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Working Paper

Chair for Insurance Economics
Friedrich-Alexander-University of Erlangen-Nürnberg

Version: August 2011

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ABSTRACT

Definitions of pooling effects in insurance companies may convey the impression that the achieved risk reduction effect will be beneficial for policyholders, since typically a) lower premiums are paid for the same safety level with an increasing number of insureds, or b) a higher safety level is achieved for a given premium level for all pool members. However, this view is misleading and the purpose of this paper is to reexamine this apparent merit of pooling from the policyholder's perspective. This is achieved by comparing several valuation approaches for the policyholders' claims using different assumptions of the individual policyholder's ability to replicate the contract's cash flows and claims. The paper shows that the two considered definitions of risk pooling do not offer insight into the question of whether pooling is actually beneficial for policyholders.

JEL-Classification: D46; G13; G22

Keywords: Risk Pooling, Theory of Risk, Risk Valuation

1. INTRODUCTION

Risk pooling in insurance companies is often referred to as the "production law" of insurance. Articles and standard textbooks on insurance and risk theory may thereby convey the impression that risk pooling in insurance companies (group balance concept) generates an additional value for policyholders. The reason for this can be outlined as follows. Actuarially calculated premiums are usually given by the value of expected losses plus a risk premium, or safety loading, to achieve a given safety level for the portfolio of the insured. Hence, with an increasing community of insureds, the actuarially calculated individual premium generally decreases, while simultaneously, the insurance company's safety level remains constant. On the other hand, for a given individual premium, an increasing number of pool members implies

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that the ruin probability converges to zero, i.e., an increasing safety level in the pool can be achieved (see e.g., Smith and Kane (1994), Albrecht (1982, 1990) for both definitions of risk pooling).

The purpose of this paper is to reexamine this apparent merit of pooling—also called group balance concept—for policyholders by analyzing this issue in more depth. To identify in which situation pooling is beneficial for policyholders, we compare several valuation approaches with different assumptions on an individual policyholder's ability to replicate the contract's cash flows and claims. The key point in this analysis is, thus, the question to what extent individual policyholders can achieve advantages from diversification obtained at the company's level.

In the literature, several particular aspects of risk pooling have been considered. For instance, cases are examined in which pooling effects, and thus risk diversification, are achieved depending on the loss distribution, dependence structure, number of pool participants, risk measures and premium calculation schemes (see, e.g., Albrecht, 1982, 1984; Zigenhorn, 1990; Cummins, 1991). Furthermore, positive safety loadings for premiums in the pool are derived based on the ruin probability by arguing that the insurer will certainly become insolvent if the premium is based on the expected loss only (see Bühlmann, 1996). In a similar setting to the present paper, Borch (1990) shows that a change in the premium level in a pool with homogenous risks does not influence the utility in the specific case of μ/σ preferences from the insurer's perspective in the context of pooling. Cummins (1991) illustrates that in the case of independent and identically distributed risks, the insurer's total buffer fund, necessary to ensure a given safety level (e.g. an acceptable ruin probability), goes to infinity with an increasing number of pool members, while the required buffer for each policy goes to zero. This implies that a policy premium approximately equal to the expected loss should be sufficient to provide the desired safety level. The buffer fund could be composed of, for example, equity capital or, for a one-period mutual insurer, be provided by policyholders. However, Cummins (1991) does not explicitly evaluate the policyholder and shareholder claims and, furthermore, does not focus on the question as to whether risk pooling per se, or the fact that the policy premium can be reduced with an increasing number of policyholders, is of value for the insured.

The aim of this paper is thus to extend and combine previous work by focusing on the merits of pooling claims (using the two definitions above) from the policyholder's perspective using different valuation approaches. The valuation approaches and observations are summarized as follows. The article considers a mutual insurance company where policyholders are both debt and equity holders. Starting with actuarial pricing, a premium is calculated for a given safety level determined by a fixed ruin probability. When paying the premium, policyholders also acquire a claim on the surplus (excess of asset value over claims payments). Hence, they possess a shareholder stake as well as a debt holder position. The description of pooling effects in such a company may give the impression that the group balance concept will, in general, be beneficial for policyholders, since lower premiums are paid for the same safety level with an increasing number of insureds.

However, this view is misleading as the policyholder's *value* of claims will strongly depend on the model framework, e.g., on individual preferences or policyholders' ability to replicate the contract's cash flows. If policyholders can fully replicate cash flows, the value of their claims positions can be evaluated using a financial pricing approach assuming that requirements spanning, information, no arbitrage, and competitivity are satisfied. With an increasing number of insureds, the decreasing premium can be separated in a decreasing equity position value and the value of the debt holder position, which approaches the present value of losses. Overall, the value of equity and debt positions calculated using present value will sum up to the initial premium paid by the policyholder for all n, and only the partition between equity claims and debt holder's claims is altered. Thus, in this setting, no additional value is generated through diversification on the company's level as it can equivalently be achieved by policyholders on an individual level in such a model setup.

Pooling may imply additional value from the policyholder's perspective, if replication or diversification is not achievable for policyholders as it is for the insurance company (see Borch, 1990). To examine this case, we additionally focus on a different resultant valuation approach and consider the case where individual policyholders cannot diversify at all. We use the concept of utility functions to analyze whether in this case pooling is of value from the perspective of the policyholder. Results then depend on different assumptions of initial wealth and degree of risk aversion and illustrate that an increasing number of pool members and the naïve diversification is – in contrast to the financial approach – beneficial for the policyholder.

We can further highlight that the premium level in both model frameworks (i.e., with or without the possibility to replicate further cash flows) plays no role in possible advantages from risk pooling. More precisely, the fulfillment of conditions under which the two definitions of risk pooling hold true—fixing the safety level and reduced premiums or fixing the premium and increasing the safety level for an increasing number of pool members—do not provide any information on whether pooling is actually beneficial from the policyholder perspective or not.

The paper is structured as follows. Section 2 provides a discussion of previous literature and Section 3 presents the base case with the individual and collective perspective on risk pooling claims. An analysis of the merits of pooling claims from the policyholder's perspective is conducted in Section 4, and Section 5 concludes the paper.

2. LITERATURE REVIEW

This section provides an overview of previous literature on pooling claims and summarizes their main results. In particular, as described above, two definitions of risk pooling have been extensively discussed in the literature. Both definitions are based on actuarially calculated premiums that are given by the value of expected losses plus a safety loading to achieve a given safety level for a pool of contracts. In *Case A*, diversification effects imply that the individual premium decreases with an increasing community of insureds in the pool when fixing the ruin probability. In *Case B*, the ruin probability decreases if the individual premium is fixed.

Smith and Kane (1994) present and discuss both *Cases A* and *B*. They show, using examples, that pooling is beneficial for policyholders with respect to the ruin probability of the pool. However, an evaluation of the policyholders' contract cash flows is not conducted in this context. In regard to *Case A*, William, Smith, and Young (1995, pp. 274-276), as well as Smith and Kane (1994), further clarify that pooling effects do not constitute a necessary precondition to conduct insurance business, as long as the insurance seller holds sufficient equity capital relative to the maximum loss. In this case, individual risks can also be insured without pooling.

Daykin, Pentikaïnen, and Pensonen (1994, pp. 155-170) describe *Case B*, which is also analyzed in Powers, Venezian, and Juca (2003), with a single-period ruin probability model from a regulator perspective, and show how to improve risk management programs. They derive necessary and sufficient conditions for the ruin probability to converge to zero and analyze requirements for the normal approximation for different assumptions with respect to the insurer's capital supply, the underwriting profit loading, and adverse selection.[1] In this context, they consider the case of parameter uncertainty and errors in parameter estimation and thus extend previous work by Venezian (1983), where pooling effects and safety capital are defined depending on the characteristics of uncertainty measured by the variance of the estimates, such that the insurer achieves a safety level with a given reliability. To analyze these issues, Venezian (1983) applies a normal power method and calculates the insurer's financial efficiency, which is measured by the required safety capital per insured risk (corresponding to the safety loading on the premium) in order to achieve a fixed safety level.

Case B is also analyzed in Venezian (1984), whereby risk pooling of heterogeneous (not identical but independent) risks is derived in order to increase the insurer's financial efficiency, which is beneficial by reducing capital requirements. The article further focuses on the question of fair premiums by means of price discrimination and the distribution of equity if groups of risks are not identical and pooled within one single insurer, as compared to insuring each group by a different insurer. Beard, Pentikaïnen, and Pensonen (1984) also study *Case B* and show that for a fixed ruin probability, the safety capital decreases for an increasing number of risks in the pool, thereby particularly focusing on the net retention, which in this setting is c. p. higher for larger companies. Based on Houston (1964), Cummins (1974) examines independent and identically (normally) distributed risks and a buffer fund, which serves to cover losses that exceed the expected losses and to ensure a given safety level (e.g. an acceptable ruin probability). The buffer fund per insured risk is shown to decrease for an increasing number of risks in the pool, which is also the case for non-homogenous, positively correlated risks. Cummins (1991) extends this work and – for *Case B* – demonstrates that in the case of independent and identically distributed risks, the total buffer fund goes to infinity with an increasing number of risks, while the required buffer for each individual goes to zero, which, as similarly stated by Venezian (1983, 1984), is generally considered as beneficial. In the context of tax aspects of captive insurance, Porat and Powers (1999) define both *Cases A* and *B* and clarify in respect to *Case A* that premiums and capitalization are forced by market conditions rather than being driven by the classical definitions of risk pooling.

Other work includes Heilmann (1988), where within an infinite planning horizon, a given finite reserve requires the premium to have a positive loading to avoid a ruin probability of

[1] The authors also point out that the sole consideration of the ruin probability only accounts for the underwriting profitability, thus neglecting the asset side.

100 %. The approach is based on Cramér (1955) (see also Bühlmann (1996, pp. 141 ff.) and Straub (1997, S. 37 ff.)). Furthermore, Powers (2006) discusses the empirical observation that risk pooling is not beneficial for larger companies (measured by premium volume), even though, theoretically, the premium-surplus-ratio should increase according to the law of large numbers. Powers (2006) argues that this effect might be due to disadvantages in risk selection, as growth is accompanied with an underwriting of bad risks.

Brockett (1983) emphasizes the correct use of the central limit theorem justification when calculating the ruin probability of an increasing number of independent and identically distributed risks in a pool, which represents a large deviation probability problem. In particular, the sum of independent and identically distributed risks is approximately normally distributed for a fixed large number of risks only under certain conditions for an increasing number of risks.

With respect to the benefits of risk pooling from the insurer's perspective, Diamond (1984) shows under which conditions an increase in the number of risks raises the insurer's utility, thereby also demonstrating that the risk premium per insured decreases. Furthermore, Borch (1990) demonstrates that in the case of risk pooling, the premium calculation does not influence the insurer's utility in case of μ/σ preferences. Denuit, Eeckhoudt, and Menegatti (2010) examine the benefits of risk pooling according to *Case A*, referring to Smith and Kane (1994). They assume that the insurer is Bernoulli risk-averse and, thus, requires a premium loading on the expected loss.

Consequently, previous work focuses mainly on the definitions and necessary conditions for risk pooling with respect to the insurer's safety level as well as the insurer's utility of pooling. Some papers suggest that pooling is beneficial for the insurer since the required safety capital per insured risk decreases for an increasing number of pool members. While pooling can generally be beneficial in order to satisfy regulatory requirements, for instance, this interpretation cannot be affirmed from the findings of capital market theory, as costs of capital only arise due to non-diversifiable risks, while diversifiable insurance risks are not relevant for pricing. Thus, to assess whether pooling is beneficial for stakeholders and particularly policyholders, future cash flows should be explicitly evaluated depending on assumptions on the policyholder's ability to replicate and diversify. In this context, both policyholder and shareholder positions must be taken into consideration.

Clearly, if policyholders can fully diversify and replicate their risks, a financial intermediary is not necessary and no additional value can be generated from pooling. This may differ if replication is not fully possible, thus using a preference-dependent valuation method. Furthermore, in the valuation process, the shareholder stake must be considered to ensure a fair situation (see Venezian, 1984) without arbitrage opportunities. While the kind of premium calculation scheme (pre-, post-, or mixed funding) and the amount of the premium do play a role with respect to the *definitions* of pooling (*Case*s *A* and *B*), it has no effect on the *value* of the cash flows, if policyholders receive their claims and the remainder is distributed equally among all homogenous risks (in case of a mutual insurer; similar arguments hold for a stock insurer). In summary, the two definitions of pooling do not provide a clear indication, or necessary conditions, on whether pooling (and insurance) is actually beneficial for policyholders. To answer this question, the cash flows need to be comprehensively evaluated.

3. POOLING CLAIMS: THE BASE CASE

3.1 Claims and premiums in the base case: Individual and collective perspective

As a first step, we consider the base case, where n risks (n exposure units) resulting from n policyholders are pooled in a portfolio within a specified reporting period (e.g., one year with $t = 0, 1$).[2] The number of risks n within the portfolio is deterministic. A central prerequisite in the analysis of pooling effects are the assumptions on the distribution and dependence structure of risks to assess the distribution of a sum of n risks. As it is done in this context by, e.g., Cummins (1991), we assume that the claim sizes of the n risks are independent and identically distributed (i.i.d.), following a normal distribution.

Let X_i denote the normal distributed claim size of risk i (with $i = 1,\ldots,n$) at time $t = 1$. The stochastic total claim amount S at time $t = 1$ in the pool consisting of n risks is normally distributed and given by

$$S = \sum_{i=1}^{n} X_i \ .$$

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[2] For the base case and actuarial risk pooling in general, see, e.g., Beard et al. (1984), Kaas et al. (2001), Straub (1988).

To cover the claims within the pool, the insurer collects premiums at time $t = 0$ for each risk taken. The individual premium is calculated based on an actuarial premium principle and, thus, the premium for risk i is given by the expected claim per risk and a safety loading $c > 0$:

$$\pi_i = E\left(X_i\right) + c,$$

where $E(\cdot)$ stands for the expected value. In what follows, we assume that initial contributions in the pool are compounded with a risk-free rate of $r = 0$ %. The collective premium of the pool can be calculated by

$$\pi = \sum_{i=1}^{n} E\left(X_i\right) + nc = E\left(S\right) + nc = n\pi_i,$$

which, due the assumption of identically distributed risks, corresponds to the number of risks times the individual premium.

3.2 The effect of pooling claims

When studying the effects of pooling claims for an increasing number of risks n, the choice of a suitable risk measure is crucial. In the following, we consider the ruin probability, i.e., the probability that the total premiums collected in the pool are not sufficient to cover the total claims occurred at time $t = 1$.

In general, one can distinguish two different approaches when analyzing pooling effects for an increasing number of risks in the pool. Diversification effects can either arise with a reduced premium for a given safety level of the pool (*Case A*, hereafter), or – in case premiums are fixed ex ante with $c > 0$ – with a reduction in the ruin probability within the pool (*Case B*).

Case A – Fixed ruin probability

In the first case, the effect on the individual premium necessary to ensure the desired ruin probability can be studied while requiring that the ruin probability R of the pool remains fixed at a given level $R = \varepsilon$:

$$R = P\left(S > \pi\right) \overset{!}{=} \varepsilon \quad \Leftrightarrow \quad P\left(S > E\left(S\right) + nc\left(n\right)\right) \overset{!}{=} \varepsilon .$$

Under the given assumptions, the total claim amount S in the pool follows a normal distribution and, thus, the ruin probability can be written as

$$R = 1 - N\left(\frac{E(S) + nc(n) - E(S)}{\sigma(S)}\right) = 1 - N\left(\frac{c(n)}{\sigma(X_i)} \cdot \sqrt{n}\right) \overset{!}{=} \varepsilon,$$

where N denotes the distribution function of a standard normal distribution. This is equivalent to deriving c for a given number of risks n from

$$\frac{c(n)}{\sigma(X_i)} \cdot \sqrt{n} = z_{1-\varepsilon} \quad \Leftrightarrow \quad c(n) = \frac{z_{1-\varepsilon} \cdot \sigma(X_i)}{\sqrt{n}},$$

where $z_{1-\varepsilon}$ denotes the (1–ε)-quantile of the standard normal distribution. For an increasing number of risks n in the pool, the individual premium converges towards the expected claim per risk $E(X_i)$ for a fixed ruin probability ε.

In case c is positive,[3] $\sqrt{n}$ increases with an increasing number of risks in the pool, and hence c and thus π_i can be lowered.[4] Thus, given the assumption used in this section, potential merits of pooling are often formulated in the following way: giving a constant safety level (1–ε), insurance can be provided for each pool participant at a cheaper rate if the number of participants in the pool increases.

Case B – Fixed premium

Alternatively, one can ex ante fix the individual premium with some positive safety loading $c > 0$. In this case, the ruin probability converges to zero for an increasing number of risks n in the pool:

$$R = 1 - N\left(\frac{E(S) + nc - E(S)}{\sigma(S)}\right) = 1 - N\left(\frac{c}{\sigma(X_i)} \cdot \sqrt{n}\right) \underset{n \to \infty}{\longrightarrow} 0.$$

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[3] Under the assumed normal distribution for X_i, the ruin probability ε is lower than 50% for all n.

[4] See also Cummins (1991, p. 268).

3.3 Discussion regarding *Case A* and *Case B*

Beside the discussed cases, hybrid forms of *Cases A* and *B* can be found (see, e.g., Zigenhorn, 1990). In particular, premium principles can be derived that lead to decreasing individual premiums (converging towards the expected claim per risk) for an increasing number of risks n and to a continuous improvement of the ruin probability R with $R \underset{n\to\infty}{\longrightarrow} 0$.

The pooling concept suggests that under the given assumptions, policyholders benefit from pooling claims in insurance companies. More precisely, given a fixed probability $\varepsilon < 50\%$ for the contract fulfillment, the premium decreases with an increasing number of policyholders (see *Case A*). Or, as in *Case B*, the probability of a fulfillment of the contract increases given a fixed premium (with $c > 0$) and an increasing number of risks in the portfolio.

However, in this context, the possibility that the customers may not want to purchase insurance (here: participate in a homogenous pool with $n \geq 2$) in the first place is not considered. In addition, the shareholder position is not taken into account and is, hence, not evaluated, even though the pool is solvent with probability $1-\varepsilon$. In the case of a mutual insurer, the remaining surplus is, in our case, owned by n policyholders and, hence, should be distributed to them. In the case of a stock insurer, a group that participates in the surplus without initial contribution is barely conceivable from an economic point of view, as this would imply a clear arbitrage opportunity.[5]

Furthermore, pooling effects as described above suggest that the type of premium calculation is important for pooling. For instance, the safety level c needs to be positive to achieve the pooling effects described above. However, in what follows, we compare different valuation schemes to illustrate that *merits of pooling* from the policyholders' perspective under the given assumptions do not depend on the way premiums are calculated. Even more importantly, the merits of pooling claims as defined in *Case A* and *B* do not provide any information regarding possible advantages policyholders may face when risk pooling is conducted. Thus, the following section aims to study this issue by evaluating the respective claims.

[5] See Cummins (1991, pp. 267-268).

4. AN ANALYSIS OF MERITS OF POOLING CLAIMS FROM A POLICYHOLDER'S PERSPECTIVE

4.1 The policyholder's starting point

The value of pooling from the policyholder's perspective can be derived in different ways and depends on underlying assumptions such as the policyholder's diversification opportunities. In the following, we consider the setting of a mutual insurer, where policyholders are also owners of the insurance firm.[6] The policyholder's wealth W_i, $i = 1,\ldots,n$, at time $t = 1$ can thus be described as follows:

$$W_i = A_i\left(1+r\right) - X_i - \pi_i\left(1+r\right) + I_i + E_i . \tag{1}$$

Here, A_i represents the initial capital of the policyholder i at time $t = 0$, which is compounded with the risk-free rate of return. For reasons of simplification, we again assume $r = 0$ % and, thus, omit r in what follows. The value of the investment at time 1 is reduced by the stochastic claim X_i and by the compounded premium π_i paid at time zero to insure against losses. In addition, the wealth is increased by the indemnity payment I_i given by the insurance contract as well as by the shareholder stake E_i for the surplus claim. The variables X_i, I_i, and E_i on the right hand side of Equation (1) are stochastic.

In the following, we analyze the effect of pooling on the wealth position of the policyholder. A policyholder also has the choice not to participate in pooling and, hence, not to purchase insurance. In this case, the wealth at time $t = 1$ is given by

$$W_i = A_i - X_i .$$

4.2 Considerations in a frictionless and efficient market

In a frictionless and efficient capital market, the individual is able to replicate all future cash flows by means of capital market instruments. In this setting, pooling effects may occur as described by the criteria *A* and *B* and examples laid out in Section 3. However, for the wealth position of the policyholder in a "fair" setting (in the sense of an arbitrage-free valuation), risk pooling effects – more precisely, the reduction of unsystematic risk – have no relevance. "Fair" means, in this context, that the initial insurance premium is equal to the present value

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[6] Note that the following arguments analogously hold for a stock insurer.

PV of future cash flows consisting of the stochastic indemnity payment I_i and the shareholder claim E_i:

$$\pi_i = PV\left(I_i + E_i\right). \tag{2}$$

When determining the policyholder's indemnity payments, possible default has to be taken into account by subtracting the default option value from the present value of the claims X_i to account for the cases where the insurer is not solvent and thus not able to fully cover all liabilities of the policyholders in the pool. As in Section 3, default is defined as the case where the total premium income π is not sufficient to cover the total losses S in the pool. Hence, under fair conditions, for the policyholder's indemnity payment it holds that

$$PV\left(I_i\right) = PV\left(X_i\right) - \frac{1}{n} PV\left(\max\left[S - \pi, 0\right]\right). \tag{3}$$

Thus, for every participant in the pool, the present value of the individual claims is reduced by one-*n*th of the present value of the default option in the pool.

Regarding the shareholder claim, the remaining surplus (premiums less claims) is also distributed equally to the pool members if the pool is solvent:

$$PV\left(E_i\right) = \frac{1}{n} PV\left(\max\left[\pi - S, 0\right]\right). \tag{4}$$

The requirement of a fair valuation described in Equation (2) is fulfilled, as can be seen when using the fact that $\max\left(S, \pi\right) = S + \max\left(\pi - S, 0\right) = \pi + \max\left(S - \pi, 0\right)$ to obtain

$$\begin{aligned} PV\left(I_i + E_i\right) &= PV\left(I_i\right) + PV\left(E_i\right) \\ &= PV\left(X_i\right) - \frac{1}{n} PV\left(\max\left[S - \pi, 0\right]\right) + \frac{1}{n} PV\left(\max\left[\pi - S, 0\right]\right) \\ &= PV\left(X_i\right) + \frac{1}{n} PV\left(\pi - S\right) \\ &= PV\left(X_i\right) + \pi_i - \frac{1}{n} \sum_{i=1}^{n} PV\left(X_i\right) \\ &= \pi_i. \end{aligned} \tag{5}$$

The last transformation in Equation (5) leads to the individual premium paid by all policyholders and is valid whenever the same present value calculation is used for all risks.

Equation (5) further represents the special case in which the policyholders are risk-neutral and only expected values matter. More precisely, in this case we have

$$PV\left(I_i+E_i\right)=E\left(I_i+E_i\right)=\pi_i\,. \tag{6}$$

Given Equations (5) and (6) respectively, the policyholder is indifferent as to whether he or she purchases insurance or not. In particular, the safety loading *c* (with $\pi_i = E\left(X_i\right)+c$) does not influence the wealth position of the policyholder. The policyholder stays indifferent for positive or negative values of *c*.

Reconsidering Case A – Fixed ruin probability

Since the ruin probability, *R*, of the pool is fixed to ε and the safety loading for each policyholder is positive ($c\left(n\right)$ > 0), the individual premium π_i will decrease for an increasing number of risks in the pool. Hence, the individual premium is a function of *n*. Replacing the total premiums and aggregate losses in Equation (3) leads to

$$PV\left(I_i\right)=PV\left(X_i\right)-PV\left(\max\left[\frac{1}{n}\cdot\sum_{i=1}^{n}X_i-\frac{1}{n}n\cdot\left(E\left(X_i\right)+c\left(n\right)\right),0\right]\right). \tag{7}$$

For an increasing number of i.i.d. risks (a random sample from a probability distribution *X* with finite mean and variance), the law of large numbers implies that

$$\lim_{n\to\infty}P\left(\left|\frac{1}{n}\cdot\sum_{i=1}^{n}X_i-\mu\right|<\varepsilon\right)=1$$

for $E\left(X_i\right)=\mu$, *i* = 1,…,*n* (see, e.g., Cummins, 1991). Since *c* depends on *n* in *Case A* and is decreasing to zero for large *n*, the second part of the right hand side of Equation (7) – i.e., the policyholder's part of default put option value of the pool – converges to zero. The present value of the indemnity payment will reach the present value of the individual claim as $n\to\infty$.

Analogously, the present value of the shareholder claim,

$$PV\left(E_i\right)=PV\left(\max\left[\frac{1}{n}n\cdot\left(E\left(X_i\right)+c\left(n\right)\right)-\frac{1}{n}\cdot\sum_{i=1}^{n}X_i,0\right]\right), \tag{8}$$

converges to the loading $c(n)$, which in turn decreases to zero as the number of risks in the pool increases for a given ruin probability. Therefore, in case of definition *A*, the present value of the indemnity payment converges to the present value of the individual claim and the present value of the shareholder claim becomes worthless for the policyholder for large *n*. Hence, for large *n*, the wealth position of the policyholders in the pool becomes risk-free.

To illustrate this theoretical observation, Table 1 contains a numerical example. Let $\pi_i = E(X_i) + c(n)$ denote the premium for each pool member. The safety level of the pool is fixed to 99 % (hence, $R = P(S > \pi) = 1\%$). Claims are independent and normally distributed with $E(X_i)$ = 30 and $\sigma(X_i)$ = 10, r = 0, and we assume a risk-neutral market; hence, $PV(\cdot) = E(\cdot)$.

Table 1: Premiums π_i and present values of payouts $PV(I_i)$ and $PV(E_i)$ for pooling claims for a given ruin probability of 1 % (*Case A – fixed ruin probability*)

n	1	10	50	100	1000	10000
π_i	53.2635	37.3566	33.2900	32.3263	30.7357	30.2326
$c(n)$	23.2635	7.3566	3.2900	2.3263	0.7357	0.2326
$PV(I_i)$	29.9661	29.9893	29.9952	29.9966	29.9990	29.9996
$PV(E_i)$	23.2974	7.3673	3.2947	2.3297	0.7367	0.2330
$PV(I_i + E_i)$	53.2635	37.3566	33.2900	32.3263	30.7357	30.2326

Table 1 shows that the premium per participant paid into the pool decreases for an increasing number of participants *n*.[7] With $c(n) > 0$, the pooling effect of *Case A* described in Section 3 is fulfilled. Nevertheless, no additional value is created for the policyholders through an increasing number of pool participants since $\pi_i = PV(I_i + E_i)$ holds true for any value of *n*.

Reconsidering Case B – Fixed premium

In *Case B*, the premium – and thus *c* – is fixed and does not depend on *n*. Hence, for an increasing number of risks, the default option value in Equation (7) converges to zero, since the loading *c* is assumed to be positive. In addition, the value of the shareholder claim converges to *c*.

[7] See also Cummins (1991) for a similar example.

Table 2 gives an illustration for a fixed loading of c = 0.5 for each policyholder. As in Table 1, claims are independent and normally distributed with $E(X_i)$ = 30 and $\sigma(X_i)$ = 10. The risk-free rate of return r is again set to zero and a risk-neutral market is assumed.

Table 2: Ruin probability of the pool R, present values of payouts $PV(I_i)\, PV(E_i)$ for the case of pooling claims for a given safety loading c = 0.5 (*Case B – fixed premium*)

n	1	10	50	100	1000	10000
π_i	30.5000	30.5000	30.5000	30.5000	30.5000	30.5000
c	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
$R = P(S > \pi)$	48.0061 %	43.7184 %	36.1837 %	30.8538 %	5.6923 %	0.0000 %
$PV(I_i)$	26.2556	28.9727	29.6509	29.8022	29.9923	30.0000
$PV(E_i)$	4.2444	1.5273	0.8491	0.6978	0.5077	5.0000
$PV(I_i + E_i)$	30.5000	30.5000	30.5000	30.5000	30.5000	30.5000

Comparison of Cases A and B

In both *Cases A* and *B*, the present value of the policyholder's total wealth at time t = 1, $PV(W_i)$, remains unchanged for all premium levels as long as $\pi_i = PV(I_i + E_i)$ holds true for all policyholders in the pool. This results from Equation (1), where $PV(\pi_i(1+r)) = \pi_i$ cancels $\pi_i = PV(I_i + E_i)$ out, and A and X remain unchanged. The wealth position of a policyholder could only be improved if the premium is below the present value of future payouts, $\pi_i < PV(I_i + E_i)\left(\triangleq E(X_i) + c\right)$. However, such a premium principle for some policyholders would be a disadvantage for other policyholders in the pool.

Since under this valuation principle, individuals can perfectly diversify and replicate future cash flows, pooling, or, more precisely, diversification of unsystematic risk, does not offer any additional benefit. Clearly, no additional value can be generated by means of pooling and thus, no reason for the existence of insurance institutions can be established in such a context.

In particular, it is not relevant for policyholders whether pooling effects as defined in *Case A* or *Case B* exist under the assumed setting. For instance, a calculation of a premium according to $\pi_i = E(X_i) + c$ with c < 0 is still fair from the policyholder's perspective as long as $\pi_i = PV(I_i + E_i)$ holds (see Equation (5)), even though it does not lead to a pooling effect as described in *Case B*. This is illustrated in a third numerical example provided in Table 3 with the input data from Table 2 and a safety loading of c = –1.

Table 3: Premiums π_i and payouts ($I_i + E_i$) for the case of pooling claims for a fixed premium level per of π_i = 29.00 (*Case B – fixed premium*)

n	1	10	50	100	1000	10000
π_i	29.0000	29.0000	29.0000	29.0000	29.0000	29.0000
c	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
$R = P(S > \pi)$	53.9828 %	62.4085 %	76.0250 %	84.1345 %	99.9217 %	100.0000 %
$PV(I_i)$	25.4906	28.1759	28.8004	28.9167	28.9999	29.0000
$PV(E_i)$	4.5094	0.8241	0.1996	0.0833	0.0001	0.0000
$PV(I_i + E_i)$	29.0000	29.0000	29.0000	29.0000	29.0000	29.0000

Because the safety loading c is not positive, pooling effects according to *Case B* as defined in Section 3 do not exist and the insurance company becomes insolvent with certainty for large n.[8] However, a policyholder using the present value is still indifferent with respect to a participation in the pool. Clearly, the level of c does not play a role as long as the conditions are the same for all participants and hence, all payoffs after paying the claims are homogenously distributed to the policyholders in t = 1.

4.3 The case of risk-averse policyholders

In contrast to the previous section, in what follows, we assume an incomplete market setting in which policyholders are not able to replicate future cash flows with given market instruments. Hence, a preference-dependent valuation is required.

Preference-dependent valuation

In the following, we assume that the policyholder has μ/σ-preferences. The preference function Φ of the policyholder's wealth position W_i at time t = 1 can be written as

$$\Phi = E(W_i) - \frac{a}{2} \cdot \sigma^2(W_i),$$

with a > 0 denoting the risk aversion parameter. In the following, we can see that in this setting, it is not necessary to distinguish between the risk pooling definitions according to *Case A* and *Case B.*

8 See also Bühlmann (1996) and his reasoning of a positive safety loading.

As can see from Equations (5) and (6), the premium principle $\pi_i = E(X_i) + c$ $(c \in \mathbb{R})$ implies $\pi_i = E(I_i + E_i)$.[9] Hence, the expected wealth in t = 1 of the policyholder depends neither on the value of the safety loading c nor on the number of the pooling participants:

$$E(W_i) = A_i - E(X_i) - \pi_i + E(I_i + E_i) = A_i - E(X_i).$$

Thus, the expected wealth of the policyholder is not influenced by the purchase (or not) of insurance. Hence, for the question of whether the utility level Φ can be increased via risk pooling, an analysis of $\sigma^2(W_i)$ is sufficient.

If no insurance is purchased, $\sigma^2(W_i) = \sigma^2(X_i)$. If risk pooling (and insurance) is chosen, one obtains

$$\sigma^2(W_i) = \sigma^2(-X_i + I_i + E_i).$$

In any case, $\sigma^2(W_i)$ does depend on the number of pool members but not on the premium principle. This can be shown by using the result in Equation (5). Since we have $I_i + E_i = X_i + \frac{1}{n}\cdot(\pi - S)$, the variance of the wealth of the policyholder in t = 1 can be written as

$$\begin{aligned} \sigma^2(W_i) &= \sigma^2(-X_i + I_i + E_i) = \sigma^2\left(-X_i + X_i + \frac{1}{n}(\pi - S)\right) \\ &= \sigma^2\left(\frac{1}{n}(\pi - S)\right) = \sigma^2\left(\frac{1}{n}\left(E(S) + cn - \sum_{i=1}^{n} X_i\right)\right) \\ &= \sigma^2\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2(X_i) \\ &= \frac{1}{n}\sigma^2(X_i). \end{aligned} \tag{10}$$

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9 This does not only hold true in case of a risk-neutral policyholder, as the premium can be rewritten using the risk adjustment R_{adj} (that is not influenced by changes in c) as follows:

$PV(I_i + E_i) \overset{!}{=} \pi_i = E(X_i) + c \Leftrightarrow E(I_i + E_i) + R_{adj} = E(X_i) + c$

$\Leftrightarrow E(I_i + E_i) = E(X_i) + c - R_{adj} = E(X_i) + c^*$.

For instance, using the Capital Asset Pricing Model, the risk adjustment R_{adj} is given by (λ stands for the market price of risk, M_1 denotes the value of the market portfolio in t = 1)

$R_{adj} = \lambda \operatorname{cov}(I_i + E_i, M_1)$.

A change in the safety loading c will change c^* to the same amount; however, R_{adj} is not affected.

Whenever risk pooling is conducted, it holds that $\sigma^2\left(-X_i + I_i + E_i\right) < \sigma^2\left(X_i\right)$ due to naïve diversification (for $n \geq 2$). For large *n*, $\sigma^2\left(W_i\right)$ converges to zero, and the wealth position of the policyholder becomes deterministic.[10]

Thus, the utility level can be increased with an increasing number of pool participants for risk-averse policyholders. Again, the existence of risk pooling effects according to *Case A* and *Case B* as defined in Section 3 do not provide a hint with regard to possible merits of pooling, since the way premiums are defined do not play a role, *ceteris paribus*, with respect to the policyholder's utility Φ.

The following example illustrates this point. We employ the same input data as used in the numerical example before (see Tables 1 to 3, $E\left(X_i\right)$ = 30, $\sigma\left(X_i\right)$ = 10, *r* = 0) and set the policyholder's risk aversion parameter to *a* = 2. The initial wealth of the policyholders is given by *A* = 500.

Table 4: The policyholder's utility Φ depending on the number of risks *n* in the pool

n	1	10	50	100	1000	10000
Φ	270.0000	450.0000	466.0000	468.0000	469.8000	469.9800

Table 4 shows that for large *n*, the individual policyholder's wealth position in *t* = 1 almost becomes risk-free and converges to the maximum possible utility of 470 $\left(= A_i - E\left(X_i\right)\right)$. The utility is not influenced by the premium payments π_i and even remains unchanged in the extreme case where no upfront premiums payments are required ($\pi_i = 0$ with $c = -E\left(X_i\right) = -30$). For positive values of *c*, the requirements of *Case B* can be fulfilled; however, this has no relevance from the perspective of the policyholder in this context.

In summary, it can be shown that at least in the model frameworks considered, the classical definition of merits of pooling is neither a sufficient nor a necessary condition for the existence of positive effects via risk pooling for the policyholder.

[10] Borch (1990, p. 85) shows in a reinsurance context that, *ceteris paribus*, a change in the premium level does not influence the utility of an insurance firm pooling risks.

5. SUMMARY

In this paper, we studied two specific definitions of risk pooling that are extensively examined in the literature. Firstly, we considered the case where, when fixing the safety level using the ruin probability, actuarially calculated premiums can be reduced for an increasing number of pool members, and, secondly, the case where the premium level is fixed and the ruin probability goes to zero with an increasing pool size. Both definitions imply a seeming benefit of risk pooling for the policyholder, which can be misleading. Therefore, in this paper, we revisit the merits of pooling by focusing on the policyholder's perspective in the case of a mutual insurer using different valuation approaches, thereby also taking into account both stakes of the policyholder (shareholder and debt holder position). The fundamental difference in the valuation approaches is their assumption of an individual policyholder's ability to replicate the contract's future cash flows.

We point out that if policyholders can fully replicate cash flows, the value of their claims positions can be evaluated using a present value approach in which the decreasing premium for an increasing pool size can be separated in a decreasing present value of the shareholder claim and the present value of the indemnity payment, which decreases towards the present value of the loss. Overall, however, the value of equity and debt positions always sums up to the initial contribution by the policyholder and only the partition between shareholder and debt holder's claims is altered. Furthermore, the premium level – for each participant to be paid up-front in the pool – does not play a role for the policyholder's wealth position. Hence, in this valuation framework, no additional value is generated through diversification on the company's level as it can be achieved equivalently by policyholders on an individual level.

However, pooling does imply additional value from the policyholder's perspective if replication or diversification is not achievable for policyholders as it is for the insurance company and valuation can be conducted based on utility functions. In this case, pooling is beneficial for risk-averse policyholders since the expected wealth remains unchanged, no matter whether insurance is purchased or not, while the risk decreases for an increasing number of pool members. Again, the premium level and thus the definitions of pooling *A* and *B* do not play a role, i.e. in this context, the policyholder's utility level is not influenced by the amount of premiums paid upfront in the pool as long as homogenous risks are treated identically.

In summary, the paper shows that the two considered definitions of risk pooling cannot provide insight regarding possible benefits for policyholders and that the merits of pooling need to be analyzed under different assumptions on policyholders' ability of replicating claims. Following this line of reasoning, it is not per se clear for which economically relevant question with respect to insurer/policyholder decision making the classical definitions of risk pooling allow a clear answer. The central reason for the conclusion that the classical definitions of risk pooling do not provide information whether pooling is beneficial is that the premium level and, thus, the premium's safety loading is not only irrelevant for the utility of pool members in the specific case of μ/σ preferences, as shown by Borch (1990), but in the case of all frameworks considered in this paper. Rather, premiums and capital structure can generally be considered to be driven by market forces (Porat and Powers, 1999). Furthermore, in the case of present values, the policyholder is indifferent as regards the insolvency level of the insurer, as long as the premium paid corresponds to the present value of future payoffs. However, the reduction of insolvency risk by means of pooling can be beneficial in other contexts, as an increase in the safety level can help insurers satisfy regulatory requirements (see Powers, Venezian, and Juca (2004), Venezian (1984)).

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