Risk Measurement and Management of Operational Risk in Insurance Companies from an Enterprise Perspective

Nadine Gatzert, Andreas Kolb

Working Paper

Chair for Insurance Economics
Friedrich-Alexander-University of Erlangen-Nuremberg

Version: December 2012
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ABSTRACT

Operational risk can substantially impact an insurer’s risk situation and is now increasingly in the focus of insurance companies, especially due to new European risk-based regulatory framework Solvency II. The aim of this paper is to model and examine the effects of operational risk on fair premiums and solvency capital requirements under Solvency II. In particular, three different approaches of deriving solvency capital requirements are analyzed: the Solvency II standard model, a partial internal model, and a full internal model. This analysis is not only of relevance for Solvency II, but also regarding an insurer’s Own Risk and Solvency Assessment (ORSA) that is not only planned in Solvency II, but also by the NAIC in the United States. The analysis emphasizes that diversification plays a central role and that operational risk measurement and management is highly relevant for insurers and should be integrated in an enterprise risk management framework.

Keywords: Operational risk, Solvency II, ORSA, CAPM
JEL classification: C51, G22, G31, G32

1. INTRODUCTION

In the context of new risk-based capital requirements for banks and insurers imposed by Basel II/III and Solvency II, respectively, the discussion about operational risk intensified and especially large insurers are now confronted with the need to develop and implement adequate risk measurement and management instruments to deal with operational risk. In Solvency II, operational risk is defined analogously as in Basel II/III as “the risk of loss arising from inadequate or failed internal processes, personnel or systems, or from external events. Operational risk […] shall include legal risks, and exclude risks arising from strategic decisions, as well as reputation risks” (see European Parliament and the Council, 2009, Article 13, No. 33, Article 101, No. 4). Operational risk is also of high relevance for the

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* Nadine Gatzert and Andreas Kolb are at the Friedrich-Alexander-University (FAU) of Erlangen-Nuremberg, Chair for Insurance Economics, Lange Gasse 20, 90403 Nuremberg, Germany, Tel.: +49 911 5302 884, nadine.gatzert@fau.de, andreas.kolb@fau.de.

1 See also Basel Committee (2004, p. 137). In Basel II/III, operational risk is categorized into the seven event types “internal fraud”, “external fraud”, “employment practices and workplace safety”, “clients, products, & business practice”, “damage to physical assets”, “business disruption & systems failures” and “execution,
National Association of Insurance Commissioners (NAIC), where the potential inclusion of a specific charge for operational risk within the U.S. system of risk-based capital for insurers is discussed (see Vaughan, 2009; PwC, 2012). Besides the new regulatory requirements, cases of high operational losses in the recent past also strongly emphasize the importance and considerable risk associated with operational loss events. One of the most mentioned events in this context is the bankruptcy of Barings Bank in 1995, which was followed by a $1.3 billion loss caused by its rogue head derivatives trader in Singapore.\(^2\) The potential impact of operational losses on an insurer’s risk situation is also stressed by figures regarding potential insurance fraud by policyholders, which in the German insurance market, for instance, is estimated to about €4 billion per year (see Hiebl, Roedenbeck, and Kiefer, 2012). In the third party liability insurance only, 25% of all claims are suspected to be fraudulent and for an average motor liability insurance company, losses due to fraud are estimated to €32.5 million per year (see Hiebl, Roedenbeck, and Kiefer, 2012).\(^3\) The magnitude of these operational loss events in the past strongly demonstrates the need for an adequate measurement and management of operational risks, which is also required according to the new framework Solvency II. The aim of this paper is to model and quantify the effects of operational risk from an enterprise perspective by focusing on an insurer’s pricing and solvency capital requirements under Solvency II. We thereby compare the Solvency II standard formula with a partial and a full internal model.

A large part of the academic literature concerns the modeling of operational risk. Cruz (2002), McNeil, Frey, and Embrechts (2005), Gourier, Farkas, and Abbate (2009), and Shevchenko (2010), for instance, point out the importance of extreme value theory for calculating aggregate losses by using the loss distribution approach. Another part of the literature empirically analyzes operational loss data. While most of these studies examine empirical data from the banking sector (see, e.g., Moscadelli, 2004; de Fontnouvelle et al., 2003; Dutta delivery, & process management”. This categorization of operational risk is also suggested for insurers by the German Insurance Association (see GDV, 2007, p. 10). Note that in Basel II, operational risk was introduced as a third risk class in addition to market and credit risk (see Cummins, Wei, and Xie, 2011; Kamiya, Schmit, and Rosenberg, 2011), while in insurance, a more sophisticated risk classification system would, e.g., separately define financial risks (e.g., market, credit, etc.), policyholder insurance risk (e.g., property insurance, workers compensation insurance, health insurance, etc.), business risk (e.g., management, strategy, etc.) and operational risk (consistent with Basel II and Solvency II).

\(^2\) Other examples of operational risk events include the Nasdaq odd-eighths pricing scandal in 1994 as well as the losses of Société Générale in 2008 and UBS in 2011, both due to rogue traders. Similar examples in the insurance sector include the Swiss Life investment scandal in 2002, the AIG Finite Reinsurance Accounting fraud in 2005, as well as the AIG credit default swap write-down in 2008.

\(^3\) Another major issue is fraud in the context of commissions paid to agents. For example, the bankruptcy of the German MEG AG in 2009 caused irrecoverable losses for several insurance companies due to fraud in commissions (see Altenähr, 2010).
and Perry, 2006), Hess (2011b) also investigates operational loss data for insurance companies. Several studies dealing with operational risk also assess the dependencies between the risk cells of banks, including, e.g., Böcker and Klüppelberg (2008), Ebnöther et al. (2003), Frachot, Roncalli, and Salomon (2004), and Mittnik, Paterlini, and Yener (2011). Furthermore, Hess (2011a) examines the impact of the financial crisis on operational risk, while Cummins, Lewis, and Wei (2006) focus on the market value effects of operational loss events for U.S. banks and insurers, and spillover effects of operational risk events on banks and insurers are analyzed in Cummins, Wei, and Xie (2011). Different forms of insurance contracts for operational risk are analyzed in Peters, Byrnes, and Shevchenko (2011) for the case of banks.

In this paper, we contribute to the literature by presenting a model for how to integrate operational risk from an enterprise perspective and, based on this, focus on the impact of operational risk on an insurer’s pricing and capital requirements under Solvency II. We thereby compare the Solvency II standard model with a full internal model using the risk sensitive loss distribution approach for operational risk and a partial internal model that only focuses on the operational value at risk, i.e. without taking into account diversification effects. In the analysis, we also study the impact of dependencies between operational risk and the insurer’s loss distribution, amongst others, using the concept of copulas. The model is calibrated based on empirical data from previous literature and the numerical analysis allows the identification of key characteristics that increase or decrease capital requirements above or below the static risk-based factor used for the Solvency II standard model. For insurers, these considerations are also of special relevance in the context of their Own Risk and Solvency Assessment (ORSA) as required by Solvency II’s Pillar 2 or the NAIC in the United States (see NAIC, 2011; Blanchard, 2012; Wicklund and Christopher, 2012).

One main finding is that diversification plays an important role in the quantification of operational risk and that insurers should closely monitor and manage operational risk. In particular, our results reveal that the capital requirements of the Solvency II standard model may severely underestimate operational risk. In contrast, a partial internal model that only focuses on the operational value at risk, i.e. without taking into account diversification effects, tends to overestimate the capital requirements for operational risk. In any case, operational risk measurement and management is highly relevant for insurers and should be integrated in an enterprise risk management in order to adequately control and steer an insurance company.

The remainder of the paper is structured as follows. Section 2 includes the model framework of the insurer including operational risk, premium calculation, and risk measurement. Section
3 presents the results of the numerical analyses. A discussion of further issues regarding measuring and managing operational risks is given in Section 4, while Section 5 concludes.

2. Model Framework

This section describes the model framework used to quantify the effects of operational risk on the insurer’s risk situation and solvency capital requirements. First, we specify how operational risk is modeled and illustrate the model of the insurance company. Next, fair contracts and the determination of premiums are presented, followed by a comparison of different ways of how to derive solvency capital requirements.

Modeling operational risk

To model the operational risk of an insurer, the loss distribution approach (LDA) is used (see, e.g., Gourier, Farkas, and Abbate (2009) and Hess (2011a)), which implies that the total aggregate loss is given by

\[ Z_t = \sum_{i=1}^{N_t} X_i, \]

where \( Z_t \) denotes the aggregate loss in the time interval \([0, t]\), \( N_t \) the loss frequency in the same time period and \( X_i \) the loss severity of the \( i \)-th event. Furthermore, the losses \( X_i \) are independently and identically distributed random variables and the loss frequencies and loss severities are assumed to be independent.\(^4\) The loss frequencies are modeled by a homogenous Poisson process with intensity \( \lambda > 0 \), i.e. the distribution of the frequencies is given by

\[ P_i(n) = P(N_i = n) = e^{-\lambda t} \left( \frac{\lambda t}{n!} \right)^n. \]

The severities of the claims in the upper tail of the loss distribution are described by means of extreme value theory (EVT). In previous operational risk models (see, e.g., Gourier, Farkas, \( ^4 \) The total operational risk of a bank or an insurer is then given by the aggregation of the dependent total aggregate losses (see Equation (1)) of all risk cells. To model the dependence between different operational risk cells the literature suggests splitting into models for frequency dependence and severity dependence. Hence, in this paper only one risk cell is considered and for a single risk cell the assumption of independent frequencies and severities is satisfied (see, e.g., Böcker and Klüppelberg, 2008).
and Abbate, 2009; Hess, 2011a), EVT is applied in the upper tail of the loss distribution as conventional distributions like the lognormal, exponential or gamma distribution are not capable to reproduce the heavy tails of operational losses. As EVT focuses on the tail area of a distribution, it provides a possibility to approximate losses that exceed a high threshold \( u \) by the Generalized Pareto Distribution (GPD), which is given by

\[
GPD_{\xi, \beta}(y) = \begin{cases} 
1 - \exp\left(-\frac{y}{\beta}\right), & \xi = 0 \\
1 - \left(1 + \frac{\xi y}{\beta}\right)^{-\frac{1}{\xi}}, & \xi \neq 0 
\end{cases}, \quad \beta > 0, \ y \geq 0 \text{ if } \xi \geq 0 \text{ and } 0 \leq y \leq \left(-\beta/\xi\right) \text{ if } \xi < 0,
\]

where \( \beta > 0, y \geq 0 \) if \( \xi \geq 0 \) and \( 0 \leq y \leq (-\beta/\xi) \) if \( \xi < 0 \), \( y = x - u \). The parameters \( \xi \) and \( \beta \) are called the shape and the scale, respectively, where \( \xi \) is the key parameter and determines the heavy-tailedness of the distribution.\(^5\) To model the loss severity distribution \( F(x) \), we thus fix the threshold \( u \) at the \( q \)th percentile\(^6\) and construct a spliced distribution function, where the body of the distribution, i.e. the losses below the threshold \( u \), follows a lognormal distribution \( F_{\log} \), and the tail, i.e. the losses over the threshold \( u \), is modeled with the GPD, \( F_{\text{GPD}} \), i.e. the loss severity distribution is given by

\[
F(x) = \begin{cases} 
F_{\log}(x) \cdot q, & \forall x \leq u \\
1 \cdot q + F_{\text{GPD}}(x-u) \cdot (1-q), & \forall x > u 
\end{cases}.
\]

Based on this relation, the distribution of the total aggregate loss \( Z_t \) of Equation (1) in the time interval [0, \( t \)] can be modeled by

\[
G_t(x) = P[Z_t \leq x] = \sum_{n=0}^{\infty} P[N_t = n] P[Z_t \leq x | N_t = n] = \sum_{n=0}^{\infty} P_n(t) F^{\sigma^*}(x), \quad x \geq 0, t \geq 0,
\]

\(^5\) In particular, three different cases can be distinguished: for \( \xi = 0 \), the GPD equals an exponential distribution, whereas for \( \xi < 0 \), a short-tailed Pareto type II distribution is obtained. In the case of \( \xi > 0 \), an ordinary Pareto distribution is induced and, therefore, the GPD is heavy-tailed. If the chosen threshold \( u \) is reasonably high, the theorem of Balkema and de Haan (1974) and Pickands (1975) states that the GPD is the canonical distribution for modeling excess losses over the defined threshold \( u \). Moreover, for most of the classical loss distributions, the excess distribution converges to the GPD when the threshold \( u \) is increased (see McNeil, Frey, and Embrechts, 2005, pp. 277-278), which means that the excess distribution over a high threshold \( u \) can be approximated by the GPD.

\(^6\) The choice of the threshold \( u \) is a central aspect when modeling the spliced distribution function. On the one hand, it has to be high enough to fulfill the limit law condition. On the other hand, a sufficient number of observations must be ensured to properly estimate the upper tail of the distribution function (see, e.g., Gourier, Farkas, and Abbate, 2009).
where \( F^\ast_n(x) \) denotes the \( n \)-fold convolution of \( F(x) \).

**Modeling the insurance company**

Figure 1 shows a balance sheet of the insurance company at time \( t \), where \( A_t \) is the market value of the assets, \( S_t \) denotes the value of insurance claims, and \( Z_t \) comprises the losses resulting from operational risk. The total value of liabilities is thus composed of \( S_t \) and \( Z_t \), and \( E_t \) is the company’s equity, which is determined as the difference between assets and liabilities.

**Figure 1:** Balance sheet of the insurance company at time \( t = 0, 1 \)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_t )</td>
<td>( E_t )</td>
</tr>
<tr>
<td>( S_t )</td>
<td>( Z_t )</td>
</tr>
<tr>
<td></td>
<td>( {L_t} )</td>
</tr>
</tbody>
</table>

At time zero, the insurer receives premiums \( \pi^{S_1} \) paid by the policyholders for insured losses at time \( t = 1 \), and an initial contribution by shareholders \( E_0 \). Thus, the total initial capital sums up to

\[
A_0 = E_0 + \pi^{S_1}.
\]

The initial capital is invested in the capital market, whereby a fraction \( \gamma \) is invested in high-risk assets, \( A_{0,\text{high}} = \gamma \cdot A_0 \), and the remaining part \( (1-\gamma) \) is invested in low-risk assets, \( A_{0,\text{low}} = (1-\gamma) \cdot A_0 \). Low-risk and high-risk assets are assumed to be lognormally distributed with mean \( E(A_{1,j}) \) and standard deviation \( \sigma(A_{1,j}) \), for \( j = \text{low,high} \). Thus, the total value of the asset portfolio \( A_1 \) at time \( t = 1 \) is given by

\[
A_1 = \gamma \cdot A_{1,\text{high}} + (1-\gamma) \cdot A_{1,\text{low}}.
\]

The insurer becomes insolvent if assets are not sufficient to cover the liabilities, i.e. if \( L_1 > A_1 \), as shareholders have limited liability. In this setting, operational risk will have an impact on the premium paid by policyholders for insured losses at time 1, as a higher default risk (caused by the presence of operational risk) would generally decrease the value of the insurance policy. Thus, to study the impact of operational risk on pricing and risk assessment, we compare three different cases as exhibited in Table 1.
Table 1: Overview of different assumptions regarding operational risk with respect to pricing and risk measurement

<table>
<thead>
<tr>
<th>Case No. $i$</th>
<th>Operational risk taken into account in basic pricing</th>
<th>Operational risk taken into account in risk measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Setting without operational risk)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>2 (Setting with operational risk)</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>3 (Setting with operational risk)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

First, we consider the setting without operational risk, i.e. where operational risk is neither taken into account in pricing nor in the calculation of risk measures (Case 1), i.e. $Z_1 = 0$. This serves as a reference case and allows an analysis of how risk measures are wrongly assessed if operational risk is set to 0. In the second case, operational risk is considered in the calculation of risk measures, but it is not taken into account in basic pricing (Case 2). Third, operational risk is considered in pricing as well as in the calculation of risk measures (Case 3). We hereby assume that – in a world with operational risk – operational losses are covered first, as they occur before the insurer is able to pay out the policyholders’ claims. Thus, depending on the assumptions regarding operational risk laid out in Table 1 (Cases $i = 1, 2, 3$), one can distinguish between three cases (realizations) at time 1:

1. $A_i^t < S_i + Z_i^t, A_i^t \geq Z_i^t$: the insurer is insolvent; operational losses are paid out, but insured losses can only be covered partially or not at all,
2. $A_i^t < S_i + Z_i^t, A_i^t < Z_i^t$: the insurer is insolvent; neither operational nor insured losses can be covered,
3. $A_i^t \geq S_i + Z_i^t$: the insurer is solvent; operational and insured losses can be covered.

Hence, at time $t = 1$, the operational loss claims $L_1^{Z,i}$, policyholders’ claims $L_1^{S,i}$, and the equityholders’ position $E_i$, for Cases $i = 1, 2, 3$, are given by

$$L_1^{Z,i} = \min(A_i^t, Z_i^t) = Z_i^t - \max(Z_i^t - A_i^t, 0),$$

$$L_1^{S,i} = \min(A_i^t - L_1^{Z,i}, S_i) = S_i - \max(S_i - (A_i^t - L_1^{Z,i}), 0),$$

and

$$E_i = \max(A_i^t - L_1^{L,i} - L_1^{Z,i}, 0),$$
thus summing up to $A_i^j$.

**Fair contracts and determination of premiums**

Valuation of equityholders’ and policyholders’ claims is conducted using the capital asset pricing model (CAPM) (see Gründl and Schmeiser, 2002). To ensure a fair situation from the shareholders’ perspective, the value of equityholders’ claims must be equal to their initial contribution (depending on the assumptions regarding operational risk, Cases $i = 1, 2, 3$), i.e.

$$V_t(E_i^j) = e^{-r_f} \cdot \left[ E\left(E_i^j\right) - \eta \cdot \text{Cov}(E_i^j, r_m) \right] = E_0,$$

(3)

where $V_t(.)$ stands for the valuation approach used to determine the market value at time $t$ (here by means of the CAPM), $r_m$ denotes the return of the market portfolio at time $t = 1$ and $\eta$ stands for the market price of risk, such that $\eta = \left( E(r_m) - r_f \right) / \sigma^2(r_m)$, where $r_f$ denotes the risk-free interest rate. To ensure that Equation (3) holds, the policyholders’ premiums are adjusted accordingly. This is done in two steps. First, the basic premium $\pi_i^{S_{\text{basic}}}$ is calculated by

$$\pi_i^{S_{\text{basic}}} = V_t(L_i^{S_{i}}) = e^{-r_f} \cdot \left[ E\left(L_i^{S_{i}}\right) - \eta \cdot \text{Cov}(L_i^{S_{i}}, r_m) \right].$$

Second, the fair premium $\pi_i^{S_{i}}$ is derived by adding a loading $\delta_i^{S_{i}}$,

$$\pi_i^{S_{i}} = \pi_i^{S_{\text{basic}}} \cdot \left( 1 + \delta_i^{S_{i}} \right),$$

which is calibrated such that the situation is fair from the shareholders’ perspective, i.e. that Equation (3) is satisfied.$^7$

Note that the use of an alternative paradigm for deriving the fair premium may actually imply a higher (or lower) loading and thus also a larger difference between the cases with and without operational risk. According to the three-factor model by Froot (2007), for instance, an extension of the two-factor model proposed by Froot and Stein (1998), firms do not only take into account the systematic risk factor as in the CAPM in setting premiums, but additionally

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$^7$ Note that if premiums are not adjusted, the shareholder value would decrease accordingly. In the present setting, it is thus assumed that shareholders have alternative investment opportunities in financial assets that do not involve operational risk. Hence, they would not agree to carry operational losses and require a respective adjustment of policyholders’ premiums (see also Doherty and Garven (1986) for similar arguments regarding double-taxation).
include a second factor that reflects the covariability of the product’s returns with the firm’s pre-existing portfolio of non-tradable risks (driven by the insurer’s and the customers’ aversion to insolvency risk), and a third factor that accounts for the covariance with firm-wide skewed risks. The latter takes into account that negatively skewed exposures are generally associated with higher costs and that they cause firms to conduct more aggressive reinsurance and hedging activities as well as make less aggressive and more diversified underwriting and investment decisions (Froot, 2007, p. 276). The extent of the difference in the loading when using the CAPM as compared to the Froot (2007) three-factor model would generally depend on the extent of the two additional factors that would also be driven by possible concentrations of operational risk on the balance sheet (e.g., large third-party distribution systems) as well as the correlation between operational risks and other assets or liabilities (e.g., financial guaranty underwriting). Hence, with increasing correlations and increasing risk concentrations of operational risks on the balance sheet, the fair loading would increase, which in turn would imply a reduced risk level for the insurer due to a higher premium income (given a sufficient demand by policyholders).

**Solvency capital requirements (SCR) and risk measurement**

Based on the previously described model framework and the assumptions regarding operational risk, the solvency capital requirements (SCR) can be derived. Under Solvency II, the SCR are defined as the amount of capital needed at time $t = 0$ to meet future obligations for a required safety level $\alpha$ using the value at risk (VaR) with a confidence level of 99.5% ($\alpha = 0.5\%$) on the basis of the risk-bearing capital at time $t = 1$ (see European Parliament and the Council, 2009, Article 101, No. 3). The risk-bearing capital ($RBC$) characterizes the available economic capital and is defined as the difference between the value of assets and liabilities,$^8$

$$RBC_i^t = A_i^t - L_i^t = A_i^t - S_i - Z_i^t,$$

and

$$RBC_0^i = V_0(A_i^t) - V_0(L_i^t) = V_0(A_i^t) - V_0(S_i + Z_i^t)$$

where $i = 1, 2, 3$ (see Table 1 for assumptions regarding operational risk). The solvency capital requirements are defined based on the VaR of the change of the $RBC$ over one period, where $RBC_{i,j}^t$ of Equation (4) is discounted with the risk-free interest rate $r_f$ (see, e.g., Gatzert and Schmeiser, 2008), such that

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$^8$ Under Solvency II, the difference between assets and liabilities is also called the net asset value (NAV) instead of $RBC$ (see EIOPA, 2010, p. 91-92).
As focus is laid on the impact of operational risk on an insurer’s solvency situation and solvency capital requirements, three approaches for deriving the solvency capital requirements are compared: 1) using the Solvency II standard model for operational risk ("SM"), 2) a partial internal model for operational risk ("PM"), or 3) a full internal model ("IM") for deriving the total SCR.

First, we assume that the SCR for operational risk are calculated using the Solvency II standard model as laid out in QIS 5 (see EIOPA, 2010, p. 103). In this case, capital requirements for operational risk are given by 30% of the basic solvency capital requirements ($BSCR_i$), which are calculated without taking into account operational risk, i.e. by setting $Z_i = 0$. Note that this does not affect the calculation of premiums, which is still conducted according to the three cases defined in Table 1 (i.e. depending on whether operational risk is taken into account in pricing or not). The $BSCR_i$ is thus derived by

$$BSCR_i = -VaR (e^{-r^i} \cdot RBC_{i,SM}^i - RBC_{i,SM}^0)^9$$

where $RBC_{i,SM}^i = A_i^i - S_i$ and

$$RBC_{i,SM}^0 = V_0 \left( RBC_{i,SM}^i \right) = e^{-r^i} \cdot \left[ E \left( RBC_{i,SM}^i \right) - \eta \cdot \text{Cov} \left( RBC_{i,SM}^i, r_m \right) \right].$$

The capital requirements for the operational risk $SCR_{i,SM,Op}^i$, in Cases $^i = 2, 3$ can then be calculated by multiplying the $BSCR_i$ of Equation (6) with the risk-based factor 0.3 prescribed in the Solvency II standard formula, such that

$$SCR_{SM,Op}^i = 0.3 \cdot BSCR_i, i = 2, 3. ^{11}$$

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Note that operational risk itself is not modeled in the $BSCR_i$.

In Case 1 (“without operational risk”) no additional solvency capital requirements for the operational risk need to be calculated, i.e. $SCR_{SM,Op}^{i=1} = 0$ and $SCR_{SM,ext}^{i=1} = BSCR_i$. Thus, solvency capital requirements in Case 1 are solely calculated with the Solvency II standard model, i.e. neither the partial internal model nor the full internal model are used in this case.

Under Solvency II, $SCR_{SM,Op}^i$ is defined by $SCR_{SM,Op}^i = \min(0.3 \cdot BSCR, Op) + 0.25 \cdot \text{Exp}_{ul}$ (see EIOPA, 2010, p. 103). In the present setting, this can be reduced to the formula stated in Equation (7) as $\text{Exp}_{ul}$ is equal to 0 due to consideration of a non-life insurer, and as $0.3 \cdot BSCR \geq \min(0.3 \cdot BSCR, Op)$, i.e. the real $SCR_{SM,Op}$ might even be smaller than the one we calculate. Thus, the derivation of $SCR_{SM,Op}$ may overestimate the actual one.
In Cases 2 and 3, the solvency capital requirements according to the standard model $SCR_{SM, total}^i$ are thus given by

$$SCR_{SM, total}^i = BSCR^i + SCR_{SM, Op}^i, i = 1, 2, 3.$$  

Second, we use a *partial internal model* that replaces the risk-based factor of 0.3 of the $BSCR$ with the operational value at risk (OpVaR). Here, capital requirements are calculated for operational risk only, without taking into account diversification effects (see, e.g., Böcker and Klüppelberg, 2005; Biagini and Ulmer, 2009). The OpVaR is given by the value at risk for a confidence level of 99.5\% of the change in operational losses within one period, where $Z_i$ is discounted with the risk-free interest rate $r_f$, for $i = 2, 3$ ($SCR_{PM, Op}^{i=1} = 0$). Hence, the target capital for operational risk $SCR_{PM, Op}^i$ is derived by

$$SCR_{PM, Op}^i = VaR \left( e^{-r_f} \cdot Z_i - Z_{i0} \right), i = 2, 3,$$  

with $Z_{i0} = V_0 \left( Z_i \right), i = 2, 3$. Hence, the total solvency capital requirements $SCR_{PM, total}^i$ are given by the sum of $BSCR^i$ (Equation (6)) and $SCR_{PM, Op}^i$ (Equation (8)).

$$SCR_{PM, total}^i = BSCR^i + SCR_{PM, Op}^i, i = 2, 3.$$  

Third, we calculate the solvency capital requirements with a full *internal model*, thus also taking into account diversification benefits. The risk-bearing capital $RBC_i^t$ at time $t = 1$ is calculated as the difference between $A_i^t$ and $L_i^t$, consisting of insured losses and operational losses, for $i = 1, 2, 3$. Afterwards, the solvency capital requirements $SCR_{IM, total}^i$ are calculated as defined in Equation (5), such that

$$SCR_{IM, total}^i = -VaR \left( e^{-r_f} \cdot RBC_i^1 - RBC_{0i}^1 \right), i = 1, 2, 3. \tag{9}$$  

We further calculate the residual in order to obtain the implicit operational risk capital charges $SCR_{IM, Op}^i$ by the difference between the $SCR_{IM, total}^i$ of Equation (9) and the $BSCR^i$ of Equation (6), thus

$$SCR_{IM, Op}^i = SCR_{IM, total}^i - BSCR^i, i = 1, 2, 3.$$  

Table 2 summarizes the three approaches for deriving solvency capital requirements.

\[\text{Note that in Case 1 ("without operational risk"), } SCR_{IM, total}^{i=1} = BSCR^{i=1}.\]
Table 2: Overview of the three approaches for deriving the $SCR_{k, total}^i$ and $SCR_{k, Op}^i$ ($k = SM, PM, IM$) for Cases $i = 1, 2, 3$ (see Table 1: assumptions regarding operational risk with respect to pricing and risk measurement)

<table>
<thead>
<tr>
<th>$SCR_{k, total}^i$</th>
<th>$SCR_{SM, Op}^i$</th>
<th>$SCR_{PM, Op}^i$</th>
<th>$SCR_{IM, total}^i - BSCR^i$ *</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BSCR^i$</td>
<td>$-\text{VaR}<em>\alpha \left( e^{-r_f} \cdot RBC</em>{1,SM}^i - RBC_{0,SM}^i \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$BSCR^i + SCR_{SM, Op}^i$</td>
<td>$BSCR^i + SCR_{PM, Op}^i$</td>
<td>$-\text{VaR}<em>\alpha \left( e^{-r_f} \cdot RBC</em>{1,SM}^i - RBC_{0,SM}^i \right)$</td>
</tr>
<tr>
<td>$SCR_{SM, Op}^i$</td>
<td>$0.3 \cdot BSCR^i$</td>
<td>$\text{VaR}<em>\alpha \left( e^{-r_f} \cdot Z</em>{1}^i - Z_{0}^i \right)$</td>
<td>$(SCR_{IM, total} - BSCR^i)$ *</td>
</tr>
</tbody>
</table>

*Residually derived

In addition to the $SCR$, the shortfall probability ($SP$) is calculated, which is given by

$$SP^i = P \left( A^i < L^i \right), i = 1, 2, 3.$$

3. NUMERICAL ANALYSIS

This section presents numerical results with respect to operational risk measurement and management as well as the impact of operational risk on fair premiums, using empirical parameters from previous literature. In addition, sensitivity analyses are conducted to identify key risk drivers.

Input parameters

The input parameters are summarized in Table 3. The expected value and the standard deviation of the company’s loss are based on empirical data of a medium-sized German non-life insurer as presented in Eling, Gatzert, and Schmeiser (2009). Furthermore, the expected values and standard deviations for high- and low-risk assets are based on data from representative indices (S&P 500, international government bond indices) following Gatzert and Kellner (2011). The parameters for the lognormal distribution of the operational losses as well as the parameter for the frequency of operational losses are also based on empirical data for U.S. insurance companies as presented in Hess (2011b). The parameters for scale and shape of the GPD as well as the quantile $q$ (to determine the threshold $u$) are adapted from Gourier, Farkas, and Abbate (2009). Furthermore, due to the correlation between the size of operational losses and the firm size, the simulated operational losses have to be adjusted by multiplying the operational losses at time $t = 1$ with the factor $\kappa$ to ensure that the operational
losses fit with the parameters of the insurance company. As we consider a medium-sized German insurance company in the present analyses, a factor $\kappa$ of 0.30 is applied (see Selvaggi, 2009). In addition, a truncation point $T$ is integrated that ensures that every single operational loss $X_i$ has a minimum amount, i.e. if an operational loss occurs in the time interval $[0, t]$, its amount equals the maximum of the simulated operational loss and the truncation point $T$. The implementation of a truncation point is necessary due to the fact that in empirical databases (e.g. Algo OpData and SAS OpRisk Global Data), only operational losses over a certain threshold are considered.

**Table 3: Input parameters for the basic simulation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Available equity capital at time $t = 0$</td>
<td>$E_0$ €48 million</td>
</tr>
<tr>
<td>Parameters for the lognormal distribution of the operational loss</td>
<td>$\mu_{Z_i}, \sigma_{Z_i}$ 1.52, 2.26</td>
</tr>
<tr>
<td>Scale of the GPD of the operational loss</td>
<td>$\beta$ 0.01</td>
</tr>
<tr>
<td>Shape of the GPD of the operational loss</td>
<td>$\xi$ 0.89</td>
</tr>
<tr>
<td>Quantile of the threshold $u$ for the mixed lognormal GPD</td>
<td>$q$ 90%</td>
</tr>
<tr>
<td>Frequency of operational losses</td>
<td>$\lambda$ 0.15</td>
</tr>
<tr>
<td>Truncation point</td>
<td>$T$ €0.1 million</td>
</tr>
<tr>
<td>Adjustment factor of the operational losses</td>
<td>$\kappa$ 0.30</td>
</tr>
<tr>
<td>Expected value of the company loss</td>
<td>$E(S_1)$ €110 million</td>
</tr>
<tr>
<td>Standard deviation of the company loss</td>
<td>$\sigma(S_1)$ €22 million</td>
</tr>
<tr>
<td>Expected value of high-risk assets</td>
<td>$E(A_{1,\text{high}})$ 1.12</td>
</tr>
<tr>
<td>Standard deviation of high-risk assets</td>
<td>$\sigma(A_{1,\text{high}})$ 0.23</td>
</tr>
<tr>
<td>Expected value of low-risk assets</td>
<td>$E(A_{1,\text{low}})$ 1.06</td>
</tr>
<tr>
<td>Standard deviation of low-risk assets</td>
<td>$\sigma(A_{1,\text{low}})$ 0.07</td>
</tr>
<tr>
<td>Investment in high-risk assets</td>
<td>$\gamma$ 0.25</td>
</tr>
<tr>
<td>Expected value of the market portfolio</td>
<td>$E(r_m)$ 0.08</td>
</tr>
<tr>
<td>Standard deviation of the market portfolio</td>
<td>$\sigma(r_m)$ 0.04</td>
</tr>
<tr>
<td>Market price of risk</td>
<td>$\eta$ 37.50</td>
</tr>
<tr>
<td>Kendall’s tau for low-risk and high-risk investment</td>
<td>$\rho(A_{\text{high}}, A_{\text{low}})$ 0.20</td>
</tr>
<tr>
<td>Kendall’s tau for high-/low risk assets and company losses</td>
<td>$\rho(A_{1}, S_1)$ 0.10</td>
</tr>
<tr>
<td>Kendall’s tau for high-/low risk assets and operational losses</td>
<td>$\rho(Z_i, A_1)$ -0.27</td>
</tr>
<tr>
<td>Kendall’s tau for operational losses and company losses</td>
<td>$\rho(Z_i, S_1)$ -0.05</td>
</tr>
<tr>
<td>Kendall’s tau for market portfolio and high-/low risk assets</td>
<td>$\rho(r_m, A_1)$ 0.20</td>
</tr>
<tr>
<td>Kendall’s tau for market portfolio and liabilities</td>
<td>$\rho(r_m, S_1)$ -0.20</td>
</tr>
<tr>
<td>Kendall’s tau for market portfolio and operational losses</td>
<td>$\rho(r_m, Z_i)$ -0.10</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r_f$ 0.02</td>
</tr>
</tbody>
</table>

When modeling insurance risks, it is important to account for dependence structures between the different processes (see, e.g., Gatzert and Kellner, 2011). Therefore, we explicitly model the dependence between high-risk and low-risk assets, between the losses resulting from operational risk and assets (high-risk and low-risk, respectively), between the losses resulting from operational risk and the losses resulting from the insurance policies (company losses), as
well as between the company losses and the assets (high-risk and low-risk, respectively).\textsuperscript{13} We thereby apply the concept of the Gauss copula (see McNeil, Frey, and Embrechts, 2005, p. 193).\textsuperscript{14} To calibrate the Gauss copula, Kendall’s rank correlation $\rho$ is used due to its invariance against non-linear transformations. The parameter for the correlation between the high-risk assets (equity) and the losses resulting from operational risk is based on empirical data following Cummins, Lewis, and Wei (2006), where a significant negative stock price response to operational loss events in the U.S. insurance industry is revealed.\textsuperscript{15} We further assume that there is small negative correlation between the losses resulting from operational risk and the liabilities.\textsuperscript{16} The parameters for the correlation between the market portfolio and the different relevant risk factors are adopted from Gatzert, Schmeiser, and Toplek (2011). Numerical results are based on Monte Carlo simulation with 500,000 sample paths. In addition, latin hypercube sampling is used to improve the stability of the simulation (see Glasserman, 2010, pp. 236-243).\textsuperscript{17} The parameters were then subject to sensitivity analyses.

\textit{The impact of operational risk on an insurer’s pricing, shortfall risk, and solvency capital requirements}

We first consider the three possible cases laid out in Table 1, i.e. “without operational risk” (Case 1), “with operational risk but not taken into account in basic pricing” (Case 2), and “with operational risk and taken into account in basic pricing” (Case 3). Case 1 serves as the reference case, which can be compared to the other cases in order to illustrate how an insurer’s risk level may be misestimated if operational risk is not taken into account. Table 4

\textsuperscript{13} Kahane and Nye (1975) and Cummins, Lin, and Phillips (2009), e.g., show that the correlation parameter between company losses and assets in general depends on the specific insurance line and the investment activities and can either be positive or negative. In this analysis, the correlation between assets and company losses is set to a positive value (for both high-risk and low-risk assets), but should be empirically calibrated for each individual insurance company.

\textsuperscript{14} Alternatively, varying dependence structures (e.g. $t$-copula, Archimedean copula) and non-linear dependencies may be appropriate depending on the concrete setting, as they can have a substantial impact on results (see Ai and Wang, 2012).

\textsuperscript{15} Their estimated correlation coefficient is used in this regard due to a lack of other available information and the correlation between low-risk assets and operational risk is set to the same value. Both numbers should, however, be subject to further empirical analysis, which holds for all correlations in regard to operational risk.

\textsuperscript{16} Similar to the correlation between company losses and assets, the correlation between company losses and operational risk depends on the individual situation of the insurance company and on the aggregation of the different operational risk cells. In the following, this assumption will be subject to a specific sensitivity analysis in order to study the impact of different correlations between company losses and operational risk.

\textsuperscript{17} We chose a sufficiently high number of sample paths and further implemented latin hypercube sampling to achieve low sample standard errors (e.g. for the basic simulation, the sample standard error of the risk-bearing capital amounts to 0.0341) and ensured that the results remain stable for different sets of random numbers.
displays the shortfall probabilities for basic premiums (Part a) as well as for fair premiums (Part b). When comparing Cases 1 and 2, if the insurer only requires the basic premium without additional loadings, one can observe that the shortfall probability strongly increases from 0.67% to 1.54% in the presence of operational risk. If operational risk is taken into account in pricing, the basic premium decreases due to a higher default option value,\textsuperscript{18} which in turn implies a further increase in the shortfall probability to 1.60% in Case 3.

Part b) in Table 4 displays the shortfall probability if a fair loading is added to the basic premiums that ensures that the situation is fair from the equityholders’ perspective (see Equation (3)). Operational risk is thereby always included in the calculation of the fair loading, but not necessarily in the basic premium. In the setting without operational risk (Case 1), the fair premium is considerably lower as compared to both cases with operational risk (Cases 2 and 3) and also lower than the basic premium. This also implies a slight increase in the shortfall probability from 0.67% to 0.70%. In the situation with operational risk (Cases 2 and 3), the fair premiums are equal due to the calibration of the fair loading risk even though the basic premiums differed, as shareholders require a risk-adequate compensation on their initial contribution and thereby account for operational risk. In line with this, the corresponding shortfall risk is equal as well (1.46%), which – due to the positive premium loading – is lower than in the case where the insurer only charges the basic premium (1.54% and 1.60%). Hence, if premiums are calculated fair from shareholders’ perspective, the policyholders are the first to cover the risk of operational losses.

\textsuperscript{18} When operational risk is taken into account in basic pricing, the default option $\max\left(S_i - \left( A_i - L^Z_i \right), 0 \right)$ increases and, therefore, $L^Z_i = S_i - \max\left(S_i - \left( A_i - L^Z_i \right), 0 \right)$ is decreasing, which lowers the basic premium.
Table 4: Shortfall probability for basic and fair premiums for different assumptions regarding the consideration of operational risk in pricing and risk measurement (see Table 1)

<table>
<thead>
<tr>
<th>Case 1 (without operational risk)</th>
<th>Case 2 (with operational risk but not taken into account in basic pricing)</th>
<th>Case 3 (with operational risk and taken into account in basic pricing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic premium $\pi_{S, basic}$</td>
<td>117.55</td>
<td>117.55</td>
</tr>
<tr>
<td>Shortfall probability SP</td>
<td>0.67%</td>
<td>1.54%</td>
</tr>
<tr>
<td>$\delta_{S_i}$</td>
<td>-0.6%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Fair premium $\pi_{S_i} = \pi_{S, basic} \cdot (1 + \delta_{S_i})$</td>
<td>116.86</td>
<td>118.63</td>
</tr>
<tr>
<td>Shortfall probability SP</td>
<td>0.70%</td>
<td>1.46%</td>
</tr>
</tbody>
</table>

We next study the impact of the assumptions regarding operational risk (Cases 1 to 3) on the insurer’s solvency capital requirements. In Table 5, the SCR are displayed for the basic premiums (Part a) and the fair premiums (Part b). In Case 1 (without operational risk), no risk charge for operational risk is required and, therefore, the total solvency capital requirements $SCR_{total}$ are equal to the $BSCR$ and the three approaches for deriving the $SCR$ coincide. In this Case 1, the $SCR$ for the basic premium are about 51.53 and only slightly higher in case of the fair premium with 51.54 due to the higher shortfall risk (see Table 4).

In the presence of operational risk (Cases 2 and 3), $SCR$ considerably increase for both fair and basic premiums. In Part a) of Table 5 (basic premium) and Case 2, for instance, it can be seen that when fully accounting for imperfect correlations between risk factors as in the case of the full internal model, diversification benefits between operational risks, insurance risks, and market risks result in a considerable reduction of the $SCR_{total}$ as compared to the case of the partial internal model, where no diversification benefits are taken into account. In addition, diversification effects also imply that the $SCR_{total}$ is closer to the one of the standard model. This diversification benefit amounts to 38.1% in both Cases 2 and 3, as the $SCR_{total}$ is reduced from 127.59 to 78.93 in Case 2 and from 127.60 to 78.96 in Case 3, respectively.
Table 5: The impact of assumptions regarding the consideration of operational risk in pricing on solvency capital requirements, given the basic and fair premiums in Table 4

<table>
<thead>
<tr>
<th>Case 1 (without operational risk)</th>
<th>Case 2 (with operational risk but not taken into account in basic pricing)</th>
<th>Case 3 (with operational risk and taken into account in basic pricing)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard model</td>
<td>Partial internal model</td>
</tr>
<tr>
<td>a) Basic premium $\pi_{b, \text{basic}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSCR</td>
<td>51.53</td>
<td>51.53</td>
</tr>
<tr>
<td>$\text{SCR}_{\text{Op}}$</td>
<td>0</td>
<td>15.46</td>
</tr>
<tr>
<td>$\text{SCR}_{\text{total}}$</td>
<td>51.53</td>
<td>66.99</td>
</tr>
<tr>
<td>b) Fair premium $\pi_{b} = \pi_{b, \text{basic}} \cdot (1 + \delta_{\text{b}})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSCR</td>
<td>51.54</td>
<td>51.55</td>
</tr>
<tr>
<td>$\text{SCR}_{\text{Op}}$</td>
<td>0</td>
<td>15.47</td>
</tr>
<tr>
<td>$\text{SCR}_{\text{total}}$</td>
<td>51.54</td>
<td>67.02</td>
</tr>
</tbody>
</table>

*Residually derived as $\text{SCR}_{\text{IM,Op}} = \text{SCR}_{\text{IM,\text{total}}} - \text{BSCR}$

In Part b) of Table 5, solvency capital requirements are displayed for fair premiums and show very similar results as in the case of the basic premium in Part a). In addition, when premiums are calculated fair from the shareholders’ perspective, the solvency capital requirements are equal in Cases 2 and 3 due to the fair calculation of the premium loading $\delta_{\text{b}}$ (see Table 4). Thus, fair pricing (using a fair loading) or the consideration of operational risk in basic pricing (Case 3) does not considerably impact the $\text{SCR}$, which is mainly due to the low overall shortfall probabilities in the present setting.

However, this changes when looking at a setting with higher operational risk as shown in Figure 2, where the shortfall probabilities are displayed (right column) for basic premiums (upper left graph) and for fair premiums (lower left graph) for varying operational loss intensities $\lambda$. If the insurer does not impose a fair loading (upper row), the basic premium is decreasing in Case 3 for an increasing operational loss intensity $\lambda$ as operational risk is considered in basic pricing, which increases the default risk and thus lowers the basic premium (see also Table 4). Therefore, the shortfall probability is increasing faster than in Case 2, where operational risk is not included in pricing, implying that the basic premium remains unchanged even if the operational loss intensity $\lambda$ increases. Thus, if premiums are calculated without a fair loading (that accounts for operational risk), pricing assumptions regarding the basic premium can in fact substantially impact an insurer’s risk situation. In
contrast, if premiums are calculated fair from the shareholders’ perspective (bottom row), the premium loading adjusts the basic premium for the higher number of operational losses, such that the policyholders carry the higher operational risk. Hence, the fair premiums are increasing for higher loss intensities and are equal for Cases 2 and 3 (see also Table 4), thus also implying the same shortfall probability.

As solvency capital requirements do not considerably differ in Cases 2 and 3, we now only focus on Case 3, where operational risk is taken into account in basic pricing, and further assume that the insurer imposes a fair premium loading. Figure 3 exhibits results of a sensitivity analysis regarding the $SCR_{\text{total}}$ for varying relevant parameters, including the frequency $\lambda$ of operational losses, the correlation between company losses and operational losses $\rho_{\tau}(Z_t, S_t)$, and the expected company losses $E(S_t)$.

In the upper left graph in Figure 3, the basic and the fair premiums are displayed for an increasing frequency $\lambda$ (see also Figure 2), and the upper right graph shows the corresponding $SCR$ for the standard model, the partial internal model, and the full internal model. Here, the $BSCR$ is constant and equal for all three approaches, as operational risk is not taken into account when deriving the $BSCR$ (see Equation (6)). This also implies that in case of the standard model, $SCR_{\text{SM,Op}}$ is constant as well due to the derivation by means of the risk-based factor ($SCR_{\text{SM,Op}} = 0.3 \times BSCR$). Thus, the total $SCR$ in case of the standard model does not change if the operational loss intensity is increasing.
Figure 2: Premiums (basic and fair) and corresponding shortfall probabilities for different assumptions regarding the consideration of operational risk in pricing (Cases 1 to 3, see Table 1) for varying operational loss intensities $\lambda$.

Notes: Case 1 = without operational risk, Case 2 = with operational risk but not taken into account in basic pricing, Case 3 = with operational risk and taken into account in basic pricing.

For basic premiums $\pi_{\text{basic}}^b = V_0^b(L_t^b) = e^{-\gamma t} \left[ E(L_t^b) - \eta \cdot \operatorname{Cov}(L_t^b, r_m) \right]$, fair premium $\pi_{\text{fair}}^f = \pi_{\text{basic}}^b \cdot (1 + \delta^f)$.

Compared to the standard model, the $SCR$ for operational risk derived by the partial internal model and the full internal model are increasing for an increasing operational loss intensity $\lambda$, where $\text{SCR}_{\text{PM,Op}}$ is increasing faster than $\text{SCR}_{\text{IM,Op}}$ due to the non-consideration of diversification benefits. Only for very low values of $\lambda$ does the standard model require more capital for operational risk than the full internal model and thus generally appears to underestimate operational risk (depending on the individual firm’s operational risk). In contrast, the partial internal model tends to overestimate operational risk, since it does not account for diversification effects between risk factors.
*Residually derived as $SCR_{IM,Op} = SCR_{IM,\text{total}} - BSCR$
In regard to the correlation between company losses and operational losses $\rho_{i}(Z_{i}, S_{1})$ it can be seen from the second row in Figure 3 that for varying correlations, the basic premiums and the fair premiums are slightly decreasing. As in the first row of Figure 3, the $BSCR$ and $SCR_{SM,Op}$ remain constant. Furthermore, the same holds true for the $SCR_{PM,Op}$ for operational risk derived by the partial internal model, since the calculation of the OpVaR does not account for dependencies between operational losses and company losses (see Equation (8)). The full internal model, in contrast, fully accounts for dependencies and diversification benefits, such that $SCR_{IM,Op}$ decreases for lower correlations $\rho_{i}(Z_{i}, S_{1})$. When varying the expected company losses $E(S_{1})$ (lower graph in Figure 3), the basic and fair premiums are increasing linearly for an increasing $E(S_{1})$. In the lower right graph in Figure 3 it can be seen that the $BSCR$ and the $SCR_{SM,Op}$ are also linearly increasing due to the increasing $E(S_{1})$ and since the solvency capital requirements for the operational risk in the standard model are calculated by the factor 0.3 of the $BSCR$. In case of the partial internal model, $SCR_{PM,Op}$ remains constant as it is based on the OpVaR only, which does not depend on the company’s losses. When looking at the $SCR_{IM,Op}$, we again observe that this is decreasing for an increasing $E(S_{1})$ due to diversification effects, thus implying only a slight increase in the total solvency capital requirements.

Lastly, we study the impact of the asset allocation on the fair premium, shortfall probability and $SCR$ as displayed in Figure 4 by varying the fraction $\gamma$ of high-risk assets. The results show that if $\gamma$ is increasing, the fair premium and the shortfall probability increase. With respect to the $SCR_{Op}$, the right graph in Figure 4 illustrates that in case of the partial internal model, the $SCR_{PM,Op}$ is again constant as assets do not impact the OpVaR. In case of the standard model, $SCR_{SM,Op}$ stays almost constant until $\gamma$ reaches about 0.6 and then increases, similar to the development of the shortfall probability. In comparison to this, $SCR_{IM,Op}$ is slightly increasing for a $\gamma$ smaller than 0.6 and then decreasing if $\gamma$ is higher than this value. Thus, the development of $SCR_{IM,Op}$ is opposed to the one observed in case of the Solvency II standard formula. Hence, these results again emphasize the diversification benefits that arise due to imperfect correlations between assets and operational risks. One can also observe that as in Figure 3, the standard model may underestimate operational risk and that the partial internal model overestimates it due to the non-consideration of diversification benefits, which are taken into account in case of the full internal model.
**Figure 4:** Fair premiums, shortfall probabilities and corresponding $SCR_{Op}$ for varying the fraction $\gamma$ of high-risk assets for Case 3 (with operational risk and taken into account in basic pricing)

*Residually derived as $SCR_{IM,Op} = SCR_{IM,total} - BSCR$

### 4. Implications and Further Considerations in REGARD TO MEASURING AND MANAGING OPERATIONAL RISKS

The previous analyses emphasized the importance of adequately taking into account operational risk when assessing an insurer’s risk and solvency situation, thereby using an aggregate view of operational risk without distinguishing between different risk cells. Hence, in practice, two additional central aspects must be taken into account. First, the total operational risk of an insurer is typically given by aggregating dependent losses of all risk cells. Therefore, frequency dependence and severity dependence between the different risk cells have to be considered when calculating the solvency capital requirements for the operational risk of an insurance company (see, e.g., Böcker and Klüppelberg, 2008). Hence, an accurate framework for modeling dependent risk cells for operational risk is indispensable. Furthermore, in the present setting, extreme value theory is used and, therefore, the GPD has to be estimated for independent and identically distributed losses. In addition, an adequate threshold must be chosen. This involves a tradeoff between choosing a higher threshold (that improves the approximation of the excess distribution function) and choosing a lower threshold (that reduces the amount of data available for estimating the GPD, but in turn increases the standard errors of the GPD’s estimates) (see McNeil, 1999). Second, the model needs to be adequately calibrated, which requires sufficient loss data as the basis of risk
measurement and risk management (see Kalhoff and Haas, 2004), which is also vital for an adequate estimation of the correlation structure among different risk cells (see Haas and Kaiser, 2004). Here, in principle two different sources of operational loss data are available, internal and external loss data. The creation of an internal database over time thereby provides a more reliable assessment of an individual insurer’s operational loss events if the data set becomes sufficiently large. However, internal operational loss data is often limited as operational risk includes human errors and, thus, the willingness of employees to inform about operational loss events will be one crucial success factor to create an internal database (see Kalhoff and Haas, 2004). Hence, the quantification of solvency capital requirements for operational risk remains a challenge for many insurers, also and especially due to the small sample size of internal operational loss events, which is why external databases can be integrated to gather additional loss information and to obtain a more accurate picture of operational risk. External databases are either available from providers (e.g. Algo OpData and SAS OpRisk Global Data) or consortia of insurers, which must then be adapted to the size of the individual insurance company and the size of internal operational loss events (see Selvaggi, 2009). However, the occurrence of high severity operational loss events is often kept confidential. Therefore, loss data is typically biased towards low severity losses, and the true frequency of operational loss events is underestimated due to unreported large losses.

Besides measuring operational risk, one major issue is its management, which includes prevention and insurance. Methods of preventing operational losses mainly comprise the monitoring and optimization of processes as well as the initialization of training for the employees and business continuity management. However, these methods only influence the probability of operational losses, but not the magnitude of single operational loss events (see Auer, 2008, pp. 138-141). The amount of operational losses – especially for risks with low frequency and high severity – can be reduced by developing emergency plans, for instance. Furthermore, a variety of insurance products against operational losses is available that are typically linked to the event type, as there is no coverage for operational risk in general. For example, insurance against natural hazards is available for the risk type “damage to physical assets”, fidelity insurance for “internal and external fraud”, errors and omissions insurance for

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19 Even though large operational losses should typically be known by the management, the successful creation of an internal database depends on the cooperativeness of the employees to pass on the required information. According to Kalhoff and Haas (2004), a number of companies in their study tried to motivate their employees to relay the required data through positive and negative incentive systems. However, this method still did not fully solve the problem of ensuring the transmission of information (see Kalhoff and Haas, 2004).

20 E.g., ORIC, which was launched by the Association of British Insurers in 2005 as a response to new regulations of the UK Financial Services Authority (FSA) and Solvency II.
“clients, products & business practice” or loss of profits insurance for the event type “business disruption & system failures” (see Cruz, 2002, pp. 258-259; Auer, 2008, pp. 149-150). In addition, large industrial firms (e.g. ThyssenKrupp) have recently expressed the need for insurance against “cyber-attacks”, e.g. sabotage of manufacturing facilities, which is currently not available and can be classified in the event types “external fraud” and “business disruption & system failures”. As an alternative to traditional reinsurance, operational risk can be mitigated by issuing insurance-linked securities such as cat bonds, or by means of insurance derivatives. However, at least under Basel II, these risk transfers are not accepted to reduce the solvency capital requirements (see Auer, 2008, p. 141). In addition, when transferring operational risk, the recognition of insurance mitigation in the calculation of the solvency capital requirements under Basel II/III is limited to 20% of the total operational risk capital charge (see Basel Committee, 2004, p. 148), a rule that according to Hess (2009) may also be implemented in Solvency II.

While taking preventive action and purchasing insurance against operational risk reduces the risk of monetary losses due to operational risk events, there is a considerable reputational risk associated with these events. In particular, although reputational risk is explicitly excluded in the definition of operational risk, the reputation of an insurance company can be damaged as a consequence of operational losses (see de Fontnouvelle et. al, 2006; Kamiya, Schmit, and Rosenberg, 2011). Results in a study by Fiordelisi, Soana, and Schwizer (2011), e.g., indicate that substantial reputational losses follow after operational loss events and that the highest reputational damage is caused by the operational risk type “fraud”. Moreover, Cummins, Lewis, and Wei (2006) show that after an operational loss event, the decrease in market value of banks and insurers is even higher than the pure operational loss amount. The studies of Gillet, Hübner, and Plunus (2010) and Perry and de Fontnouvelle (2005) also illustrate that at least for the event type “internal fraud”, the loss in market value is greater than the operational loss announced. Hence, especially for companies whose activities are based on trust such as banks and insurers, reputation is a key asset and, thus, prevention in regard to operational risk and especially with respect to fraud is vital (see Fiordelisi, Soana, and Schwizer, 2011). In addition, new insurance products that provide coverage for reputational losses are now offered in the market. For instance, insurance policies by Munich Re and Zurich cover profit setbacks caused by reputational damages up to €150 million, while Allianz developed a new product that covers the costs of communication in terms of press work, advertisement and media monitoring, after a reputational loss event up to €10 million

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21 As the premium of insurance typically exceeds the expected operational losses, transferring risk through insurance generally does not reduce the expected losses, but it helps stabilizing losses over time, thus reducing risk.
(see Höpner, 2012). Furthermore, as legal risks are included in the definition of operational risk under Solvency II, operational risk depends also on the country and the individual situation of the insurance company. Therefore, operational risk measurement and management should be integrated in an enterprise risk management framework that accounts for all relevant factors and dependencies as well as insurance and prevention, which can imply a positive impact on firm value (see Hoyt and Liebenberg, 2011).

5. CONCLUSION

This paper examined the impact of operational risk on fair premiums and solvency capital requirements under Solvency II. Three different approaches were used for the latter, including a full internal model and a partial internal model that only focuses on the operational value at risk (OpVaR), i.e. without taking into account diversification effects. These were compared to the Solvency II standard model that uses a risk-based factor to derive capital requirements for operational risk.

The results showed that the presence of operational risk in general does not considerably impact fair premiums (fair from the shareholders’ perspective) if the insurer’s safety level is sufficiently high. However, this observation changed for higher operational loss intensities, where neglecting operational risk in pricing severely impacted an insurer’s shortfall risk if premiums were not calculated in a fair way.

Regarding solvency capital requirements, we found that the internal model considered in this paper led to similar results as the Solvency II standard formula as long as the operational loss intensity was not too high. This is a consequence of the diversification benefits taken into account in case of the internal model that arise due to imperfect correlations between operational risks, insurance risks, and market risks. For increasing operational loss intensities, however, the standard model clearly tended to underestimate risk as capital requirements are calculated based on the fixed factor 0.3 of the basic solvency capital requirements (that do not include operational risk). Hence, both the standard model and the partial internal model are not able to reflect diversification benefits due to imperfect correlations between operational losses and insured losses. While the solvency capital requirements derived by the Solvency II standard formula and the partial internal model thus remain constant for decreasing correlations between operational losses and insured losses, the SCR derived by the full internal model decreases due to an increasing diversification benefit. The same holds true when varying the expected company losses and the fraction invested in low-risk assets, which
emphasizes the potential for diversification benefits that arise due to imperfect correlations between assets, company losses and operational risks.

Since diversification benefits are not taken into account in case of the partial internal model, which derives the operational value at risk separately, solvency capital requirements are generally overestimated in this case. One way to reduce monetary losses from operational risk is (re-)insurance, which, however, is only available for specific event types and not for operational risk in general. Furthermore, the integration of supplementary prevention methods in addition to reinsurance is vital in order to reduce the probability and magnitude of operational losses, as operational loss events can cause severe reputational damage, which can even worsen an insurer’s solvency situation beyond the operational loss amount and considerably reduce market values. Operational risk should thus be measured and managed in the context of a holistic enterprise risk management system with an internal risk-sensitive approach that accounts for dependencies between risk factors and additionally includes specific insurance and prevention programs. This is also relevant in the context of insurers’ Own Risk and Solvency Assessment (ORSA) as required according to Solvency II’s Pillar 2 and the NAIC in the United States. Amongst other aspects, future research could focus on an (empirical and theoretical) analysis of the impact of operational risk (e.g., concentrations on the balance sheet) and correlations between operational risks and the risk profile of other assets and/or liabilities on pricing, risk management, and capital budgeting decisions in general using, e.g., a multi-factor model as proposed by Froot (2007).

REFERENCES


