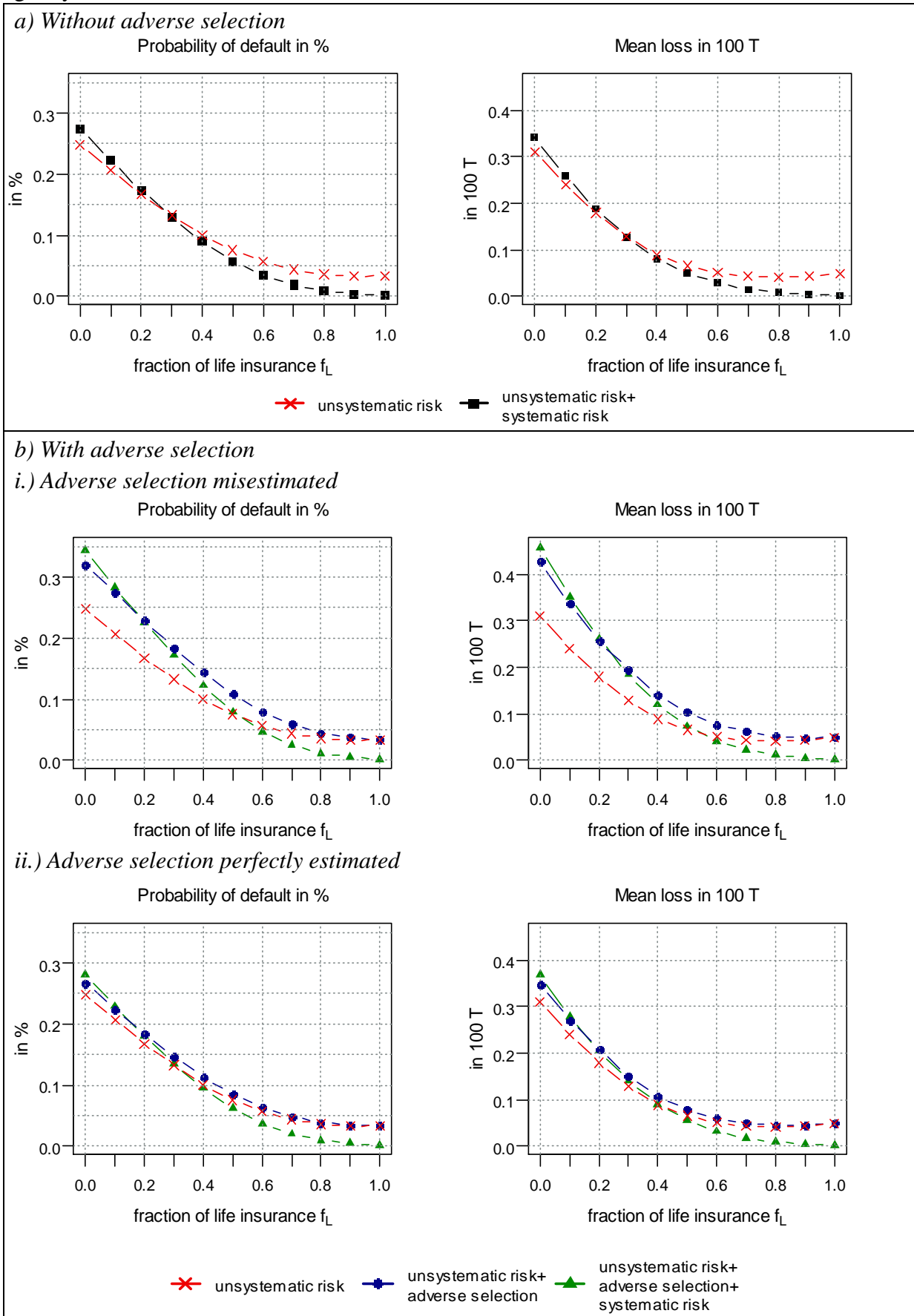
In the numerical analysis, we distinguish different cases with respect to mortality risk for pricing and risk measurement as exhibited in Table 3 (see also Table 1 for the notation of mortality rates). We first study the case without taking into account adverse selection, using only unsystematic and systematic mortality risk. Regarding the impact of adverse selection (see third row in Table 3, “With adverse selection” → “unsystematic risk + adverse selection”), the two cases concerning the ability of the insurance company to forecast and thus to take into account adverse selection in pricing are studied. If the insurer cannot account perfectly for adverse selection, the different development of mortality rates for annuitants and life insurance policyholders cannot be fully taken into account when calculating premiums and benefits. The resulting annuity is 688. Second, if the insurer is able to perfectly estimate and thus account for adverse selection effects, the resulting annuity is 663.

Table 3: Overview of assumptions on the insurer's pricing and risk measurement

	<i>Notation in figures</i>	<i>Pricing and reserving</i>	<i>Fair annuity</i>	<i>Risk measurement</i>
<i>Without adverse selection</i>	“unsystematic risk”	$q_x^A = q_x^{pop}$ $q_x^L = q_x^{pop}$ <hr/> $P_L^{unsyst} = P_L^{fair}$; $SP_A^{unsyst} = SP_A^{fair}$	748	$q_x^A = q_x^{pop}$ $q_x^L = q_x^{pop}$
	“unsystematic risk + systematic risk”	$q_x^A = q_x^{pop}$ $q_x^L = q_x^{pop}$ <hr/> $P_L^{syst} = (1 + \delta) \cdot P_L^{fair}$; $SP_A^{syst} = (1 + \delta) \cdot SP_A^{fair}$	748 ¹	$q_x^A = q_x^{pop, syst}$ $q_x^L = q_x^{pop, syst}$
<i>With adverse selection</i>	“unsystematic risk + adverse selection”	$q_x^A = \begin{cases} q_x^{ann*}: \alpha \neq 0, \beta_1 = 1, \beta_2 = 0, , \text{adv. sel. misestimated} \\ q_x^{ann}, \text{ if adv. sel. perfectly estimated} \end{cases}$ $q_x^L = q_x^{pop}$ <hr/> $P_L^{unsyst} = P_L^{fair}$; $SP_A^{unsyst} = SP_A^{fair}$	688 663	$q_x^A = q_x^{ann}$ $q_x^L = q_x^{pop}$
	“unsystematic risk + adverse selection + systematic risk”	$q_x^A = \begin{cases} q_x^{ann*}: \alpha \neq 0, \beta_1 = 1, \beta_2 = 0, , \text{adv. sel. misestimated} \\ q_x^{ann}, \text{ if adv. sel. perfectly estimated} \end{cases}$ $q_x^L = q_x^{pop}$ <hr/> $P_L^{syst} = (1 + \delta) \cdot P_L^{fair}$; $SP_A^{syst} = (1 + \delta) \cdot SP_A^{fair}$	688 ¹ 663 ¹	$q_x^A = q_x^{ann, syst}$ $q_x^L = q_x^{pop, syst}$

¹ Systematic mortality risk is considered in the premium through the loading δ .

Figure 4: Probability of default and mean loss under different types of mortality risk for a longevity scenario



The impact of unsystematic mortality risk on an insurer's risk situation

We begin by examining the impact of unsystematic mortality risk on the risk situation of an insurance company to gain insight into central effects of portfolio composition. Furthermore, this setup serves as a benchmark for further analyses. In this scenario, the only uncertainty stemming from mortality is the time of death of each policyholder, realized with a certain known probability distribution and identical for all policyholders (independent of the purchased product). Figure 4 shows the results of this analysis, displaying the absolute level of risk measured by the probability of default and the mean loss. As shown in Part a) of Figure 4, the risk of a portfolio of insurance liabilities can generally be considerably decreased through portfolio composition, particularly by selling more life insurance contracts than annuities.²⁶

The impact of systematic mortality risk on an insurer's risk situation

Systematic mortality risk is considered based on different scenarios, where Figure 4 displays results for the longevity scenario.²⁷ Without adverse selection (Figure 4 Part a)), the impact of a systematic change in mortality on a portfolio of life insurance contracts ($f_L = 1$) is greater than the impact on a portfolio of only annuities ($f_L = 0$), which is due to the different types of insured risks.²⁸ However, the impact on a portfolio of annuities is still not negligible but amounts to an increase of 10.3% for the probability of default and to 9.8% for the mean loss. This opposed reaction of life insurance and annuities in response to systematic mortality risk and the negative correlations between cash flows induced by systematic mortality risk (see Figure 2) create natural hedging opportunities that can immunize the risk of an insurance company against changes in mortality (at the intersection points, where the risk level remains unchanged despite the unexpected common factor impacting mortality).

Further analyses for the neutral scenario and the mortality scenario showed that even though the mean life expectancy is not impacted in case of the former, life insurance contracts are considerably more sensitive towards systematic mortality risk.²⁹ This result is also supported by the mortality scenario, where the probability of default increases from 0.03% (in Figure 4 a) to almost 2%

²⁶ The exact portfolio composition for which the insurer's risk is minimized depends on input parameters and contract characteristics. For example, if the term life insurance is financed through a single premium, the risk level for a portfolio with only term life insurance is higher as compared to a portfolio with only annuities.

²⁷ First, the longevity scenario corresponds to a mean increase in the remaining life expectancy of a 65 year old man of about 1.9 years in the year 2012 from 18.5 years to 20.4 years. Second, the mortality scenario implies a mean decrease in the remaining life expectancy of about 1.8 years to 16.7 years. Third, the realization of ε_τ is not restricted in the "neutral" scenario, and since $E(\varepsilon_\tau) = 0$, the mean life expectancy is not impacted.

²⁸ Life insurances constitute a low-probability risk, which are more heavily impacted by a change in mortality rates than annuities, which constitute a high-probability risk, see, e.g., Gründl, Post, and Schulze (2006).

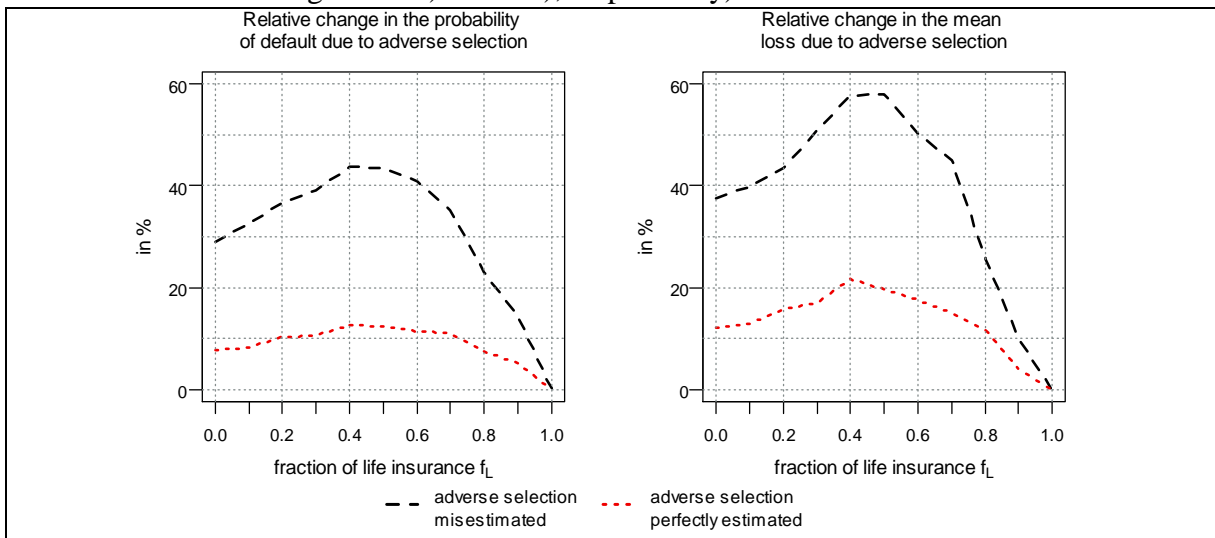
²⁹ For a portfolio with only annuities, in contrast, the risk level decreases slightly due to the inclusion of a loading for systematic mortality risk. Thus, in this case, the required premium is sufficiently high to cover the costs of systematic mortality risk for a portfolio with only annuities.

for a portfolio with only life insurance. Thus, while under unsystematic mortality risk, a portfolio of only life insurance contracts implies a lower risk level than annuities in the considered examples, the sensitivity of life insurance towards systematic mortality risk is considerably higher as compared to a portfolio of annuities.

The impact of adverse selection on an insurer's risk situation

Regarding the impact of adverse selection on an insurer's risk situation, which is induced by mortality heterogeneity among individuals and asymmetric information between insurer and insureds, in addition to Figure 4 b), Figure 5 exhibits the relative change in the risk of an insurance company due to the presence of adverse selection (perfectly estimated or misestimated) as compared to the case where only unsystematic risk is included.

Figure 5: Maximum range of risk due to adverse selection (Figure 4 b) when adverse selection is misestimated or estimated perfectly (difference between “unsystematic” and “unsystematic + adverse selection” in Figure 4 b.i) and b.ii), respectively)³⁰



When comparing the difference between the case where only unsystematic risk is considered (“unsystematic”) and the case with adverse selection (estimated perfectly or misestimated, “unsystematic + adverse selection”), Figure 4 b) and Figure 5 show that the risk level considerably increases when taking into account adverse selection. This is true even if adverse selection is perfectly estimated by the insurer as illustrated in Figure 5, line “adverse selection perfectly estimated.” In this case, the difference to the situation with only unsystematic risk still constitutes an increase of 7.8% in the default probability for a portfolio of annuities ($f_L = 0$). For mixed portfolios, the increase in risk due to the inclusion of perfectly estimated adverse selection can be even

³⁰ The relative change is calculated as the relative increase in the risk measure due to adverse selection, e.g. for the probability of default, $\left(PD_{with\ adverse\ selection}^{unsys} - PD_{without\ adverse\ selection}^{unsys} \right) / PD_{without\ adverse\ selection}^{unsys}$.

up to 21.7% in case of the mean loss. The risk level increases by a fairly large amount if adverse selection is misestimated, which is mainly due to a lower than predicted mortality in the annuity portfolio, leading to a greater than expected cash outflow. For example, for a portfolio with only annuities ($f_L = 0$), the default probability increases by about 30% (see Figure 5, left graph, “adverse selection misestimated”). The increase in the mean loss for the same portfolio even corresponds to more than 35% and can rise to almost 60% for mixed portfolios (see Figure 5, right graph), which emphasizes the importance of properly forecasting not only the mortality of the population as a whole, but especially the relationship between annuitant mortality and population mortality, as an underestimation of annuitant mortality leads to severely increased risk.

Thus, adverse selection, even if it can be perfectly forecasted and taken into account in pricing, can considerably increase the risk level of an insurance company, especially when considering mixed portfolios. However, the results also emphasize that the impact of adverse selection can be significantly decreased through a reliable forecast of the relationship between annuitant mortality and mortality of the population as a whole (and thus the mortality of term life insurance policyholders), which stresses the importance of developing models for better forecasting this relationship. Including systematic risk in addition to adverse selection (line “unsystematic + adverse selection + systematic” in Figure 4 b)) shows that if adverse selection is misestimated (Figure 4 b.i)), the risk level is considerably higher compared to the case of perfect estimation and included in pricing (Figure 4 b.ii)), while systematic mortality risk has only a slight impact.³¹

3.3 The impact of mortality risk on an insurer’s risk management

We next study the effect of mortality risk components on the effectiveness of risk management. Regarding the mortality contingent bond, the loading is set to $\lambda = 35$ bp (see Cairns et al. (2005))³² and the scaling parameter for the reference population is $n_{ref}(0) = 1$ Mio, with x equal to initial age of annuitants. Furthermore, we assume that the insurer generally purchases one MCB with a volume of 1,000 per bond for each annuity sold, i.e. $M_{bond}^{unsys}(t) = n_B \cdot 1,000 = (1 - f_L) \cdot n(0)$

³¹ When considering a portfolio with younger term life insurance policyholders ($x = 30$), older annuitants ($x = 70$), and a shorter duration of contracts ($T = 30$), the impact of adverse selection is further increased. For example, for a portfolio consisting of 50% term life insurance and 50% annuities, the mean loss increases by about 100% if adverse selection is misestimated, and by about 13% if it is perfectly estimated as compared to 73% and 9%, respectively. In addition, all results shown are based on a single portfolio of life insurance contracts consisting of annuities as well as term life insurance, where all policyholders belong to the same cohort. Taking into account continuing business activity, i.e. the repeated sale of insurance contracts in which the current mortality can be more fully acknowledged, may decrease the effects explained above.

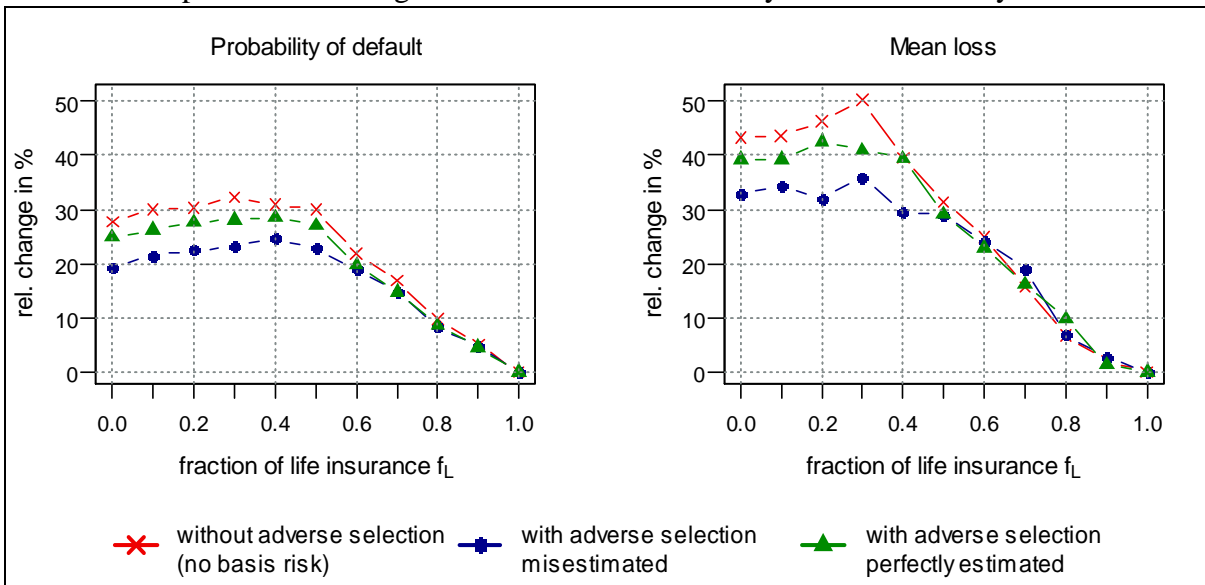
³² The size of the loading does not substantially impact the results as shown in sensitivity analyses.

· 1,000, and thus hedges 10% of its annuity business.³³ The coupon payment C corresponding to a volume of 1,000 is $C = 75$ and the premium is 1,000 (1,036) under unsystematic (systematic) mortality risk.³⁴

The impact of basis risk on the effectiveness of MCBs in reducing the risk level

In this subsection, we examine the effectiveness of MCBs for reducing the risk level of an insurance company in the absence of systematic mortality risk. The use of MCBs regarding the impact of systematic mortality risk is analyzed in the second subsection. Figure 6 shows results for different assumptions regarding the MCB and mortality risk for the probability of default and the mean loss. The line “without adverse selection (no basis risk)” shows the effectiveness of an MCB under “ideal” circumstances, in which the probability distribution of mortality for the population underlying the MCB and for the hedged insurance portfolio is identical, is examined (i.e., without basis risk). In this setup, any deviations in the mortality of the hedged and the underlying reference population are only due to unsystematic deviations in realized mortality between the population as a whole underlying the MCB and mortality within the insurance portfolio.

Figure 6: The impact of risk management using mortality contingent bonds (MCBs) under different assumptions concerning adverse selection without systematic mortality risk³⁵



³³ This number is somewhat arbitrary, but since we are merely interested in the relative effectiveness of an MCB under different assumptions concerning mortality, the different scenarios have to be comparable in terms of amount of purchased hedging instruments.

³⁴ For instance, for a portfolio with 50% annuities and 50% term life insurance, the insurance company purchases bonds with a total volume of $M_{bond}^{unsyst} = 0.5 \cdot 10,000 \cdot 1,000 = 5$ Mio, which is equal to a total coupon payment of $5,000 \cdot C = 5,000 \cdot 75 = 375,000$ (to be weighted with the percentage of survivors in the underlying population).

³⁵ The relative change shown by the dotted line is the relative reduction in the risk measure achievable through the use of MCBs, e.g. for the probability of default, $(PD_{without\ MCB}^{unsyst} - PD_{with\ MCB}^{unsyst}) / PD_{with\ MCB}^{unsyst}$.

The results show that a significant risk reduction can be achieved through the use of MCBs. For a portfolio consisting of only annuities ($f_L = 0$), for which MCBs with a total volume of $M_{bond}^{unsyst} = 10$ Mio are purchased, the probability of default is reduced by -27.8% (i.e., -0.05 percentage points) and the mean loss by -43.3% (i.e., -94 T). This observation indicates that MCBs are more effective in reducing the severity of default than in reducing the frequency of default. In terms of a relative risk reduction, the effectiveness of MCBs can be further enhanced through portfolio composition, despite purchasing fewer MCBs. In particular, for the probability of default, the maximum risk reduction of -32.2% is achieved for a portfolio with 70% annuities ($f_L = 30\%$), for which MCBs with a total volume of $M_{bond}^{unsyst} = 0.7 \cdot 10 \text{ T} \cdot 1,000 = 7$ Mio are purchased. With respect to the mean loss, the use of MCBs leads to the highest risk reduction of -50.2% for a portfolio with 70% annuities ($f_L = 30\%$).

Turning to the effectiveness of MCBs in reducing the level of risk of the insurance company under basis risk, the results vary depending on the insurer's ability to estimate adverse selection. For a portfolio comprised only of annuities ($f_L = 0$), i.e., a typical pension fund, for example, without basis risk, MCBs imply a reduction of 27.8% in the probability of default, while the probability of default can only be decreased by 19.4% in the case of a misestimated adverse selection used in pricing, which constitutes a significant loss in efficiency as compared to the case without adverse selection. However, if the insurer is able to estimate adverse selection perfectly and takes this knowledge into account in pricing, the loss in efficiency compared to the case where no basis risk is included can be reduced, but only by around 3 percentage points in the case of an annuities portfolio. Thus, this result emphasizes the importance of accounting adequately for basis risk effects when determining the amount of risk management needed to achieve a desired risk level.

Due to the higher level of risk and the loss in efficiency of MCBs under basis risk, more MCBs need to be acquired to achieve the same amount of risk reduction in the presence of basis risk, which comes with greater cost for transferring this risk to the capital market. To gain an impression about the risk management costs associated with basis risk (if adverse selection effects are perfectly accounted for), one can calculate the additional volume of MCBs needed to reach the same level of risk as when basis risk is absent, i.e. when the probability distributions of the mortality of annuitants and term life insurance policyholders are identical. For example, for a portfolio of 50% term life insurance ($f_L = 0.5$) and 50% annuities, MCBs with a volume of $M_{bond}^{unsyst} = 0.5 \cdot 10,000 \cdot 1,597 = 7.99$ Mio are needed under basis risk to achieve the same probability of default that would otherwise be achieved through purchasing bonds with a volume of only $M_{bond}^{unsyst} = 0.5 \cdot 10,000 \cdot 1,000 = 5$ Mio when no basis risk is modeled. This corresponds to an increase in the volume of risk management of almost 60%. For the mean loss, the volume has to be increased by even 65.6% to $M_{bond}^{unsyst} = 0.5 \cdot 10,000 \cdot 1,656 = 8.28$ Mio. For a portfolio with annuities only ($f_L =$

0), an increase in the volume of MCBs by 36.3% (41.0%) for the probability of default (mean loss) would be necessary.

The impact of basis risk on the effectiveness of MCBs for hedging systematic risk

In the literature, MCBs are also described as an important tool in reducing the impact of systematic mortality risk. Here, the longevity scenario is considered since the “survivor bond” by Blake and Burrows (2001) was proposed for hedging the longevity risk inherent in annuities and pensions. The results for this analysis are displayed in Table 4 for a portfolio consisting of only annuities. In the case without basis risk, MCBs can be used to reduce the impact of systematic mortality risk on an insurer’s risk situation. This is particularly evident for the mean loss, which in the presence of systematic mortality risk can be reduced by almost 40% in the present setting by purchasing mortality contingent bonds with a volume of $M_{bond}^{syst} = 10,36$ Mio, using the same coupon as in the case of unsystematic mortality risk. This result confirms the previous finding that MCBs prove more useful in hedging the severity of default as compared to the frequency of default. The risk reduction effect is almost as strong in the case where adverse selection is perfectly estimated and priced by the insurer (reduction of 38%), even though the implied change in mortality differs for the hedged population and the reference population underlying the MCB.

Table 4: Probability of default and mean loss for a portfolio of only annuities ($f_L = 0$) including systematic mortality risk (longevity scenario)

Portfolio of annuities only	Without adverse selection (no basis risk)		With adverse selection (in the presence of basis risk)			
			misestimated		perfectly estimated	
	PD	ML	PD	ML	PD	ML
Without MCB	0.27%	342 T	0.34%	458 T	0.28%	369 T
With MCB	0.22%	245 T	0.29%	349 T	0.23%	267 T
Relative reduction through MCB*	23.8%	39.4%	17.7%	31.2%	22.5%	38.0%

* The relative reduction is defined as $\frac{(PD_{without\ MCB}^{syst} - PD_{with\ MCB}^{syst})}{PD_{with\ MCB}^{syst}}$ and $\frac{(ML_{without\ MCB}^{syst} - ML_{with\ MCB}^{syst})}{ML_{with\ MCB}^{syst}}$.

However, if adverse selection is misestimated and not perfectly taken into account in pricing, the effectiveness of MCBs is considerably dampened. An additional analysis of mixed portfolios further shows that under unexpected low mortality, the insurer’s risk level is further decreased,

since unexpected low mortality leads to lower payout for term life insurance contracts and, at the same time, a higher payments from the MCB.³⁶

The effectiveness of natural hedging under adverse selection

Insurance companies can use the opposed reaction of term life insurances and annuities in response to a change in mortality to hedge the impact of systematic mortality risk using natural hedging. This subsection studies the impact of adverse selection on the effectiveness of this risk management strategy (see Table 5). Since the neutral scenario does not imply a change in the expected life expectancy, only the longevity and mortality scenario are considered. The optimal fraction of life insurance contracts f_L corresponds to the intersection points of the respective risk measure, e.g. of the lines “unsystematic” and “unsystematic + systematic” in Figure 4 a). At these points, for the given portfolio composition, the risk level remains unchanged for the modeled unexpected changes in mortality. The immunization is thereby driven by the negative correlation between cash flows for death benefits and for annuities over the contract term (see Figure 2), which is especially pronounced for later contract years and contributes to the immunizing effects utilized in natural hedging.

Table 5: Fraction of life insurance at which the impact of systematic mortality risk on the risk situation of an insurance company is immunized

<i>Fraction of life insurance f_L to immunize portfolio</i>	<i>Without adverse selection</i>		<i>With adverse selection</i>			
			<i>misestimated</i>		<i>perfectly estimated</i>	
	PD	ML	PD	ML	PD	ML
<i>Longevity scenario</i>	27.2%	30.0%	18.7%	22.0%	15.5%	18.6%
<i>Mortality scenario</i>	21.3%	20.3%	21.0%	19.5%	21.0%	20.1%

If no adverse selection is assumed in pricing and risk measurement, the insurance company can eliminate the impact of unexpected low mortality on the probability of default by signing about 27.2% life insurance contracts in case of the longevity scenario (30.0% when using the mean loss as the relevant risk measure). In case of the mortality scenario, the optimal fraction of life insurance contracts is reduced to 21.3% (20.3% for the mean loss). The presence of adverse selection leads to a change in the optimal fraction of life insurance contracts, at which the risk of an insurance company is immunized against unexpected low mortality. This is particularly evident in case of the longevity scenario. If adverse selection is not correctly priced, the optimal fraction is reduced from 27.2% to 18.7% for the probability of default and from 30.0% to 22.0% for the mean loss. Thus, despite the greater implied change in life expectancy of annuitants, less life insurance

³⁶ In contrast, in case of an opposed change in mortality rates, e.g. because the rate of mortality improvement is overestimated, the purchase of MCBs would lead to a further increase in the risk, since payments from the MCBs decrease at the same time that payouts for term life insurance increase.

contracts are needed to eliminate this effect. In the case of a perfectly forecasted adverse selection, the fraction of life insurance contracts is further decreased to 15.5% and 18.6%, respectively. For the mortality scenario, in contrast, the impact of adverse selection is overall negligible and the optimal fraction of life insurance remains almost constant. Compared to the longevity scenario, f_L is considerably lower for the mortality scenario, which can be explained by the greater sensitivity of life insurance towards systematic mortality risk in the mortality scenario. Our immunization approach is similar to Gatzert and Wesker (2010) and Wang et al. (2010) and can also be compared to the results in Cox and Lin (2007). The results in Wang et al. (2010) differ from the results found in the present setting in that the fraction of life insurance contracts is generally higher. For instance, for a 10% mortality shift, the optimal product mix proportion, i.e. the optimal proportion of life insurance liabilities, lies between 30% and 35% life insurance.³⁷ In Cox and Lin (2007), past mortality shocks are used in an example to illustrate the effectiveness of natural hedging. First, a positive shock to mortality is modeled based on the average life expectancy improvement rate in historical mortality tables. In this case, the deviation between the present value of benefits and premiums for both products can be eliminated completely through portfolio composition by signing about equal amounts of life insurance and annuity business. Second, as an example for a bad shock, two different epidemic scenarios are modeled based on the 1918 flu epidemic. While in one scenario, natural hedging can contribute to a considerable reduction in cash flow volatility, in the other scenario an optimal ratio of annuity to life insurance business of about 80% to 90% is found. In Gatzert and Wesker (2010), slightly higher optimal portfolio fractions are found than those in the present analysis.

The differences regarding the optimal portfolio mix in previous literature and the present findings generally arise due to various reasons. First, the definition of the product mix proportion differs, e.g. using the value of liabilities (Wang et al. (2010)) instead of the number of contracts. Second, in contrast to previous studies, adverse selection effects are explicitly modeled and taken into account in the present analysis.³⁸ Additional reasons include the different natural hedging approaches (e.g. using durations), the implementation of systematic mortality risk (e.g. constant shock versus common stochastic factor), the risk measure (e.g., immunization of liabilities as compared to immunizing the insurance company as a whole), as well as differences in the product characteristics. The comparison emphasizes that the optimal portfolio composition depends on various assumptions and the concrete definition of natural hedging, issues that need to be addressed by insurers when setting a risk strategy. However, our results can still be considered to be

³⁷ The product mix proportion ω_{life} is thereby defined as $\omega_{life} = V^{life}/V$, with V as the total liability and V^{life} as the life insurance liability (see Wang et al. (2010, p. 476)).

³⁸ Wang et al. (2010) also acknowledge the importance of accounting for adverse selection effects. They conduct sensitivity analysis through implementing different mortality shifts for annuitants and life insurance and do not focus on a separate model.

generally in accordance with previous findings, especially in that the optimal portfolio composition consists of a lower percentage of life insurance in the portfolio as compared to annuities.

Overall, our findings indicate that in the present setting, natural hedging can be an effective risk management tool to immunize the risk situation of an insurance company against changes in mortality and thus systematic risk, even if annuitants and life insurance policyholders do not experience the exact same impact. However, adverse selection should be taken into account when analyzing the impact of portfolio composition, for instance in the sense of sensitivity analysis.

Simultaneous consideration of MCBs and natural hedging

While a portfolio with about 30% life insurance contracts is immunized against the modeled longevity scenario, this portfolio comes with a higher absolute level of risk than portfolios with a higher percentage of life insurance (see Figure 4). Thus, the insurance company faces a trade-off between risk minimization and immunization. In light of the significant uncertainty accompanying mortality predictions, the immunization effect should not be neglected as a potentially very effective method for hedging longevity risk, especially in view of the scarceness of alternative instruments. A potential strategy for the insurance company to overcome this trade-off, i.e., to simultaneously immunize an insurance company against changes in mortality *and* reach a desired risk level, can be to combine the two presented risk management strategies MCB and natural hedging. Since MCBs reinforce the mortality risk to which an insurance company is exposed, the amount of MCBs purchased has an impact on the portfolio composition at which an insurance company is immunized against changes in mortality. Thus, we simultaneously calculate the amount of MCBs needed to achieve a desired risk level and the fraction of life insurance necessary to immunize the desired risk level under different assumptions concerning adverse selection and thus basis risk. In this setup, we assume that the insurer intends to achieve a probability of default of 0.1% and a mean loss of 50 T.

Table 6: Portfolio composition and amount of MCB to simultaneously achieve a certain risk level and immunize this risk level against systematic mortality risk (longevity scenario)

	<i>Without adverse selection (no basis risk)</i>		<i>With adverse selection (in the presence of basis risk)</i>			
			<i>misestimated</i>		<i>perfectly estimated</i>	
	$PD^{syst} = 0.1\%$	$ML^{syst} = 50 T$	$PD^{syst} = 0.1\%$	$ML^{syst} = 50 T$	$PD^{syst} = 0.1\%$	$ML^{syst} = 50 T$
<i>Fraction of life insurance</i>	26.1%	28.1%	17.2%	20.4%	16.1%	17.6%
<i>MCB volume</i> $M_{bond}^{syst}(t)$	$(1 - 26.1\%) \cdot 10 T \cdot 1,378 = 10.18 \text{ Mio}$	$(1 - 28.1\%) \cdot 10 T \cdot 2,574 = 18.51 \text{ Mio}$	$(1 - 17.2\%) \cdot 10 T \cdot 3,498 = 28.96 \text{ Mio}$	$(1 - 20.4\%) \cdot 10 T \cdot 4,491 = 35.75 \text{ Mio}$	$(1 - 16.1\%) \cdot 10 T \cdot 2,551 = 21.40 \text{ Mio}$	$(1 - 17.6\%) \cdot 10 T \cdot 3,646 = 30.04 \text{ Mio}$
<i>Coupon C</i>	99.59	185.99	252.77	324.55	184.36	263.50

Thus, Table 6 shows that in the present setting, when no basis risk is assumed, the insurance company should sell about $f_L = 26\%$ life insurance contracts (and 74% annuities) and purchase MCBs with a volume of 10.18 Mio to achieve a default probability of 0.1% and to simultaneously immunize this default probability against the modeled shock to mortality. With perfectly estimated adverse selection and thus in the presence of basis risk, the optimal fraction of life insurance contracts decreases by 10 percentage points in case of the PD^{sys} , while the amount of MCBs needs to be increased to 21.40 Mio to achieve and immunize the desired risk level. When adverse selection is not correctly forecasted, the volume of MCBs has to be increased even further to 28.96 Mio, while the fraction of life insurance is slightly higher than in the case when adverse selection was perfectly forecasted. These results are in line with those in the previous subsection, in that under adverse selection, a smaller fraction of life insurance contracts is needed to eliminate the impact of unexpected low mortality on the insurer's risk situation and that more MCBs are needed to achieve a certain risk level.

3.4 Sensitivity Analyses

To examine the robustness of the results with respect to input parameters, sensitivity analyses were conducted. Concerning the loading δ for systematic mortality risk, we followed Gründl, Post, and Schulze (2006) and changed the loading to 0.5% and 5% (instead of 1%). The loading has a significant impact on the risk situation of the insurance company under systematic mortality risk. By demanding a loading of $\delta = 5\%$ instead of $\delta = 1\%$, the probability of default can be reduced in the longevity scenario by about 30% for a portfolio with only annuities and even by more than 70% for a portfolio with only life insurance contracts. When reducing the loading to 0.05%, the effects are reversed and the insurer's risk level increases under systematic mortality risk. However, the size of the effects is smaller as compared to a loading of $\delta = 5\%$. E.g., the mean loss for a portfolio with only annuities increases by about 5% in the longevity scenario.

When reducing the loading λ of the MCB from 35 bp to 20 bp, the premium under systematic mortality risk decreases from $\Pi_{x,T}^{sys} = 1,036$ to $\Pi_{x,T}^{sys} = 1,020$, which results in a lower risk of the insurance company under systematic mortality risk when MCBs are used for risk management. However, the effect is rather small. In contrast, a higher fraction of high risk assets in the investment portfolio considerably increases the risk situation of the life insurer. For example, decreasing the fraction of low risk assets α from 80% to 50% almost doubles the risk of an insurance company for a portfolio with only annuities and can more than triple it for mixed portfolios. The portfolio composition for which the risk of the insurance company is immunized against the modeled longevity scenario still ranges between 20% and 30% life insurance in the case without adverse selection, while under adverse selection, the fraction of life insurance contracts needed for an immunization decreases for a riskier asset strategy. Thus, as shown in Gatzert and Wesker

(2010), the asset allocation should be taken into account when determining the optimal portfolio composition. Concerning the impact of adverse selection, the relative increase in risk due to adverse selection is less pronounced for a riskier asset allocation. However, if adverse selection is misestimated, the increase in risk can still amount to more than 30% and is thus not negligible. Furthermore, the effectiveness of MCBs for lowering the risk level of an insurance company is slightly reduced for a riskier asset strategy.

Finally, setting the dividend to shareholders to zero ($r_e = 0\%$) considerably decreases the risk level of an insurance company as reserves can be built up faster. For a portfolio with only annuities and without adverse selection, for instance, the mean loss and the probability of default are reduced by about 20% for $r_e = 0\%$. In addition, the impact of adverse selection decreases as well, but to a minor extent.

4. SUMMARY

In this paper, we examine the impact of three different components of mortality risk – unsystematic mortality risk, adverse selection and systematic mortality risk – as well as the basis risk in longevity hedges resulting from adverse selection on a life insurer's risk level using U.K. data. Furthermore, we study the effectiveness of two risk management strategies, including natural hedging and the purchase of mortality contingent bonds (MCBs), in the case of a two-product life insurance company offering annuities and term life insurance contracts.

Our results show that under unsystematic mortality risk, the insurer's risk level can generally be reduced by means of portfolio composition. Taking into account adverse selection in addition to unsystematic mortality risk implies a substantial increase in the risk of an insurance company. However, the impact of adverse selection can be considerably reduced through a correct forecast of the relationship between life insurance policyholder mortality and annuitant mortality, i.e., under perfect information about adverse selection. Concerning the impact of systematic mortality risk, term life insurances are much more strongly affected than are annuities, which is due to the different types of risks insured.

Turning to the effect of the three mortality risk components on an insurer's risk management, our findings demonstrate that mortality contingent bonds can contribute to a major reduction in the risk level, even in the presence of basis risk, i.e., if the implied change in mortality is not identical for the underlying and the hedged population due to mortality heterogeneity and adverse selection effects. However, the extent of the risk reduction achievable with a certain volume of MCBs decreases substantially due to basis risk and if adverse selection is not correctly forecasted. Furthermore, an improvement in the efficiency of MCBs for mixed portfolios of term life insurances

and annuities can be observed. Thus, our results emphasize the importance of these factors, which should be taken into account when determining the volume of risk management activities needed to achieve a desired safety level.

Regarding the usefulness of risk management for reducing the impact of systematic mortality risk, our findings show that the effectiveness of MCBs is not severely hampered if adverse selection is correctly accounted for, i.e., under perfect information by the insurance company about annuitant mortality. This is true despite the presence of basis risk. The impact of unexpected low mortality on the mean loss, i.e., the severity of default, can be reduced by about one third through the use of MCBs if adverse selection is assumed absent or forecasted perfectly. Turning to the effectiveness of natural hedging under systematic mortality risk for eliminating the impact of an unexpected change in mortality, our observations show that despite the different implied level of mortality as well as speed of mortality improvement in the insurance portfolio, natural hedging can still be a feasible and important risk management tool against unexpected changes in mortality. However, in particular in the longevity scenario, adverse selection needs to be taken into account in determining the proper portfolio composition to immunize a portfolio against changes in mortality.

Our results indicate that for an insurance company selling different types of life insurance products, besides correctly forecasting the mortality of the population in general, which has been given great attention in recent years, the correct forecasting of the relationship between annuitant mortality and the mortality of the population as a whole is crucial. Here, further research, especially concerning the development of this relationship over time, seems necessary to enable insurance companies to conduct efficient risk management with respect to mortality risk.

REFERENCES

- Barbarin, J. (2008): Heath–Jarrow–Morton Modeling of Longevity Bonds and the Risk Minimization of Life Insurance Portfolios, in: *Insurance: Mathematics and Economics*, 43(1), 41–55.
- Bayraktar, E. and Young, V. (2007): Hedging Life Insurance with Pure Endowments, in: *Insurance: Mathematics and Economics*, 40(3), 435–444.
- Biffis, E., Denuit, M. and Devolder, P. (2009): Stochastic Mortality under Measure Changes, in: *Scandinavian Actuarial Journal*, 1651–2030.
- Björk, T. (2004): *Arbitrage Theory in Continuous Finance*, 2nd edition, Oxford University Press, Oxford.
- Blake, D. and Burrows, W. (2001): Survivor Bonds: Helping to Hedge Mortality Risk, in: *Journal of Risk and Insurance*, 68(2) 339–348.
- Blake, D., Cairns, A., Dowd, K. and MacMinn, R. (2006): Longevity Bonds: Financial Engineering, Valuation and Hedging, in: *Journal of Risk and Insurance*, 73(4), 647–672.
- Brouhns, N., Denuit, M. and Vermunt, J. K. (2002a): A Poisson log-bilinear Regression Approach to the Construction of Projected Lifetables, in: *Insurance: Mathematics and Economics*, 31(3), 373–393.
- Brouhns, N., Denuit, M. and Vermunt, J. K. (2002b): Measuring the Longevity Risk in Mortality Projections, in: *Bulletin of the Swiss Association of Actuaries*, 2002(2), 105–130.
- Cairns, A., Blake, D., Dawson, P. and Dowd, K. (2005): Pricing the Risk on Longevity Bonds, in: *Life and Pensions*, October, 41–44.
- Cairns, A., Blake, D., Dowd, K., Coughlan, G. D., Epstein, D., Ong, A., and Balevich, I. (2009): A Quantitative Comparison of Stochastic Mortality Models Using Data from England and Wales and the United States, in: *North American Actuarial Journal*, 13(1), 1–35.
- Cohen, A. and Siegelmann, P. (2010): Testing for Adverse Selection in Insurance Markets, in: *Journal of Risk and Insurance*, 77(1), 39–84.
- Coughlan, G., Epstein, D., Ong, A., Sinha, A., Hevia-Portocarrero, J., Gingrich, E., Khalaf-Allah, M. and Joseph, P. (2007): *LifeMetrics*, A Toolkit for Measuring and Managing Longevity and Mortality Risks, Technical Document, JPMorgan Pension Advisory Group.
- Coughlan, G. D., Khalaf-Allah, M., Ye, Y., Kumar, S., Cairns, A., Blake, D. and Dowd, K. (2010): Longevity hedging: A framework for longevity basis risk analysis and hedge effectiveness, The Pensions Institute, Discussion Paper PI-1013.
- Cox, S. H. and Lin, Y. (2007): Natural Hedging of Life and Annuity Mortality Risk, in: *North American Actuarial Journal*, 11(3), 1–15.
- Dawson, P., Dowd, K., Cairns, A. and Blake, D. (2011): Survivor Derivatives: A Consistent Pricing Framework, in: *Journal of Risk and Insurance*, 77(3), 579–596.

- Finkelstein, A. and Poterba, J. M. (2002): Selection Effects in the United Kingdom Individual Annuities Market, in: *Economic Journal*, 112(476), 28– 50.
- Gatzert, N. and Kling, A. (2007): Analysis of Participating Life Insurance Contracts: A unification Approach, in: *Journal of Risk and Insurance*, 74(3), 547–570.
- Gatzert, N. and Wesker, H. (2010): The Impact of Natural Hedging on a Life Insurer's Risk Situation, Working Paper, Friedrich-Alexander University of Erlangen-Nuremberg.
- Gründl, H., Post, T. and Schulze, R. (2006): To Hedge or not to Hedge: Managing Demographic Risk in Life Insurance Companies, in: *Journal of Risk and Insurance*, 73(1), 19–41.
- Hanewald, K. (2011): Explaining Mortality Dynamics: The Role of Macroeconomic Fluctuations and Cause of Death Trends, in: *North American Actuarial Journal*, forthcoming.
- Hanewald, K., Piggot, J. and Sherris, M. (2011): Individual Post-retirement Longevity Risk Management under Systematic Mortality Risk, Australian School of Business Research Paper No. 2011ACTL10.
- Hanewald, K. Post, T. and Gründl, H. (2011): Stochastic Mortality, Macroeconomic Risks, and Life Insurer Solvency, in: *Geneva Papers on Risk and Insurance - Issues and Practice*, forthcoming.
- Human Mortality Database: University of California, Berkeley, and Max Planck Institute for Demographic Research (Germany), available at: www.mortality.org or www.humanmortality.de (data downloaded on 01/16/2012).
- Lee, R. D. and Carter, L. R. (1992): Modeling and Forecasting U.S. Mortality, in: *Journal of the American Statistical Association*, 87(419), 659–671.
- Li, J. and Hardy, M. (2009): Measuring Basis Risk involved in Longevity Hedges, presentation at: Fifth International Longevity Risk and Capital Markets Solutions Conference.
- Lin, Y. and Cox, S. H. (2005): Securitization of Mortality Risks in Life Annuities, in: *Journal of Risk and Insurance*, 72(2), 227–252.
- Lin, Y. and Cox, S. H. (2008): Securitization of Catastrophe Mortality Risks, in: *Insurance: Mathematics and Economics*, 42(2), 628–637.
- Milevsky, M. M. and Promislow, D. (2003): Can Life Insurance be used to Hedge Payout Annuities?, Part 1: Modeling Longevity Risk, Working Paper.
- Ngai, A. and Sherris, M. (2011): Longevity Risk Management for Life and Variable Annuities: The Effectiveness of Static Hedging using Longevity Bonds and Derivatives, in: *Insurance: Mathematics and Economics*, 49(1), 100–114.
- Plat, R. (2009): Stochastic Portfolio Specific Mortality and the Quantification of Mortality Basis Risk, in: *Insurance: Mathematics and Economics*, 45(1), 123–132.
- Stevenson, M. and Wilson, A. (2008): Mortality of Public Sector Scheme Pensioners 2005-2007 Update. Presented to the Institute of Actuaries of Australia 4th Financial Services Forum.

Sweeting, P. (2007): Pricing Basis Risk in Survivor Swaps, Discussion Paper PI-0622, The Pensions Institute, London.

Wang, J. L., Huang, H.C., Yang, S. S. and Tsai, J. T. (2010): An Optimal Product Mix for Hedging Longevity Risk in Life Insurance Companies: The Immunization Theory Approach, in: *Journal of Risk and Insurance*, 77(2), 473–497.

Wetzel, C. and Zwiesler, H.-J. (2008): Das Vorhersagerisiko der Sterblichkeitsentwicklung - Kann es durch eine geeignete Portfoliozusammensetzung minimiert werden?, in: *Blätter der DGVM*, 29(1), 73–107.

Wills, S. and Sherris, M. (2010): Securitization, structuring and pricing of longevity risk, in: *Insurance: Mathematics and Economics*, 46(1), 173–185.

APPENDIX A

Figure A.1: Estimated value of $\exp(a_x)$ and b_x over all ages

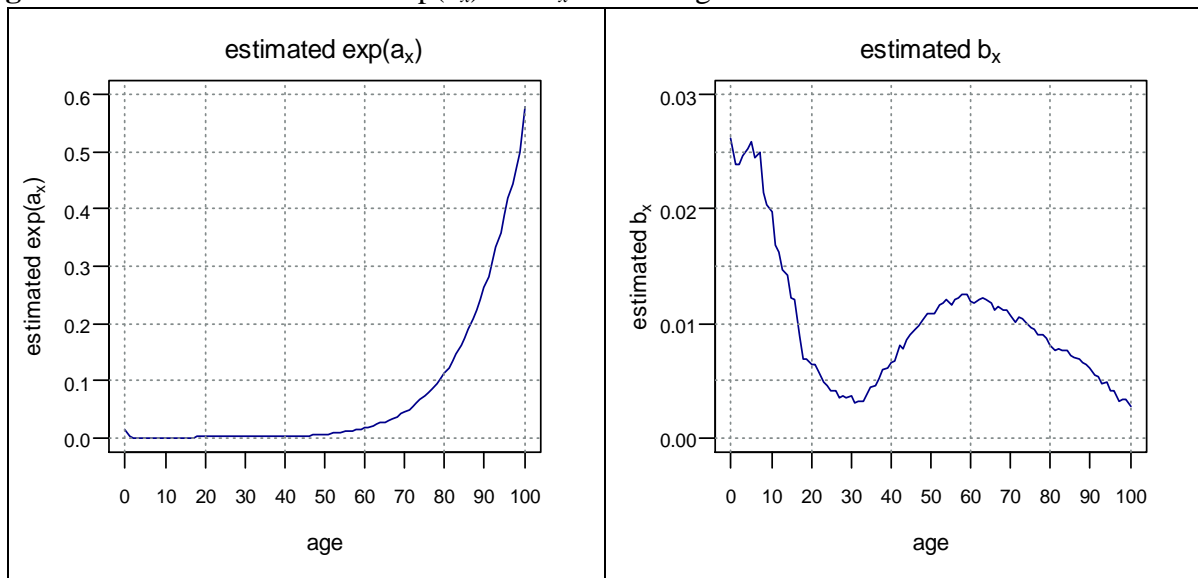


Figure A.2: Level of estimated mortality index k_t and forecasted values of k_t

