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Working Paper

Chair for Insurance Economics
Friedrich-Alexander-University of Erlangen-Nuremberg

Version: May 2012

THE IMPACT OF NATURAL HEDGING ON A LIFE INSURER'S RISK SITUATION

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ABSTRACT

Systematic mortality risk, i.e. the risk of unexpected changes in mortality and survival rates, can substantially impact a life insurers' risk and solvency situation. By using the "natural hedge" between life insurance and annuities, insurance companies have an effective tool for reducing their net-exposure. The aim of this paper is to analyze this risk management tool and to quantify its effectiveness in hedging against changes in mortality with respect to default risk measures. To achieve this goal, we model the insurance company as a whole and take into account the interaction between assets and liabilities. Systematic mortality risk is considered in two ways. First, systematic mortality risk is modeled using scenario analyses and, second, empirically observed changes in mortality rates for the last 10-15 years are used. We demonstrate that the consideration of both the asset and liability side is vital to obtain deeper insight into the impact of natural hedging on an insurer's risk situation and show how to reach a desired safety level while simultaneously immunizing the portfolio against changes in mortality rates.

Keywords: Longevity risk, mortality risk, natural hedging, life insurance, risk management

JEL Classification: G22, G23, G32, J11

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1. Introduction

In recent years, life expectancy in all industrialized countries has risen steadily. This poses a serious problem to pension funds and life insurance companies selling annuities, since the payouts for these products might be higher than expected.[1] At the same time, worldwide pandemics, such as the swine and bird flu, have appeared more frequently and spread more rapidly. A serious pandemic might lead to severely increased mortality,[2] implying high losses for a life insurance company selling life insurance contracts with a death benefit payment. This risk of unexpected high or low mortality is one of the major risks to which a life insurance company is exposed through its insurance portfolio and is of high relevance, because instruments to hedge this risk through an external partner are scarce. However, life insurance companies can use the "natural hedge", which is the opposed reaction in the value of liabilities and in the amount of benefit payouts between term life insurance products and annuities in response to shifts in mortality, to lower their net-exposure. The aim of this paper is to quantify the impact of natural hedging on a life insurance company's insolvency risk using a multi-period model framework that takes into account assets and liabilities. In contrast to previous literature, we further focus on two issues simultaneously, namely the question of how to *immunize* an insurer's solvency situation against specific changes in mortality and, at the same time, *fix the absolute level of risk*, which we illustrate by means of the insurer's investment strategy.

Instruments for hedging mortality and longevity risk, especially the possibility of hedging through capital markets, have been discussed thoroughly in the literature. A comprehensive overview about potential and existing capital market instruments is given in Blake, Cairns, and Dowd (2006a). Examples for these instruments are mortality contingent bonds, first introduced by Blake and Burrows (2001) as "survivor bonds", mortality swaps, described by Cox and Lin (2007) as a natural hedge between companies, mortality options and mortality futures. As an application of these instruments, Luciano, Regis, and Vigna (2011a) propose a Delta-Gamma hedge technique for mortality risk in endowment insurance contracts by means of longevity bonds. They additionally hedge stochastic interest risk through purchasing longevity and zero coupon bonds and extend their work in Luciano, Regis, and Vigna (2011b) by combining Delta-Gamma hedging using the natural hedge between life insurance contracts and annuities. Pricing of these instruments is discussed in, e.g., Lin and Cox (2008), and the

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[1] Some life insurance companies already claim to make losses on their annuity portfolio, because their policyholders are living too long (see Blake, Cairns, and Dowd (2006a, p. 154)).

[2] Cowley and Cummins (2005) additionally refer to the increased likelihood of terrorism in this context (p. 202).

current state of the market for these instruments is described, e.g., in Blake et al. (2009). An alternative for hedging mortality risk was proposed by Dahl (2004), who suggests linking premiums and/or benefit payments to realized mortality in the population, thus transferring the risk of an increase in the general life expectancy back to the insured.

Different aspects of natural hedging have already been discussed in the literature as well. Cox and Lin (2007) use empirical data on market quotes for single premium immediate annuities to show that they are lower for insurance companies offering life insurance and annuities at the same time as compared to "one-product" insurance companies. Bayraktar and Young (2007) follow a similar approach as in Cox and Lin (2007). To study the effect of natural hedging, they use the instantaneous Sharpe ratio to price pure endowments and life insurances jointly and show that the price for a portfolio of m life insurances and n pure endowments is lower than the sum of the prices of a portfolio of m life insurances and a portfolio of n pure endowments. Gründl, Post, and Schulze (2006) assume a shareholder value maximizing strategy and compare the effects of different risk management strategies on shareholder value during one period with a discrete mortality model. In their model framework, natural hedging is the preferred risk management tool only under certain circumstances and in others might even decrease shareholder value. Gatzert and Wesker (2011) examine the impact of different mortality risk components on an insurer's risk situation and risk management, thereby also considering natural hedging and mortality contingent bonds. Their results show that adverse selection can play an important role when determining an optimal portfolio composition to immunize a portfolio against changes in mortality, especially in case of a longevity scenario. Wang et al. (2010) focus on the change in the value of liabilities due to changes in mortality rates and discuss an immunization strategy for this risk through portfolio composition. They apply the concept of duration to mortality and derive an optimal liability mix, which is characterized by a portfolio-mortality-duration of zero. The aim of Wetzel and Zwiesler (2008) is to find a (liability) variance minimizing product mix in a stochastic interest and stochastic mortality framework. They show that the mortality variance, which is the variance due to fluctuations in mortality, can be reduced by more than 99% through portfolio composition. However, while Wang et al. (2010) and Wetzel and Zwiesler (2008) show that natural hedging can significantly lower the sensitivity of an insurance portfolio with respect to mortality risk, both concentrate on the liability side and do not take into account the asset side.

We will expand their viewpoint by considering the insurance company as a whole by taking into account both, assets and liabilities, as well as their interaction. In contrast to e.g. Gründl, Post, and Schulze (2006), in considering assets and liabilities we further focus on the insurer's risk situation in a multi-period setting over 35 years, which corresponds approximately to the usually observed duration of a life insurance contract and spans the majority of annuity contracts as well. We dynamically take into account possible default over time, as the timing of payouts can significantly influence risk during the observation period. Furthermore, in contrast to, e.g., Gatzert and Wesker (2011), we show how to obtain a desired safety level while simultaneously immunizing a portfolio against changes in default risk. The procedure is illustrated by varying the insurer's investment strategy. Overall, this approach allows a more comprehensive view of a life insurer's long-term risk situation and the risk reduction effect attainable by means of natural hedging. It is particularly useful in light of Solvency II to provide insight into the possible reduction of solvency capital requirements and long-ranging effects of management decisions concerning portfolio composition. It is further relevant for the calculation of the Market Consistent Embedded Value (MCEV), where diversification benefits between non-hedgeable risks may be taken into account, given that they are identifiable and quantifiable.[3]

The distribution of mortality is based upon the extension of the Lee-Carter (1992) model by Brouhns, Denuit, and Vermunt (2002a), which has slightly more attractive theoretical features than the original model.[4] To quantify the impact of systematic mortality risk, i.e. unexpected changes in mortality rates, on the solvency situation of the insurance company and thus the extent to which this impact can be hedged through natural hedging and to deduce the optimal ratio of life insurance contracts to hedge against systematic mortality risk, a simulation approach is used. We thereby distinguish between a longevity and a mortality scenario. Our results show that, in the present setting, systematic mortality risk can be hedged by selling about 15 – 20% term life insurance and 85 – 80% annuities depending on the risk measure and the considered scenario. Additionally, we apply this approach to realized changes in mortality rates, thus incorporating adverse selection by differentiating between mortality rates of annuitants and life insurance policyholders, and find that these can be hedged as well, but that the optimal hedge ratio, at which the impact of changing death and survival probabilities can be eliminated completely in our model setup, is dependent upon the exact realization of the in- or

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[3] The MCEV is a concept to measure shareholder value of life insurance companies introduced by the CFO Forum. This is a group formed by the Chief Financial Officers of leading European insurance companies (amongst others, Allianz, AXA, BNP Paribas, Generali, Munich Re, Swiss Re, Zurich) with the stated goal to "influence the development of financial reporting, value based reporting, and related regulatory developments for insurance enterprises on behalf of its members" (see http://www.cfoforum.nl/).

[4] This mortality model is taken as an example and can as well be replaced by other stochastic mortality models, depending on the concrete application of the approach (and the respective country). For instance, according to a quantitative comparison study by Cairns et al. (2009), a variation by the Cairns, Blake, and Dowd (2006b) two-factor model is very suitable to explain improvements in mortality rates in England.

decrease in mortality rates. We further compare the case of continued underwriting activities to the single portfolio case and find that the more general case of continued business activities only results in minor changes. Our main finding is that by selling 15% to 20% life insurance contracts, depending on the exact changes in mortality and the risk measure chosen, the impact of mortality risk can be reduced significantly. The optimal hedge ratio depends on the considered scenario for systematic mortality risk, the exact realization of mortality improvements and the investment strategy of the insurance company, each of which should be taken into account by life insurers when writing new business.

The remainder of this paper is structured as follows. Section 2 presents the model framework, including the model of the insurance company, mortality assumptions and modifications, product characteristics and relevant risk measures. The results of the numerical analysis are laid out in Section 3 and Section 4 concludes.

2. MODEL FRAMEWORK

This section describes the model framework used to examine the effects of natural hedging on an insurer's portfolio consisting of term life insurance and annuities. To measure the effectiveness of natural hedging, assets, liabilities and consequently the possibility of default are taken into account.

Modeling mortality risk

As the basis for death and survival probabilities, we use the extension of the Lee-Carter (1992) model by Brouhns, Denuit, and Vermunt (2002a) to estimate and project future mortality. Depending on the respective country, other mortality models may be more appropriate to adequately forecast mortality rates of the population.[5] Thus, the following analysis can as well be conducted using other stochastic mortality models and the model used here can be considered as an example, as our aim is to focus on the general approach with respect to natural hedging, fixing the level of risk and immunizing this risk level against unexpected changes in mortality rates, i.e. systematic mortality risk.

The Lee-Carter (1992) model consists of a demographic and a time series part, where the central death rate or force of mortality of an x year old male in year τ $\mu_x(\tau)$ is modeled through

5 In general, model risk can play an important role in mortality projections (see Coppola et al., 2011).

$$\ln\left[\mu_x(\tau)\right] = a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau} \iff \mu_x(\tau) = e^{a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau}},$$

where k_τ is a time varying index that shows the general development of mortality over time, a_x and b_x are time constant parameters indicating the general shape of mortality over age and the sensitivity of the mortality rate at age x to changes in k_τ, respectively, and $\varepsilon_{x,\tau}$ is an error term with mean zero and constant variance. Brouhns, Denuit, and Vermunt (BDV) (2002a) propose a modification to the model, which results in slightly more attractive theoretical properties. The realized number of deaths at age x and time τ, $D_{x,\tau}$, is modeled as

$$D_{x,\tau} \sim Poisson\left(E_{x,\tau} \cdot \mu_x(\tau)\right) \text{ with } \mu_x(\tau) = e^{a_x + b_x \cdot k_\tau}, \tag{1}$$

where $E_{x,\tau}$ is the risk exposure at age x and time τ defined as $E_{x,\tau} = \left(n_{x-1}(\tau-1) + n_x(\tau)\right)/2$ and $n_x(\tau)$ is the number of persons (i.e., the population size) still alive at age x and the end of year τ.[6] An important advantage of the BDV (2002a) model is that the restrictive assumption of homoscedastic errors made in the Lee-Carter (1992) model is given up. Furthermore, the resulting Poisson distribution is well suited for a counting variable such as the number of deaths (see Brillinger (1986)). The model can be estimated via the Maximum-Likelihood approach using a uni-dimensional Newton method as proposed by Goodman (1979).[7] Since a_x and b_x are time constant, they can be used directly in forecasting mortality rates. However, since k_τ is time-varying, one needs to obtain forecasts of k_τ for predicting future mortality. Lee and Carter (1992) propose to fit an appropriate ARIMA process on the estimated time series of k_τ

$$k_\tau = \underbrace{\phi + \alpha_1 \cdot k_{\tau-1} + \alpha_2 \cdot k_{\tau-2} + \ldots + \alpha_p \cdot k_{\tau-p} + \delta_1 \cdot \varepsilon_{\tau-1} + \delta_2 \cdot \varepsilon_{\tau-2} + \ldots + \delta_q \cdot \varepsilon_{\tau-q}}_{\hat{k}_\tau} + \varepsilon_\tau = \hat{k}_\tau + \varepsilon_\tau .$$

using Box-Jenkins time series analysis techniques. The obtained parameters of the ARIMA process can then be used to forecast $\hat{k}_\tau$, and, thus, $\mu_x(\tau)$.[8] Based on the estimated $\mu_x(\tau)$, the one-year death probability $q_x(\tau)$, which is the probability that an x-year old male in year τ will die within the next year, given he has survived until age x-1, can be calculated using $q_x(\tau) = 1 - \exp\left(-\mu_x(\tau)\right)$ (see Brouhns, Denuit, and Vermunt (2002a, p. 376)).[9] The respective one-year survival probability of an x-year old male policyholder is $p_x(\tau) = 1 - q_x(\tau)$, and

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[6] For simulation purposes, $E_{x,t} = -n_x(t-1) \cdot q_x / \ln(p_x)$ is used instead (see Brouhns, Denuit, and Vermunt (2002b)).

[7] Standard Maximum-Likelihood methods are not feasible due to the bilinear term $b_x k_t$.

[8] The error term ε_τ is set equal to zero in forecasting since $E(\varepsilon_\tau) = 0$.

[9] In this calculation, we assume a piecewise constant force of mortality $\mu_x(\tau)$.

the probability that an *x*-year old male policyholder will survive the next *n* years, ${}_n p_x$, can be calculated as ${}_n p_x = \prod_{i=1}^{n} p_{x+i-1}$. In the remainder of the paper, we omit the indicator τ to simplify the notation; however, all mortality rates are dependent on age and year.

It has been observed in the past that mortality rates do not remain constant but are subject to random, unexpected changes that arise due to, e.g., common factors that impact all individuals in a similar way, thus causing dependencies between lives that cannot be diversified through enlarging the portfolio size (see e.g. Biffis, Denuit, and Devolder (2010), Wills and Sherris (2010), Gatzert and Wesker (2011)). This can in general be attributed either to unexpected environmental or social influences, impacting mortality positively or negatively,[10] or to wrong expectations about future mortality due to estimation errors.[11] The impact of this so called systematic mortality risk is defined and accounted for differently in the literature. Hanewald, Piggot, and Sherris (2011) model systematic (longevity) risk as uncertain changes in mortality applying to all individuals, while Evans and Sherris (2009) define it as the uncertainty in future survival probabilities, which imply dependencies between lives due to a common improvement in mortality rates across individuals. In particular, while mortality risk may not be hedgeable in financial markets, insurers can reduce it by means of, e.g., natural hedging, reinsurance, asset-liability management, or mortality swaps (see Cox and Lin (2007)).

To gain comprehensive insight into the effectiveness of natural hedging for diversifying this risk, we follow the approach in Gatzert and Wesker (2011) and make different assumptions concerning the realization of the change in mortality rates using the absolute value of the factor ε_τ, which impacts mortality at all ages in year τ and thus causes dependencies between lives.[12] We thereby distinguish between a "longevity scenario" with unexpected low mortality, and a "mortality scenario" with unexpected high mortality, by defining

$$k_\tau^{longevity} = \hat{k}_\tau - s \cdot \left|\varepsilon_\tau\right| \text{ and } k_\tau^{mortality} = \hat{k}_\tau + s \cdot \left|\varepsilon_\tau\right|,$$

where *s* is a scaling factor that influences the extent of the implied change in life expectancy and allows a more detailed insight into the impact of mortality and longevity scenarios. While this procedure affects mortality at all ages, it does not lead to an identical change of mortality at all ages due to the multiplication with the term b_x. Therefore, the simulated change in the

10 Additionally, certain other macroeconomic variables might have an influence on mortality (see, e.g., Hanewald (2010)).

11 An example of a potential source of estimation error is the choice of the appropriate sample period, since k_τ is rather sensitive towards the specified period.

12 Due to the assumed ARIMA process for k_τ, subsequent years are also impacted by the realization of ε_τ, see Gatzert and Wesker (2011).

mortality rate for an x-year old male is consistent with the sensitivity of mortality at this age observed in the past. The deduced mortality rates based on k_{τ}^{i} for a given scaling factor will be referred to as q_x^i and ${}_n p_x^i$, for i = *longevity, mortality*, respectively. The initial mortality rates forecasted using the BDV (2002a) model are denoted by i = *initial*.

Second, recently observed changes in mortality rates are modeled by simulating the change from the old mortality tables by the Continuous Mortality Investigation (CMI) dating from the year 1992 to the updated mortality tables from the year 2000. To take into account the possibility that mortality for life insurance policyholders and annuitants might experience a different change in mortality, here referred to as adverse selection (see, e.g., Brouhns, Denuit, and Vermunt (2002a)), we use different mortality tables for these two populations of insured. Thus, we are able to analyze the usefulness of natural hedging under actually realized changes in mortality and in the presence of adverse selection.

Model of a life insurance company

A simplified balance sheet of the modeled insurance company at time t is shown in Table 1, where $A^i(t)$ is the market value of the assets of the company at time t, $L^i(t)$ is the value of total liabilities for the term life insurances and annuities, and $E^i(t)$ is the equity of the insurance company, which is residually determined as the difference between assets and liabilities. The development of these accounts depends on the mortality assumption and the scenario considered, i.e. whether the initial death rates are assumed or the longevity or mortality scenario, i = *initial, longevity, mortality*.

Table 1: Balance sheet of the insurance company at time t depending on mortality assumptions (i = *initial, longevity, mortality*)

Assets	**Liabilities**
$A^i(t)$	$E^i(t)$
	$L^i(t)$

A default occurs when assets are not sufficient to cover liabilities at time t, i.e., when $L^i(t) > A^i(t)$. In this situation, the insurance company does not hold sufficient assets to cover its future payment obligations and the company is consequently shut down. However, we will assume that the insurance benefits acquired by the policyholders are guaranteed by an external institution, which takes over payment of the benefits in case of default (see, e.g., Gatzert and Kling (2007)). Thus, the policyholders are not affected by the possibility of default and the value of their contracts does not depend upon the corresponding probability of default.

With term life insurance, a constant death benefit *DB* is paid out in case the insured dies during the contract term. We assume that policyholders pay a constant annual premium *P* at the beginning of the year, while the death benefit *DB* is paid at the end of the year in which the insured dies. With an annuity, a constant annual benefit *a* is paid at the end of each year as long as the policyholder is alive in return for a single premium *SP*, which is paid in $t = 0$. The duration of both contract types is random, as it depends upon the individual time of death, which is limited by the maximum age implied by the stochastically forecasted mortality rates ($\omega = 100$) and the contractually defined time to maturity. The initial investment made by the shareholders of the insurance company is denoted by $E(0)$. In return for their investment, the shareholders receive a constant dividend[13] *div* in each year the company is still active, i.e. has not yet defaulted. In case of default, the shareholders lose their investment, but, due to their limited liability, do not have to compensate the difference between $A^i(t)$ and $L^i(t)$.

Assets $A^i(t)$ are invested in the capital market, and, since we are merely interested in the development of the assets at an aggregate level, their composition is not considered here. Following most of the literature dealing with the valuation of insurance liabilities (e.g. Grosen and Jørgensen (2000, 2002)), we assume that the value of the asset portfolio evolves according to a geometric Brownian motion,

$$dA^i(t) = \mu \cdot A^i(t) \cdot dt + \sigma \cdot A^i(t) \cdot dW^P(t), \text{ for } i = \textit{initial, longevity, mortality},$$

where μ is the drift of the assets, σ the asset volatility, and W^P a standard Brownian motion under the real-world measure *P* on the probability space $(\Omega, \mathcal{F}, P)$, where $\mathcal{F}$ is the filtration generated by the Brownian motion. The solution of this stochastic differential equation is given by (see Björk (2009))

$$A^i(t) = A^i(t-1) \cdot \exp\left[\mu - \sigma^2/2 + \sigma\left(W^P(t) - W^P(t-1)\right)\right], \; i = \textit{initial, longevity, mortality}.$$

for a given $A(0)$. The asset base is influenced by premium payments, death benefit and annuity payments to the policyholders as well as dividend payments and mortality assumptions. Figure 1 exhibits the evolution of these payments and their influence on the asset base, where t^- denotes that payments are made in the beginning of the year *t*, while t^+ denotes those made

13 One way to define the dividend amount is to calculate it in a way that the shareholders expect to receive the risk-free rate (under the risk-neutral measure *Q*) or the market rate of return (under the real world probability measure *P*) on their initial investment each year. In this framework, the shareholders assume a yearly probability of default of 1%, thus the dividend can be calculated using $r \cdot E_0 = (1-0.01) \cdot div + 0.01 \cdot (-E_0)$ in the risk-neutral world. Under the real-world measure *P*, *r* is substituted by the market rate of return, r_m.

at the end. The number of survivors in the annuities portfolio at the beginning of year t is denoted by $n_A^i(t)$, and the number of survivors in the life insurance portfolio by $n_L^i(t)$, both dependent on the actual mortality experience in the portfolio (i = *initial, longevity, mortality*). Hence, $d_L^i(t) = n_L^i \cdot (t-1) - n_L^i(t)$ is the number of deaths within the t-th year in the life insurance portfolio, for which the insurance company has to pay out death benefits. Assuming that the total number of contracts initially sold is equal to n, the product mix is thus determined by the portion of life insurance contracts sold, denoted by d, such that, at time 0, the portfolio consists of $n_L(0) = d \cdot n$ life insurance and $n_A(0) = (1-d) \cdot n$ annuity contracts, where n = $n_A(0) + n_L(0)$.

In t = 0, the asset base consists of the initial equity by the shareholders and the premiums paid by the policyholders. At the end of each year, annuities have to be paid to all holders of annuities who are still alive at the end of each year and death benefits have to be paid to all heirs of life insurance policyholders who died within each year. Thus, assets depend crucially upon the composition of liabilities and thus on the product-mix decision (d).

Figure 1: Evolution of asset base depending on mortality assumptions (i = *initial, longevity, mortality*)

$t = 0^+$	$t = 1^-$	$t = 1^+$	$t = 2^-$	…	$t = 10^+$	$t = 11^-$	…
$+ E_0$	$- n^i_A(1) \cdot a$	$+ n^i_L(1) \cdot P$	$- n^i_A(2) \cdot a$	…	$+ n^i_L(10) \cdot P$	$- n^i_A(11) \cdot a$	…
$+ n_A(0) \cdot SP$	$-d^i_L(0) \cdot DB$		$-d^i_L(1) \cdot DB$			$- d^i_L(10) \cdot DB$	
$+ n_L(0) \cdot P$	$- div$		$- div$			$- div$	

Valuation

To determine a fair combination of benefits and premiums, we use risk-neutral valuation. We thus calculate the expected cash flows under the risk-neutral measure Q and discount them with the risk-free rate r. Mortality and market risks are assumed to be independent[14] (see, e.g., Carriere (1999, p. 340), Gründl, Post, and Schulze (2006)), and we assume that the insurance

[14] The assumption of independence between market and mortality risk is also supported by empirical studies. Hanewald (2010), for instance, analyzes the relationship between macroeconomic variables and the mortality index k_τ in several countries. While there are significant correlations for the sample as a whole, for the period from 1980 to 2006, the assumption of significant correlations between the two time series cannot be supported by the data. Furthermore, Ribeiro, and di Pietro (2009) study the correlations between longevity risk and the prices of equities and bonds. Even though longevity risk is exploited twofold, no significant correlations between stock and bond prices and longevity can be found in either case.

company does not demand a risk-premium for mortality risk.[15] Therefore, the risk-neutral martingale measure Q is identical to the objective probability measure P with respect to mortality.[16]

Premiums and benefits of the insurance contracts are calculated using the equivalence principle. Hence, the insurance company calculates the premiums and benefits such that the expected benefit payments and the expected premium payments are equal at the inception of the contract. The value of each contract is set equal to a fixed amount M. Thus, using the risk-free rate r, the premium P and death benefit DB for the term life insurance can be calculated as

$$\sum_{t=0}^{T-1} P \cdot {}_t p_x^{initial} \cdot (1+r)^{-t} = \sum_{t=0}^{T-1} DB \cdot {}_t p_x^{initial} \cdot q_{x+t}^{initial} \cdot (1+r)^{-(t+1)} \stackrel{!}{=} M \tag{2}$$

and the single premium SP and annuity a using Equation (3) as

$$\sum_{t=0}^{T-1} a \cdot {}_t p_x^{initial} \cdot (1+r)^{-(t+1)} = SP \stackrel{!}{=} M. \tag{3}$$

Here, the initially forecasted mortality rates from the BDV (2002a) model are used in pricing and reserving, since the modeled scenarios for systematic mortality risk represent unexpected changes in mortality. Since the volume of the expected premium and benefit payment for annuities and life insurance contracts is identical, and the total number of contracts n (sum of life insurance and annuities) sold by the insurance company is fixed, the volume of the insurance portfolio does not vary for different portfolio compositions. This is intended to ensure comparability between portfolios in terms of the present value of cash in- and outflows and to thus isolate the effect of portfolio composition.

The value of liabilities for one term life insurance can be calculated as

$$B_L(t) = \sum_{s=0}^{T-t-1} \left[DB \cdot {}_s p_{x+t}^{initial} \cdot q_{s+x+t}^{initial} \cdot (1+i)^{-(s+1)} - P \cdot {}_s p_{x+t}^{initial} \cdot (1+i)^{-s} \right] \tag{4}$$

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15 This can be incorporated by, e.g., adding a loading on the actuarially fair premium, such that the insurer demands an additional premium for systematic mortality risk (see, e.g., Gatzert and Wesker (2011)). The general results of natural hedging presented in the numerical analysis section remain robust in this case, while only the level changes.

16 See, e.g., Bacinello (2003, p. 468), Dahl (2004, p. 124), Gatzert, Hoermann, and Schmeiser (2009, p. 890), Gründl, Post, and Schulze (2006).

and the value for one annuity contract is given by

$$B_A(t) = \sum_{s=0}^{T-t-1} a \cdot {}_s p_{x+t}^{initial} \cdot (1+i)^{-(s+1)} \quad . \tag{5}$$

The value of liabilities for the whole portfolio is thus attained by multiplying Equations (4) and (5) with the respective number of contracts that are still active at time t, i.e.,

$$L^i(t) = n_A^i(t) \cdot B_A(t) + n_L^i(t) \cdot B_L(t), \;\; i = initial, longevity, mortality\,,$$

which depends on the mortality scenario.

Risk measurement

To date, to the best of our knowledge, the literature on natural hedging has focused mainly upon the reaction of the liability side in response to shifts in mortality and how these can be balanced out through portfolio composition. Although we conduct this study as part of our analysis, for the main part, we are interested in the riskiness and solvency situation of the insurance company as a whole. Therefore, we first use the static measure of the expected benefit payouts in t = 0, i.e. the contractual payment obligations CP of the insurance company for different survival and mortality rates to analyze the impact of a change in mortality rates on the liability side. The contractual payment obligations (CP) for given death and survival probabilities are a linear function in the fraction of life insurance contracts d of the form

$$\begin{aligned} CP^i &= d \cdot n \cdot \sum_{t=0}^{T-1} DB \cdot {}_t p_x^i \cdot q_{x+t}^i \cdot (1+r)^{-(t+1)} + (1-d) \cdot n \cdot \sum_{t=0}^{T-1} a \cdot {}_t p_x^i \cdot (1+r)^{-(t+1)} \\ &= d \cdot n \cdot V_L^i + (1-d) \cdot n \cdot V_A^i = n \cdot V_A^i + d \cdot n \cdot \left(V_L^i - V_A^i\right), \end{aligned}$$

i = *initial*, *longevity*, *mortality*. V_A^i are the deterministic contractual payment obligations for one annuity subject depending on the scenario i = *initial*, *longevity*, *mortality*, and V_L^i is defined analogously for one term life insurance. Since the evolution of assets is considerably influenced by the product mix d, in addition to the static liability side measure, we further focus on different risk measures, which explicitly take into account the interaction between assets and liabilities and the possibility of default, capturing different default characteristics, including the probability of default and the mean loss. Under the real-world measure P, the probability of default (ruin probability) is given by

$$PD^i = P\left(T_d^i \le T\right), \tag{6}$$

where T_d^i is defined as $T_d^i = \inf\left\{t : A^i\left(t\right) < L^i\left(t\right)\right\}$ (see, e.g., Gerstner et al. (2008)), i = *initial*, *longevity*, *mortality*. In the numerical analysis, the probability of default is further divided by the number of years, such that it can be interpreted as the mean annual probability of default for the next T years. The second risk measure is the mean loss, which is calculated as the discounted expected loss in case of default, thus (in contrast to the probability of default) taking into account the extent of the default,

$$ML^i = E\left[\left(L^i\left(T_d^i\right) - A^i\left(T_d^i\right)\right)\cdot\left(1+r\right)^{-T_d^i}\cdot 1\left\{T_d^i \le T\right\}\right], \; i = initial, longevity, mortality. \tag{7}$$

Natural hedging

To hedge against systematic mortality risk, the optimal portfolio composition has to be determined (for otherwise fixed contract and asset characteristics) using the natural hedge between term life insurances and annuities. This way, the insurer can immunize its portfolio against unexpected changes in mortality rates. The optimal hedge ratio d^* is thus defined as the percentage of life insurance contracts at which the respective risk measure R (= PD, ML) does not change. It is hence given by the root of the function f

$$f^j\left(d\right) = \Delta R\left(d;\mu_x^{initial}\left(\tau\right);\mu_x^j\left(\tau\right)\right) = R\left(d;\mu_x^{initial}\left(\tau\right)\right) - R\left(d;\mu_x^j\left(\tau\right)\right) \overset{!}{=} 0, \tag{8}$$

where j = *mortality ,longevity*.

3. Numerical Analyses

The numerical analysis is conducted in two steps. In the first step, the fair benefit and premiums are calculated analytically using Equations (2) and (3) and the estimated population mortality. In the second step, the obtained parameters are used in a simulation analysis under different assumptions concerning realized mortality to analyze the effect of natural hedging on an insurer's risk situation in the sense of scenario analyses.

Mortality estimation and projections

The estimation of mortality is based on the number of deaths and exposure to risk for the United Kingdom from 1950 to 2009 available through the Human Mortality Database. The estimated demographic parameters of the BDV (2002a) model are displayed in the Appendix in Figure A.1. The mean central death rate for age x, $\exp(a_x)$, increases steadily in age; b_x on the other hand indicates the sensitivity of the central death rate at age x towards changes in the time trend k_τ and is, for adult ages, highest around the ages 50-70, indicating that improvements in life expectancy in recent years are mainly due to decreases of mortality rates at "higher" ages as already stated by Blake and Burrows (2001, p. 346). The estimated mortality trend k_τ as well as the forecasted values obtained by applying Box-Jenkins time series analysis techniques on the estimated process of k_τ are shown in Figure A.2 in the Appendix. Time series analysis indicated an ARIMA (0,1,0) model[17] with drift equal to ϕ = -1.5403 (standard error 0.3056) and the standard error of ε_τ is estimated as 2.3474.

Input parameters

Until otherwise stated, we will assume a risk-free rate of r = 3%, r_m = 5%,[18] an asset volatility σ = 10% and an asset drift of μ = 6%. The male term life insurance policyholders acquire the policy at age x = 30 for a period of 35 years in the year τ = 2012. The male annuity policyholders purchases the lifelong annuity at age x = 65 at the same time τ.[19] The actuarial interest rate is also set to i = 3%, and the present value of each contract is equal to M = 1,000. The parameters are chosen to illustrate central effects and are subject to robustness checks and sensitivity analyses. Using Equations (2) and (3) with M = 1,000 and the input parameters described above, the resulting fair premium P for the term life insurance is 46 and the death benefit DB is 25,032. The fair annuity amount a is 75 and the single premium SP is 1,000. These numbers refer to a contract with expected premium payments and expected benefit payout of 1,000 and are based on the initially assumed mortality rates forecasted through the BDV (2002a) model. We will assume that the portfolio of the insurance company consists of a total of n = 100,000 contracts written in t = 0. We then consider different portfolios that vary only in portfolio composition, i.e. in the fraction of life insurance contracts $d \in [0,1]$. Thus, the number of life insurance contracts sold in t = 0 is $d \cdot n = n_L(0)$ and the number of annuities is $(1-d) \cdot n = n_A(0)$. Thus, each portfolio has the same total expected benefit payout of $n \cdot 1{,}000 = 100\ \mathit{Mio.}$, independent of portfolio composition.

Monte-Carlo methods are employed to assess the risk and solvency situation of the insurance company. To improve comparability of results, we use the same sequence of random numbers to simulate the number of deaths at each time t for each simulation run and the same 100,000 simulation runs for the evolution of the asset base for each portfolio.[20] The number of deaths

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[17] The Schwarz as well as the Akaike information criterion indicated a more complex model for the ARIMA time series. However, subsequent residual analysis using Box-Ljung test as well as ACF and PACF analysis showed no significant residual autocorrelation.

[18] The market rate of return r_M is relevant only for the size of the dividend in the present setting.

[19] Regarding the input parameters, sensitivity analyses were conducted to ensure that the results are stable.

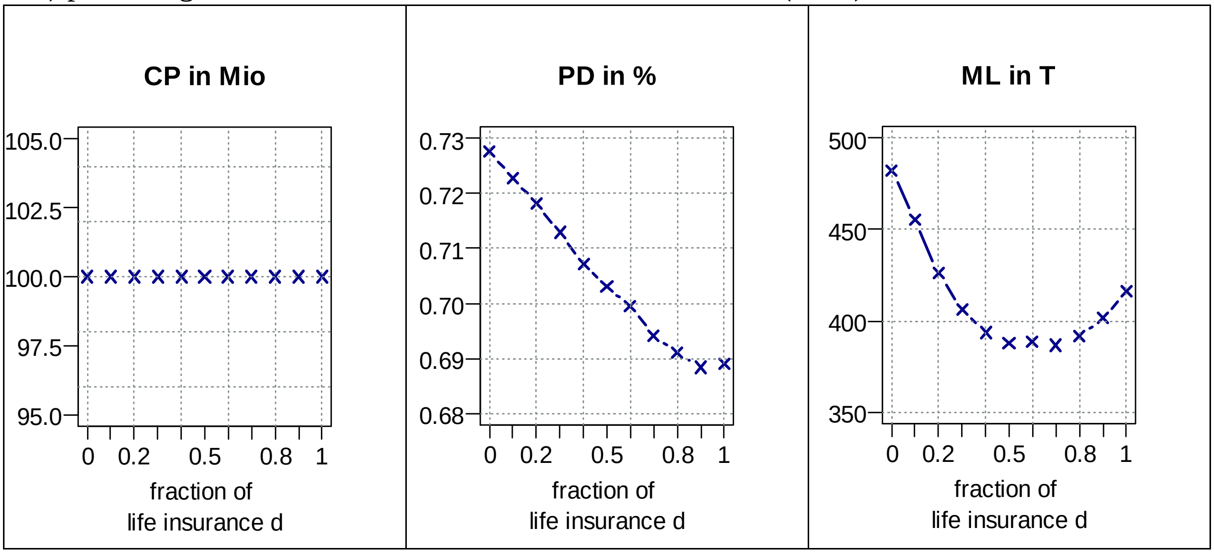
[20] The results are robust for different sets of random numbers.

are simulated using the inverse transform method for the Poisson distribution (see Glasserman (2008, pp. 54–58)).

Risk of an insurance company without systematic mortality risk

As a benchmark, we calculate the risk measures under the initial mortality rates used in benefit and premium calculation (forecasted using the BDV (2002a) model, s = 0), which are assumed to be equal to realized mortality in the insurance portfolio. Even though the expected benefit payouts are equal to 100 Mio. for all portfolios and independent of the portfolio composition, Figure 2 shows very different results in regard to the risk situation for different portfolios, resulting from the timing of cash flows.

Figure 2: Contractual payout obligations (CP), probability of default (PD), and mean loss (ML) plotted against the fraction of life insurance contracts d (s = 0)



For the chosen discrete values, the mean loss is minimal approximately for d = 0.7 and the probability of default for d = 0.9, while a portfolio consisting only of annuities (d = 0) leads to the highest risk for both risk measures in the considered setting, where term life insurances are sold against annual premiums. The risk reduction effect that can be attained via portfolio composition ranges between 5% and 20%, depending on the risk measure, and is thus not negligible.[21] These results imply that the riskiness of an insurance company is not only affected by the absolute value of payment obligations, which are identical for every portfolio as can be seen in Figure 2, but also by the characteristics of these payments, such as the timing,

[21] The overall level of risk can generally be reduced by means of reinsurance or other risk transfer instruments. Furthermore, these instruments might also lead to a shift in the risk minimizing portfolio and impact the optimal hedge ratio discussed in the subsequent section.

which in turn has a substantial impact on the development of the asset base (see Figure 1). The risk reduction effect arises as the payouts for annuities decrease over time, while the payouts for life insurance rise almost exponentially. Since these effects counterbalance each other, the payouts for the mixed portfolios are smoother, which contributes to the risk reduction effect observed earlier. The present results are used as a benchmark for the following analysis, where we concentrate on the relative change of the two risk measures and the contractual payment obligations in response to a change in mortality with respect to the risk measures shown above.

The effect of natural hedging on the liability side

In the following, numerical results for the riskiness of the insurance company in response to systematic mortality risk scenarios are shown. The longevity scenario with a scaling factor of s = 1 corresponds to a mean increase in the remaining life expectancy of a 65 year old man of about 1.9 years in the year 2012 from 18.5 years to 20.4 years,[22] while the corresponding mortality scenario implies a mean decrease in the remaining life expectancy of about 1.8 years to 16.7 years. The different risk measures for these scenarios are displayed in Figure 3, including the expected discounted benefit payouts (CP, Part a)), the probability of default (PD, Part b)), and the mean loss (ML, Part c)).

The comparison clearly reveals the difference between CP and the two risk measures, in that the consideration of the liability side shows a linear relation for varying d, while the risk measures exhibit a non-linear relation. However, all three graphs show at least some similar tendencies. In particular, the results reflect a greater sensitivity of term life insurance contracts to mortality risk, since the relative change for both risk measures and the CP is more severe if the insurance company sells only life insurance (d = 1) as compared to a portfolio of only annuities (d = 0). Taking the expected discounted benefit payouts (CP, see Part a) in Figure 3) as an example, for a portfolio of term life insurance, the longevity scenario leads to a decrease in the expected benefit payouts of about -34.3% , while the mortality scenario leads to an increase of about 63.0%. Thus, in the mortality scenario, the insurance company will have to increase its reserve by about 60% in t = 0 to be able to satisfy the increasing payouts for death benefits. For annuities, the reaction of the contractual payment obligations to changing mortality rates is considerably smaller. The different considered scenarios imply a change in expected benefit payouts of -7.2% and 7.3%. Similar results can be observed for the two risk measures in Parts b) to c). These results indicate that the value of the CP and the insurer's

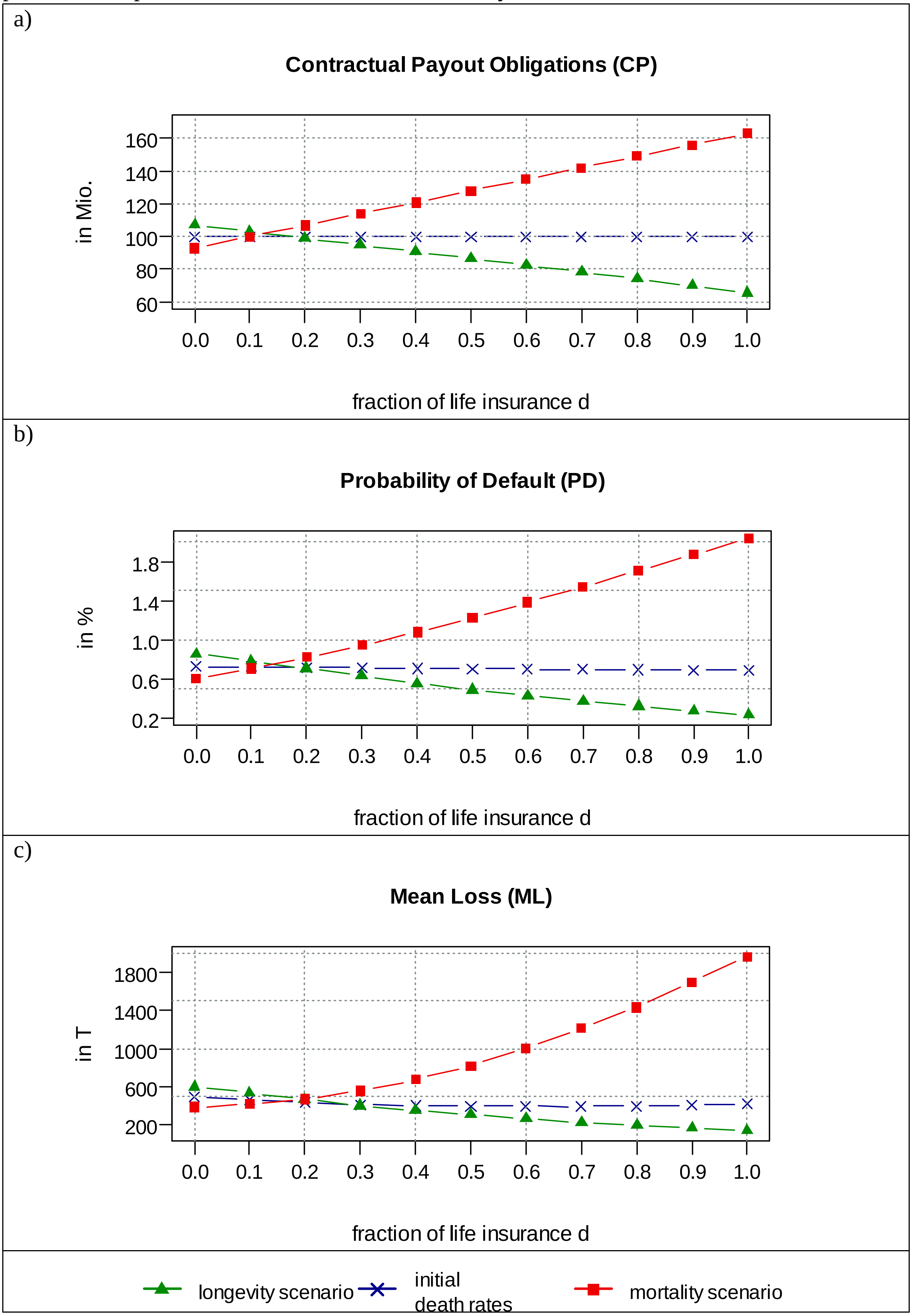
[22] The life expectancy $e_x(t)$ is given by $e_x\left(t\right)=\sum_{k\geq 0}\prod_{j=0}^{k} p_{x+j}\left(t+j\right)$ (see Brouhns, Denuit, and Vermunt (2002b)).

default risk are more sensitive to changes in mortality for a portfolio of life insurance contracts than that for annuities, since the former corresponds to a low probability risk, which is more prone to changing mortality rates. In contrast, annuities correspond to a high probability risk, since survival rates are higher than corresponding mortality rates.[23] Therefore, to hedge against the modeled scenarios for systematic mortality risk, the fraction of annuities generally should be greater than the fraction of life insurance to counterbalance their greater sensitivity. The optimal hedge ratios at which the risk situation of the life insurance company does not change in response to a change in mortality (intersections between risk measure curves in Figure 3 of initial death rates and the risk measure curves for the longevity and the mortality scenario, respectively), confirm this presumption and imply that, in the present setting under the stated assumptions and the annual term life level premiums, the insurance company should write approximately four to five annuities for every life insurance contract sold.

However, since the interaction between assets and liabilities and the dynamic evolution of payments are not taken into account in the calculation of the contractual payment obligations (CP), they allow only partial insight into the effect of natural hedging with regard to an insurer's risk situation. In particular, when comparing the impact of mortality risk on the default risk of an insurance company (using PD or ML) with the impact on the contractual payout obligations (CP), one notices that the impact of mortality risk on the company as a whole far exceeds the impact on the value of the contractual payout obligations. For example, for the longevity scenario, the probability of default increases from 0.73% to 0.85%, which corresponds to a change of 17.5% (compared to the situation with initial death rates forecasted using the BDV (2002a) model) for the portfolio of annuities (i.e. $d = 0$) as compared to a change of 7.3% (100 Mio. vs. 107 Mio.) for the contractual payment obligations (CP). For the mean loss, the relative change is even greater. This indicates that the consideration of the contractual payment obligations (CP) on the liability side alone may severely underestimate the true impact of mortality risk on an insurance company's risk situation.

[23] These results are approximately in line with those found in the previous literature. See, for example, Antolin (2007), Cox and Lin (2007), Gründl, Post, and Schulze (2006).

Figure 3: Contractual payout obligations, probability of default and mean loss plotted against portfolio composition for different realized mortality



Furthermore, the impact of systematic mortality risk upon the probability of default (PD) for a portfolio of life insurance is greater for an upward move in mortality rates than for a downward move (+195.0% for the mortality scenario as compared to -67.0% for the longevity scenario), while, for the portfolio of annuities, the effect is reversed (-17.0% for the mortality scenario as compared +17.5% for the longevity scenario). This effect is even stronger for the mean loss (+373.8 for the mortality scenario as compared to -68.1% for the longevity scenario and a portfolio of only life insurance, -21.4% for the mortality scenario as compared to +23.9% for the longevity scenario and a portfolio consisting only of annuities) and can again be ascribed to the different types of risk that are insured. The results imply that a high probability risk is more severely impacted by a decrease in the respective probabilities than by an increase, while the effect is reversed for low probability risks. This indicates that the risk of a change in mortality outweighs the chances that are connected with a move in mortality, since the increase in the riskiness in response to a bad shock outweighs the decrease in response to a good shock.

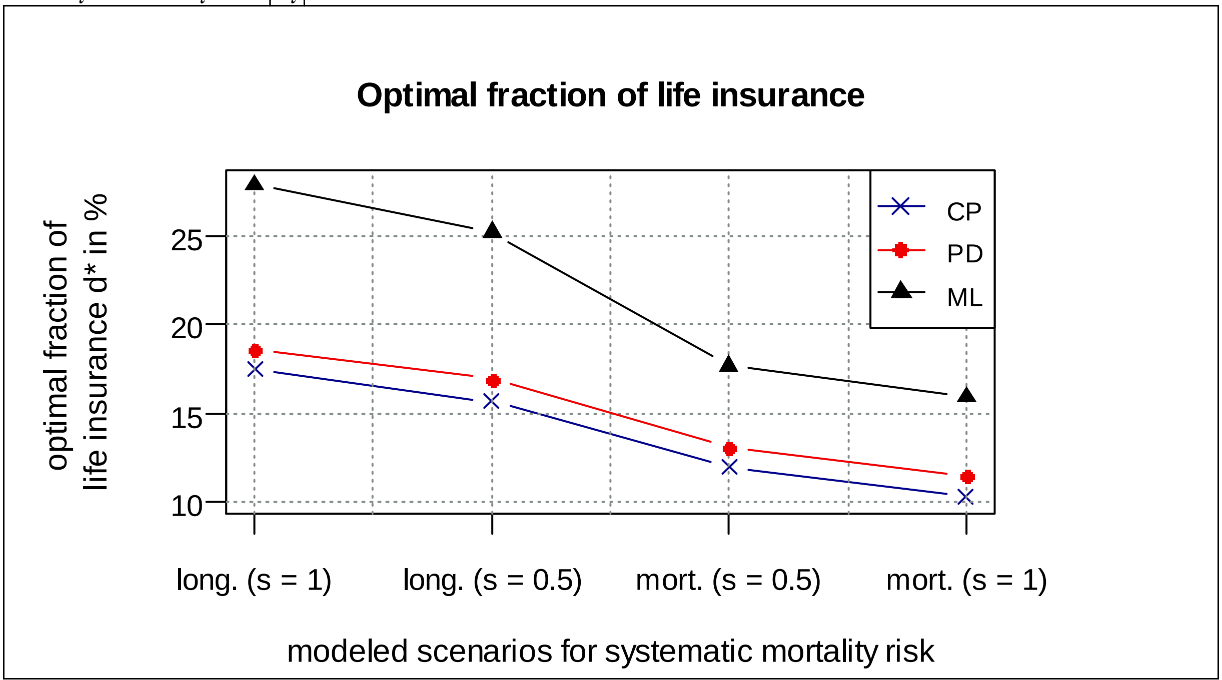
Determining an optimal portfolio-mix with natural hedging

Overall, the findings in Figure 3 show that systematic mortality risk with the implied change in mortality rates can lead to severely increased risk. The extent of this increase is subject to portfolio composition and can be reduced considerably by combining term life insurance and annuities. Thus, one faces a trade-off between a risk-minimizing portfolio composition for given mortality rates, which are sensitive to shifts and may thus imply a much higher default risk level in case of changes in mortality than the ones originally anticipated, and a portfolio, which is immunized against the modeled systematic mortality risk scenarios but leads to a higher absolute level of risk. In light of the significant uncertainty in projecting mortality rates, which makes managing mortality risk crucial for a life insurance company, as well as the scarceness of alternative instruments to hedge against mortality risk, we will first concentrate on the immunization effect of natural hedging and secondly show how to proceed to immunize a portfolio at a certain desired safety level for a given scenario of systematic mortality risk.

We thus next display the optimal ratio of life insurance contracts to hedge against the modeled scenarios for systematic mortality risk in Figure 4 (for a given asset portfolio) as implied by Equation (8)). The optimal hedge ratio d^* corresponds to the respective intersection of the risk measure curves for the modeled scenarios for systematic mortality risk with the risk measure curve for the initial death rates forecasted using the BDV (2002a) model in Figure 3. In addition, to gain further insight into the modeled scenarios and into the effect of the extent of the change in life expectancy, different scaling factors s are assumed for the longevity and the mortality scenario with s = 0.5 and s = 1, respectively. The scaling factor s influences the extent of the implied change in life expectancy. In case of the longevity scenario, for instance, a scaling factor of s = 1 leads to a mean increase in life expectancy of a 65 year old of 1.9 years, while a factor of s = 0.5 implies an increase of only 0.9 years.

Figure 4 shows that in general the optimal hedge ratio is higher for the longevity scenario as compared to the mortality scenario. In line with the previous results this implies that, in order to hedge the impact of the mortality scenario, i.e. a decrease in life expectancy, less life insurance has to be written than in case of a longevity scenario. The values for d^* for the two risk measures PD and ML lie within the range of 11% – 28% and are thus generally higher than the ones for the CP ranging between 10% and 17%. In general, when considering the insurer's risk situation, one seems to need a higher percentage of life insurance to hedge against a downward move in mortality, i.e. a longevity scenario, which is in line with the observation that annuities are more sensitive with respect to these changes in mortality rates. Therefore, more life insurance contracts are needed to balance out this effect.

Figure 4: Optimal d^* for hedging against changes in the discounted expected benefit payouts (CP), the probability of default (PD) and the mean loss (ML) in response to the modeled scenarios for systematic mortality risk for different scaling factors s, with $k_\tau^{longevity} = \hat{k}_\tau - s \cdot |\varepsilon_\tau|$ and $k_\tau^{mortality} = \hat{k}_\tau + s \cdot |\varepsilon_\tau|$



Overall, the difference in the optimal hedge ratios for the considered scenarios is almost twelve percentage points for the ML and seven percentage points for the PD, which is not negligible considering the volume and number of life insurance contracts and annuities. Thus, the choice of an adequate risk measure that considers both assets and liabilities is vital, and, to hedge against systematic mortality risk, the exact realization of the change in mortality is important. Nevertheless, the impact of an unknown change in mortality on the payout obligations and the risk situation of a life insurance company can still be greatly reduced in the present analysis through natural hedging by writing approximately 20% life insurance contracts when hedging the ML and 15% life insurance contracts for hedging the PD.

The impact of the investment strategy on optimal portfolio-mix

In a second step, we propose a procedure to reach a desired safety level, while simultaneously choosing an optimal portfolio-mix by varying the asset base. Hence, in Part a) of Figures 5 and 6, we investigate the impact of the investment strategy on the optimal hedge ratio and then calculate the corresponding level of risk using PD and ML for the optimal hedge ratio derived in the first step (Part b) of Figure 5 and 6) for a longevity scenario and a mortality scenario. For completeness, the optimal hedge ratio d^* to hedge against changes in the value of the contractual payment obligations CP is displayed as well, which is independent of the investment strategy and thus constant. Seven different investment strategies from low risk/low return (μ = 4%, σ = 2%, implying a Sharpe ratio of β = 0.50) to high risk/high return (μ = 10%, σ = 26%, implying a Sharpe ratio β = 0.27) are considered to conduct a sensitivity analysis.[24]

The results show how the investment strategy of the insurance company substantially influences the optimal hedge ratio d^* (see Part a) of Figure 5 and 6). In turn and as demonstrated in Figure 1, the portfolio composition has an impact on the development of the assets due to an altered timing and amount of cash-in- and outflows. These complex interactions make an interpretation of results difficult. An increase in the expected return and standard deviation of the investment portfolio leads to an increase in the optimal hedge ratio, i.e. more life insurances are needed to hedge against the modeled systematic mortality risk. This effect is less pronounced when hedging the probability of default than for the mean loss. However, even for the probability of default in case of the longevity scenario in Figure 6, a change from the

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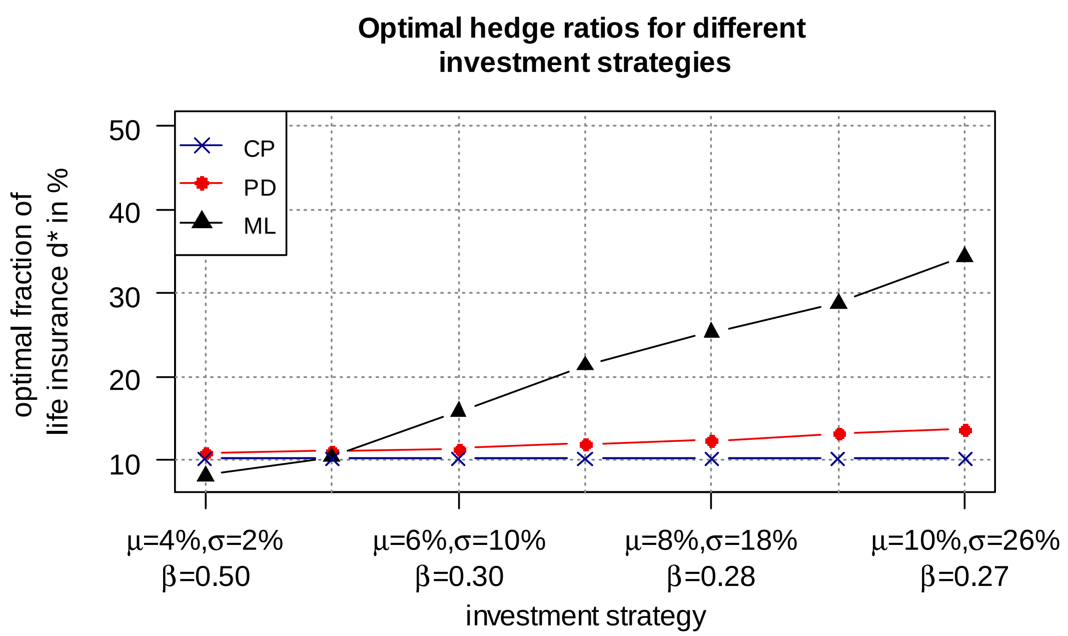
[24] The investment strategies assumed here are passive with a constant expected return and standard deviation over the contract term. The Sharpe ratio $\beta = (\mu - r) / \sigma$ provides an indication for the riskiness of the investment strategy, as it relates excess return to risk and since funds with the same Sharpe ratio imply the same evolution of the underlying assets (see Gerrard, Haberman, and Vigna, 2004). Results may differ when taking into account dynamic solvency driven investment strategies. However, the results displayed here indicate that, with a dynamic investment strategy, each regrouping of asset investments should be accompanied by an analysis of portfolio composition to ensure that systematic mortality risk is still hedged successfully.

formerly assumed investment strategy with μ = 6% and σ = 10% to the more conservative strategy with μ = 4% and σ = 2% leads to a decrease in the optimal hedge ratio of approximately 1 percentage point, while d^* decreases by about 12 percentage points when hedging the mean loss. For the mortality scenario in Figure 5, the effect is smaller for both, the probability of default and the mean loss, for which the optimal hedge ratio decreases by about 7.7 percentage points. For a riskier investment strategy of μ = 10% and σ = 26% the described effect is even more pronounced. The optimal hedge ratio increases by more than 5 percentage points in the longevity scenario (Figure 6) for the probability of default and by more than 20 percentage points for the mean loss compared to the formerly assumed investment strategy with μ = 6% and σ = 10%. The results imply that annuities are more severely impacted by the combination of riskier investment and mortality risk than term life insurances and that the risk measure plays an important role. Hence, the fraction of term life insurances to hedge a portfolio against a given change in mortality rates should increase for riskier investment strategies. Since the value of liabilities is not affected by the investment strategy, this effect is not captured when considering the CP. Thus, especially for insurance companies with a riskier investment strategy, the consideration of the liability side alone can lead to a misestimation of the optimal hedge ratio. For example, for the longevity scenario and an investment strategy with μ = 10% and σ = 26% (Figure 6), the difference in the optimal hedge ratio between the CP and the considered risk measures is about 6 percentage points for the PD and more than 30 percentage points for the ML.

In Part a) of Figures 5 and 6, d^* is defined as that fraction of life insurance at which a given risk level is not sensitive to the modeled changes in mortality rates for different investment strategies. As an example of an adjustment of parameters to achieve a certain desired risk level, which can then be immunized against shifts in mortality, we next calculate the corresponding default risk of the insurance company for the optimal hedge ratio d^* for investment strategies and risk measures given in Part a) of Figure 5 and 6 and display the resulting (corresponding) risk levels in Part b) of Figures 5 and 6. For both, the PD and the ML, for the low risk/low return combinations, the effect of the small expected return outweighs and increases the risk of the insurance company, while, for the high risk/high return combinations, the effect of the increased volatility implies an increase in risk.

Figure 5: Optimal hedge ratios *d** and the corresponding risk level for different investment strategies in the mortality scenario (*s* = 1) for probability of default, mean loss, and contractual payout obligations

a)



b)

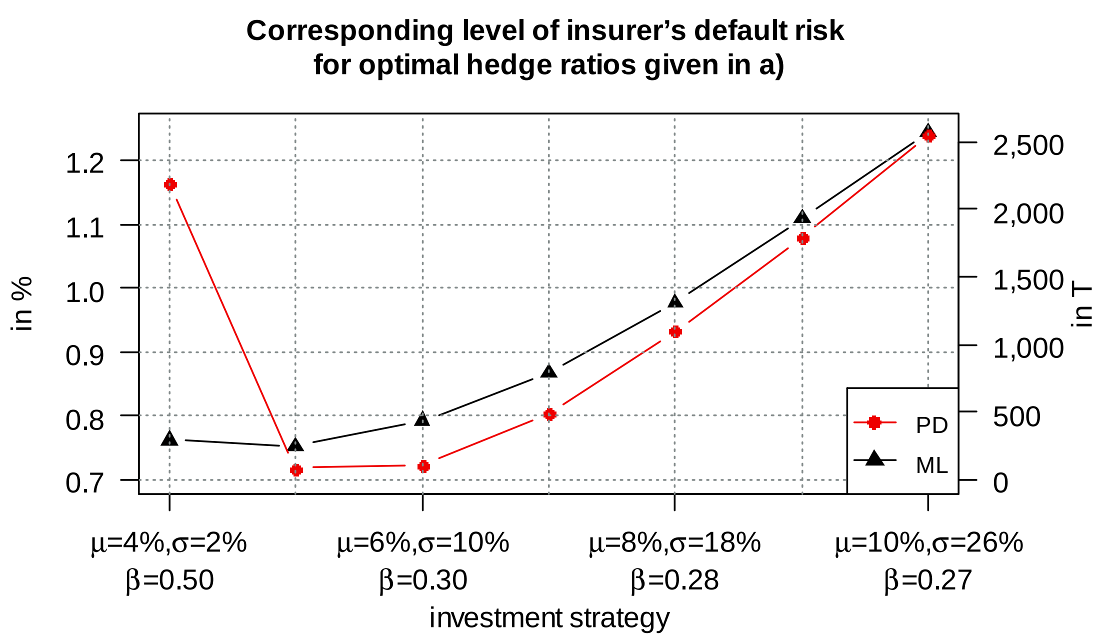
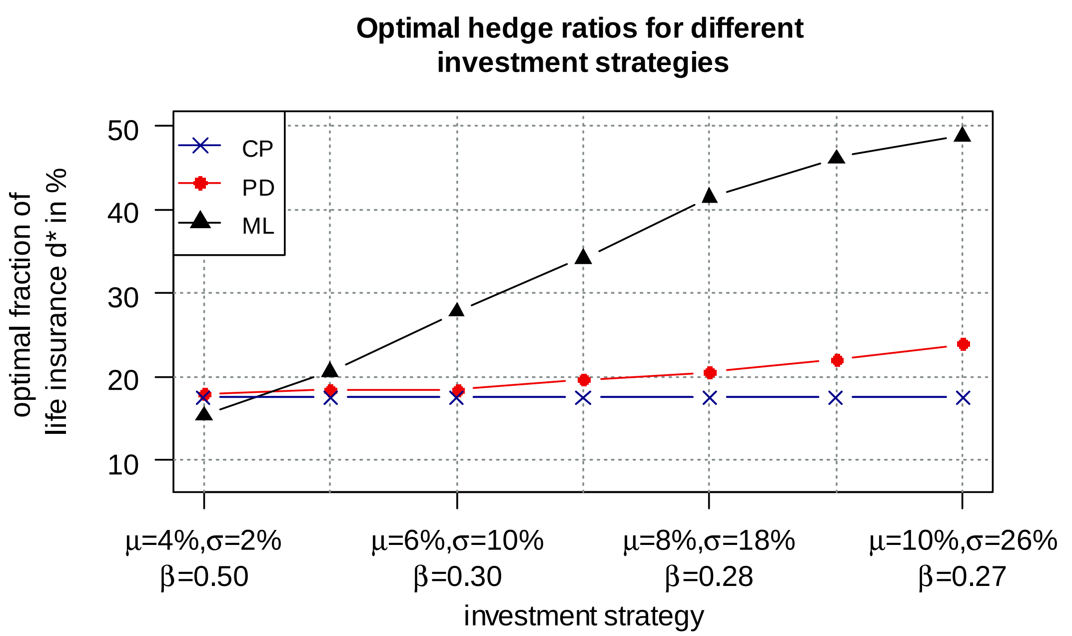
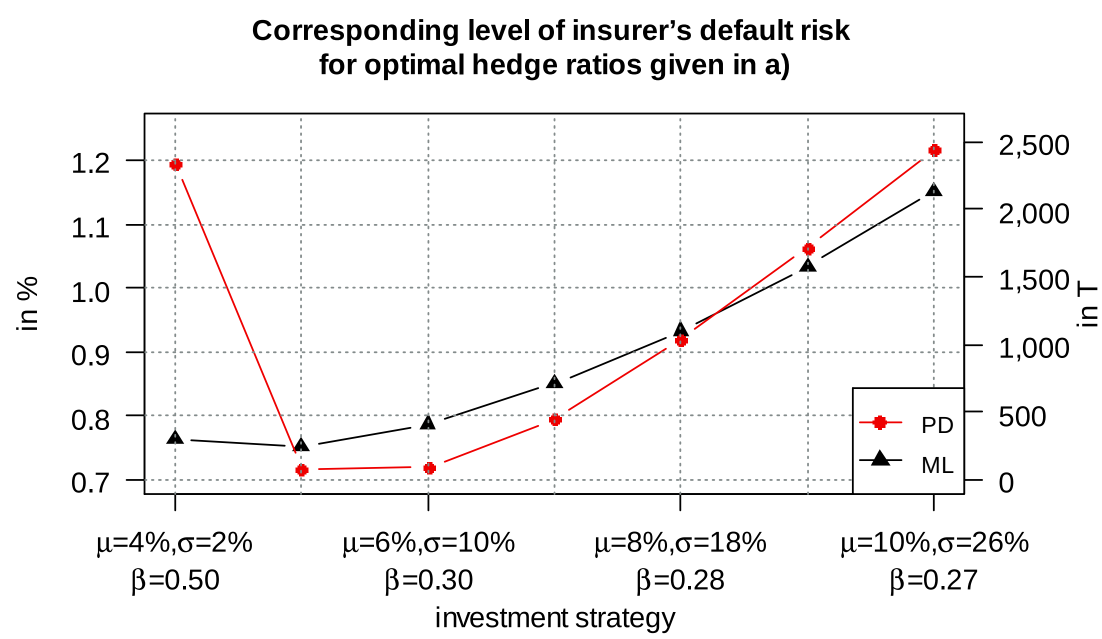


Figure 6: Optimal hedge ratios *d** and the corresponding risk level for different investment strategies in the longevity scenario (s = 1) for probability of default, mean loss, and contractual payout obligations

a)



b)



Thus, based upon Part b) of Figures 5 and 6, one can decide on the desired risk level[25] for a chosen risk measure and then turn to Part a) of Figures 5 and 6 to read out the corresponding d^*, thus immunizing this risk level against changes in mortality. These results underline that the asset side and thus the intended investment strategy should be taken into account when determining the optimal hedge ratio and when analyzing the effect of natural hedging on an insurer's risk situation. It can even be used to choose a desired safety level while simultaneously ensuring an optimal portfolio-mix. Our findings indicate that a rather conservative investment strategy leads to a lower percentage of term life insurances, while an aggressive strategy implies that fewer annuities should be sold. As an alternative to managing the asset base, life insurers can also arrange contract characteristics, reinsurance, or leverage[26] to achieve a certain safety level, while simultaneously hedging against changes in mortality rates by portfolio composition.

In general, one can deduce that, in our model set-up, systematic mortality risk can be hedged completely, but that the optimal hedge ratio depends on the direction and the extent of the change in life expectancy and on the insurer's investment strategy. The level of risk, however, is driven mainly by the investment strategy and is not very sensitive to changes in the systematic mortality risk scenario when the portfolio composition is calibrated to immunize the given risk level (see Part b) of Figure 5 and 6). Thus, while the considered scenario for systematic mortality risk has an impact upon the optimal portfolio composition, it does not substantially influence the level of risk of the immunized portfolio, if the latter is calibrated accordingly. To check the robustness of this approach with respect to the assumption of systematic mortality risk scenarios, in the following, we examine the effect in response to actually experienced changes in mortality rates.

Natural hedging under realized changes of mortality

An often cited argument against the effectiveness of natural hedging is that improvements in life expectancy do not stem from a uniform decrease in mortality rates over all ages, e.g. in the last decade improvements in life expectancy have essentially been observed at older ages (see e.g. Blake and Burrows (2001, p. 346)). Since in general life insurance contracts mature when the insured enters retirement, the improvement in mortality rates beyond the usual retirement age will not influence the value of life insurance liabilities. Therefore, a life insurance company would not be able to hedge its exposure to systematic mortality risk through

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[25] The investment strategy is merely one example of how to adjust the risk level. The risk level can also be influenced as already mentioned by contract parameter, premium loadings, or equity capital.

[26] See e.g. Gründl, Post, and Schulze (2006).

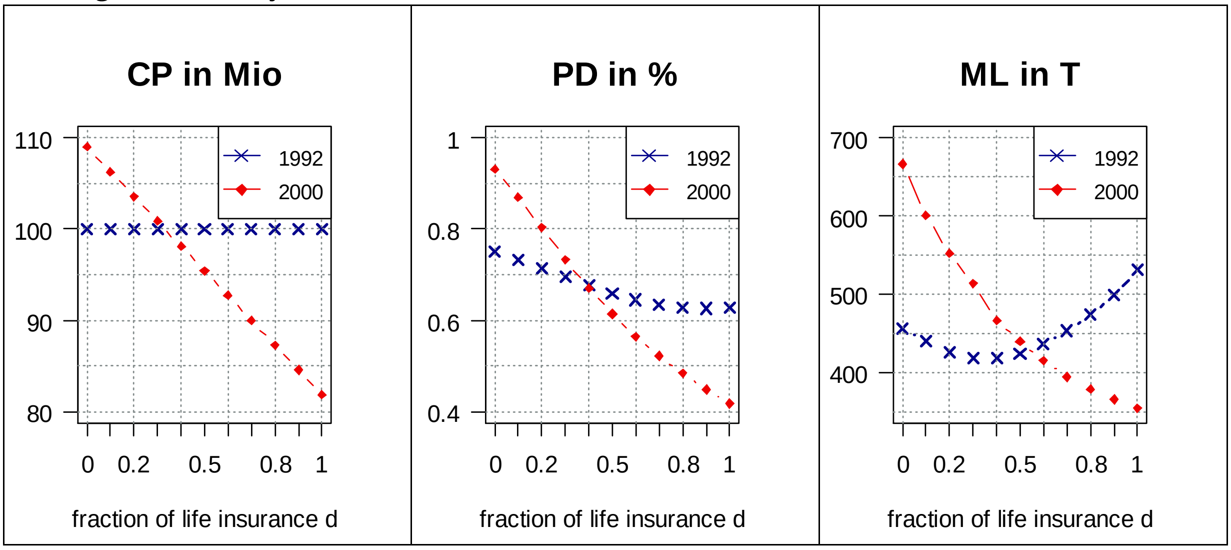
natural hedging. To assess the validity of this argument in more detail in the present setting, we apply our model to recently observed changes in mortality and analyze the effectiveness of active product mix under these mortality assumptions.

To reflect an insurance company's actual experience of changing mortality, an approach taking into account empirically observed changes in mortality and mortality adverse selection—which does not only refer to the fact that the level of mortality between term life insurance policyholders and annuitants might differ but also refers to the risk that the development of mortality rates might not be identical—is appropriate. We thus apply our model to the changes observed in the UK and the Republic of Ireland between 1992 and 2000 by using the mortality tables of the Continuous Mortality Investigation[27] (CMI), thereby distinguishing between tables for annuitants and life insurance policyholders and thus accounting for adverse selection. Here, the exchange of the old for the new life tables accounts for an increase in life expectancy in the last decade, i.e., a longevity scenario. Hence, realized mortality will be lower than assumed mortality, thus increasing annuity payouts and decreasing term life insurance payouts.

The fair premium and fair benefits are calculated using the death rates from the 1992 table. The resulting annuity is 81, while the annual premium for the term life insurance contracts is 46 and the death benefit 19,739. These input parameters are used for the same simulation analysis conducted before. The numerical results for the two risk measures and the CP as well as the optimal hedge ratio d^* for the experienced changes in mortality between 1992 and 2000 as implied by the CMI tables are shown in Figure 7. The blue line with stars shows the risk of an insurance company when mortality is equal to initially assumed mortality, while the red dash-dotted line reflects the new risk situation when realized mortality is equal to the rates implied by the 2000 mortality table and thus lower than expected. In line with the analysis in the previous section, the random numbers of deaths at age x, D_x, is simulated from a Poisson-distribution with $D_x \sim Poisson\left(E_x \cdot \mu_x\right)$, where the exposure to risk E_x is determined by the size of the insurance portfolio and μ_x is calculated by $\mu_x = -\ln\left(1-q_x\right)$ with q_x given by the CMI tables.

[27] The CMI is an institution which carries out mortality research for the population of the UK and the Republic of Ireland on the basis of data provided by life offices. It regularly publishes mortality tables for the population of, e.g., life insurance policyholders and pensioners, taking basis risk into account. CMI's life tables have also been adopted by the Actuarial Profession in the UK.

Figure 7: Contractual payout obligations, probability of default and mean loss subject to portfolio composition and optimal hedge ratio *d** for a change of mortality corresponding to the exchange of mortality table CMI from 1992 to 2000



1992 - 2000	CP	PD	ML
*d**	33.1%	38.9%	53.1%

Comparing these results with the ones presented earlier, the higher sensitivity of a portfolio of life insurance (d = 1) to changes in life expectancy compared to a portfolio of annuities (d = 0) is considerably reduced or even eliminated, depending on the risk measure. The relative change in the probability of default is 24.1% for a portfolio of annuities and -33.6% for a portfolio of term life insurances, while the change in a portfolio of life insurance (d = 1) was multiple times that of a change in a portfolio of only annuities (d = 0) before. For the mean loss, the effect is even reversed such that a portfolio of annuities is more sensitive to changes in mortality (+46.0% for a portfolio with only annuities as compared to -33.3% for a portfolio with only life insurance). This can be ascribed to the realized improvement in mortality rates implied by the change in mortality tables, because improvements in mortality rates occurred mainly at older ages in the last decade. Therefore, the mortality rates relevant to life insurance changed less than those applicable to annuities. Despite these different changes in the respective relevant range of mortality rates, the impact of the change on the expected benefit payouts and on the risk of the life insurance company can still be hedged. The optimal hedge ratio *d** increases substantially and now lies between 33% and 53% depending on the risk measure chosen (compared to 10% – 28% in the previous setting). The optimal hedge ratio calculated in this setting is considerably larger than the ones observed for the previously modeled scenarios. However, since one cannot generalize this result based on one example, further analyses seem necessary to examine in depth the impact of the exact changes of mortality and adverse selection on the optimal hedge ratio (see Gatzert and Wesker, 2011).

In particular, differences compared to the previous results may stem from adverse selection effects and the different implementation of systematic mortality risk. The modeled scenarios for systematic mortality risk in the previous settings imply a gradual decrease (increase) of mortality below (above) the expected level, since the error term ε_τ impacts mortality cumulatively over time. Thus, this implies a larger deviation between expected mortality and realized mortality towards the end of the contract term, which impacts life insurance more severely than annuities due to the different timing of payouts, and thus leads to a lower fraction of life insurances needed for natural hedging. In the empirical analysis, using the data from CMI, in contrast, we assume an immediate decrease in mortality to the level implied by the 2000 CMI table, which is not entirely realistic since these changes occurred gradually during the covered time span. However, due to lack of data for the years in between, this effect cannot be taken into account. Thus, this assumption contributes to the higher hedge ratios found in this analysis since the effect of the greater deviation of mortality towards the end of the contract term is not reflected. The differences in results might further be reducible when implementing a different mortality model that specifically fits U.K. mortality data, as, e.g., the Cairns, Blake, and Dowd (2006b) two-factor model. Yet, our analysis of the effectiveness of natural hedging under empirical changes in mortality rates still and already demonstrates that changes in mortality can be diversified by an active product mix management, even if an improvement does not occur evenly across ages and the life insurance and annuities tables are impacted differently. The optimal hedge ratio, however, does vary depending on the exact realized changes, indicating that the quality of an estimate about the expected improvement in mortality rates will be vital for a perfect hedge of mortality risk. If this estimate is not available or is unreliable, the effect of mortality risk can still be reduced substantially by means of natural hedging by signing approximately 20% life insurance contracts and, by conducting scenario analyses as presented in the present analysis, the extent of deviations in risk measures can be estimated to obtain a more holistic picture of the life insurer's risk situation.

Further analyses and robustness checks

To check the robustness of our results with respect to the restrictive assumption of business being written only once, we repeat our analysis for the situation in which the insurance company writes business repeatedly every five years. We thereby first investigate the impact of the modeled scenarios if premiums and benefits *are not* adapted to new mortality rates, and, second, the reaction in the risk measures when benefits and premiums *are* adapted to new mortality rates for new business immediately, i.e., for business written from t = 5 on. The results of these analyses are shown in Figure A.3 in the Appendix.

Our main finding is that the natural hedging effects between term life insurance and annuities do not substantially differ compared to the setting where business is written only once. In particular, the reaction in all risk measures is more severe when new business is written and premiums and benefits are not adapted (for example, for the mortality scenario, the probability of default increases more than twofold for a portfolio of only life insurance). Considering the intersection points of all graphs, these are shifted towards more annuities and are roughly at d = 40% for the probability of default and d = 50% for the mean loss (see Figure A.3 a)). The same is true for the case in which the contract parameters are adapted to realized mortality. While the impact of mortality risk can be reduced considerably by immediately adjusting premiums and benefits to realized mortality, the impact of mortality risk is still not negligible. For instance for a portfolio with only life insurance, the mean loss still changes by -67.9% and +281.3% in response to the modeled scenarios, which for the longevity scenario is approximately the same as in the single portfolio case (-67.0%), while for the mortality scenario, the impact is still greater than in the single portfolio case (+195.0%). The intersection and thus the hedge ratio at which mortality risk can be hedged decreases as compared to the case when premiums and benefits are not adapted and thus moves closer to the optimal hedge ratio observed for the single portfolio case (see Figure A.3 b)).

Thus, our investigation confirms that the effectiveness of natural hedging as discussed in the previous subsections is in general not restricted to the single portfolio case and that in the present model setup, the consideration of a single portfolio seems sufficient to obtain central insight on the effectiveness of natural hedging.

4. SUMMARY

In this paper, we investigate the effectiveness of natural hedging for immunizing a portfolio of actuarially modeled term life insurance and annuities against changes in mortality rates in a multi-period setting over 35 years. We extend previous analyses by including assets and liabilities in a dynamic framework and explicitly focus on assessing the risk situation of a life insurer with respect to mortality risk, thereby using the probability of default and mean loss. Systematic mortality risk is first assessed by taking into account the uncertainty in the time trend, where scenario analyses were conducted using a longevity scenario, i.e. an increase in life expectancy, and a mortality scenario, resulting in a lower than expected life expectancy, and, second, by comparing actually experienced changes in mortality rates. To isolate the effect of natural hedging on risk, we fix the total contract volume with regard to the present value of premiums and benefits for different portfolio compositions of term life insurance and annuities. In this setting, we determine the optimal product mix that eliminates the risk of changes in the respective risk measure for changes in mortality rates. In addition, we simultaneously consider the absolute level of risk associated with an optimal, immunizing portfolio by means of the insurer's asset portfolio.

Our results demonstrate that the impact of mortality risk on the company as a whole is considerably higher than its impact on contractual payout obligations on the liability side, which has been focused in previous studies. In the considered example, the modeled longevity scenario implies a change of 17.5% in the probability of default for a portfolio of annuities as compared to a change of 7.3% for the contractual payment obligations. This indicates that the one-sided consideration of the latter may seriously misjudge the true impact of mortality risk on the risk situation of a life insurance company.

Furthermore, the overall risk of an insurance company (absolute level and with regard to changes in mortality rates) can be considerably reduced through portfolio composition due to the adverse time structure of payouts for term life insurance and annuities, but the risk-minimizing portfolio and the portfolio hedging against shifts in mortality are not identical, implying that the insurer consequently faces a trade-off between risk minimization and mortality hedging. In this work, we focus on the effectiveness of natural hedging for immunizing a portfolio against mortality risk due to the scarceness of alternative instruments to manage and the potential severe impact of this risk. We thereby extend previous studies by showing how to choose a desired company safety level by adjusting the investment strategy while simultaneously having an immunizing portfolio.

One main finding is that the insurer's initial investment strategy can have a substantial impact on the effectiveness of natural hedging and should thus be taken into account when determining the optimal natural hedge ratio. In particular, a conservative asset management generally requires a lower portion of term life insurances while a more aggressive investment strategy implies that more term life insurances should be sold. Even though the absolute level of risk associated with an immunizing portfolio composition can vary substantially for different investment strategies and different risk measures, it is relatively stable when comparing different scenarios for systematic mortality risk for which the portfolio is immunized. Thus, while the considered scenario has an impact upon the optimal portfolio composition, it does not substantially influence the level of risk of the immunized portfolio.

To check the robustness of the results based upon two scenarios for systematic mortality risk, we further studied the effect of natural hedging when using actually experienced changes in mortality rates. The general observations do not much differ from the previous ones, implying that, in our setting, adverse selection is in general not an impediment to the effectiveness of natural hedging. However, the optimal hedge ratio still varies considerably depending upon the exact realized changes in age groups, indicating that the quality of an estimate about the expected improvement in mortality rates will be vital for the hedging success with regard to mortality risk. In this context, future research could further study the impact of adding reinsurance and other risk transfer instruments in the context of natural hedging, since this may reduce the overall risk level and shift the optimal hedge ratio. This might be particularly helpful if a change in the composition of an insurance portfolio is not easily achievable in sales. In any case, the consideration of natural hedging in immunizing a portfolio against changes in mortality appears vital for insurers, and risk management programs should take these considerations into account to avoid adverse effects from systematic mortality risk.

Overall, our analysis emphasizes the importance of natural hedging for managing systematic mortality risk, as changes in mortality rates can be diversified by an active product mix management, even if it does not occur evenly across all ages. Furthermore, the consideration of assets and liabilities and the comparison of different risk measures by means of scenario analyses allowed new insight in the effectiveness of natural hedging and the immunization of a fixed risk level, which is of relevance not only in the context of diversification benefits in Solvency II that reduce solvency capital requirements, but also when calculating the Market Consistent Embedded Value of life insurers.

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APPENDIX

Figure A.1: Estimated value of exp(a_x) and b_x over all ages

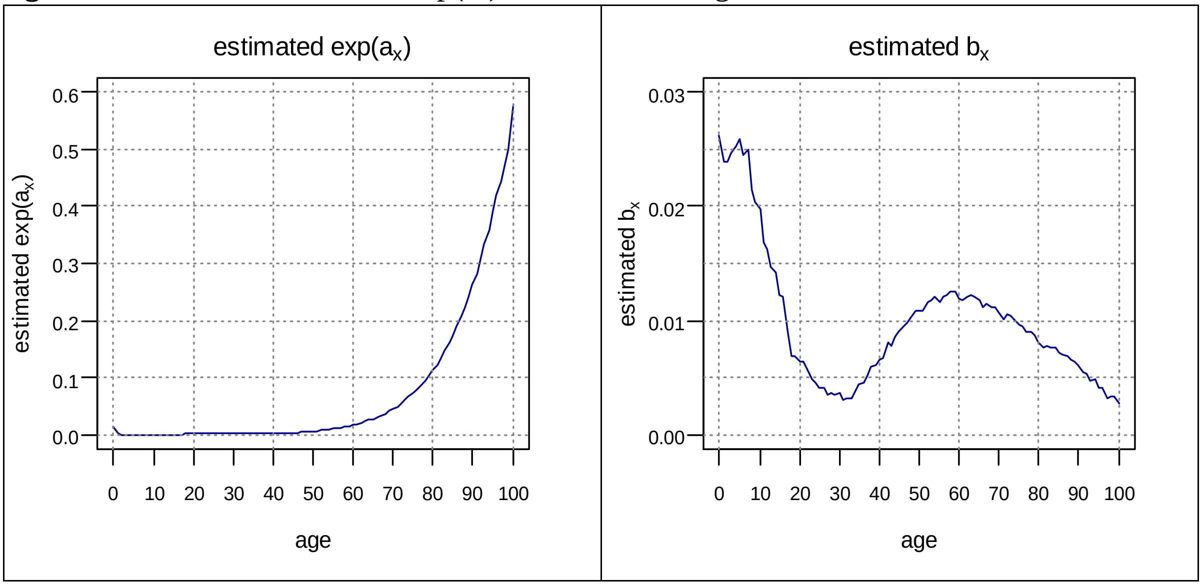


Figure A.2: Level of estimated mortality index k_τ and forecasted values of k_τ

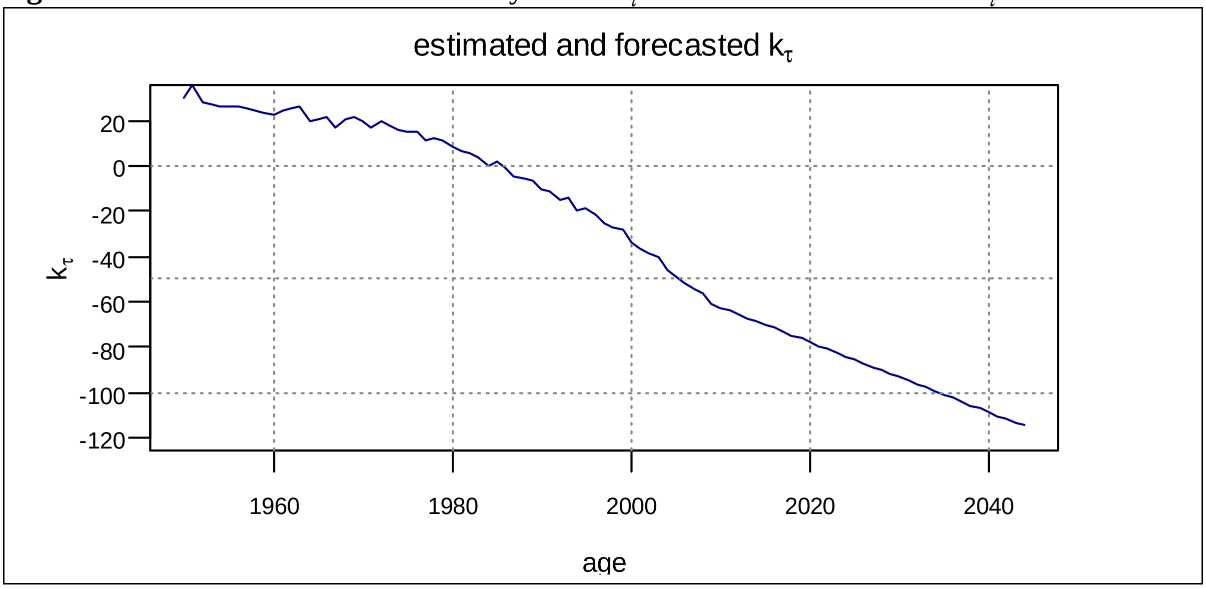
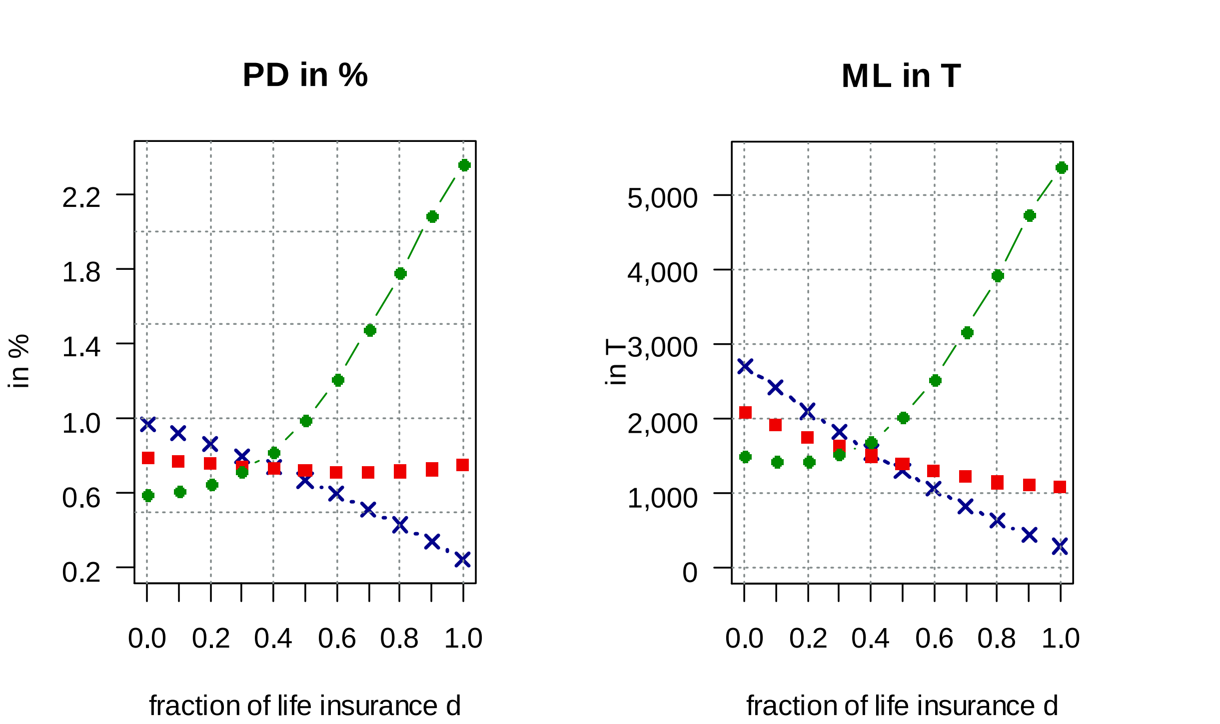


Figure A.3: Probability of default and mean loss subject to portfolio composition for a longevity and a mortality scenario with and without adapting premiums and benefits to new mortality rates when new business is written every five years

a) Premiums and benefits *are not* adapted to new mortality rates



b) Premiums and benefits *are* adapted to new mortality rates