FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

RECHTS- UND WIRTSCHAFTS-
WISSENSCHAFTLICHE FAKULTÄT

On the Relevance of Premium Payment Schemes for the Performance of Mutual Funds with Investment Guarantees

Nadine Gatzert

Working Paper

Department of Insurance Economics and Risk Management
Friedrich-Alexander-University of Erlangen-Nürnberg

Version: August 2013

ON THE RELEVANCE OF PREMIUM PAYMENT SCHEMES FOR THE PERFORMANCE OF MUTUAL FUNDS WITH INVESTMENT GUARANTEES

Nadine Gatzert [*]

JEL Classification: D81, G11, G13, G22

ABSTRACT

In financial planning, customers are typically confronted with choosing a premium payment scheme when investing in a mutual fund, which is often equipped with an investment guarantee to provide downside protection. Guarantee costs may thereby also be charged differently depending on the provider. In this paper, we investigate the impact of the premium payment method on different performance measures for a mutual fund with an investment guarantee. We compare a fund with annual and upfront premiums as well as constant guarantee costs versus the guarantee price as an annual percentage fee of the fund value, always ensuring that the present value of premium payments is the same for all product variants. We further study the relevance of the guarantee level and the contract term. Our results emphasize that even though the present value of premiums paid into the contract is the same, the type of premium (upfront versus annual) as well as the type of guarantee cost (upfront versus annual fee) have a considerable impact on the performance. Providers can thus make a product more attractive for consumers by individually adjusting the premium scheme depending on their preferences and by making the resulting risk-return-profile transparent, while keeping the other contract characteristics unchanged (e.g. extent of the guarantee).

Keywords: Investment guarantees, mutual fund, risk-return profiles, guarantee costs, performance measures

1. INTRODUCTION

In recent years, mutual funds and unit-linked life insurance products with investment guarantees have become increasingly important in the banking and insurance industry, especially against the

[*] Nadine Gatzert is at the Friedrich-Alexander-University (FAU) Erlangen-Nürnberg, Chair for Insurance Economics and Risk Management, Lange Gasse 20, 90403 Nuremberg, Germany. Email: nadine.gatzert@fau.de, Tel. +49 911 5302 884. The author would like to thank an anonymous referee and the editor for valuable comments on an earlier version of the paper.

background of the demographic development and old-age provisions.[1] The maturity payoff of these products mainly depends on the riskiness of the underlying fund and the type of guarantee included and has been subject to considerable research in the past. In particular, risk-return profiles of different products are regularly compared with the intention to provide information for customers when making financial decision. In this paper, we extend previous work by analyzing the relevance of the premium payment method for otherwise fixed contract characteristics and show how this can substantially impact the terminal payoff distribution, which is important for customers with different risk-return preferences.

Asset guarantees in unit-linked life insurance products were first analyzed in Brennan and Schwartz (1976) and Boyle and Schwartz (1977). In the context of equity-indexed annuities and thus perpetual American options, Gerber and Shiu (2003b) focus on dynamic fund protection, while Lachance and Mitchell (2003) and Kling, Russ, and Schmeiser (2006) examine the pricing of interest-rate guarantees in government-subsidized pension products in a Black-Scholes framework. Boyle and Tian (2009) derive optimal parameters of equity-indexed annuities based on the maximization of an investor's expected utility. With focus on the pricing and performance of mutual funds with guarantees, Gatzert and Schmeiser (2009) compare the risk and return profiles of mutual funds with a lookback guarantee and an interest-rate guarantee and further show that the underlying fund's investment strategy has a considerable impact on the results by contrasting a conventional fund with a constant proportion portfolio insurance managed fund. Graf, Kling, and Russ (2011) also derive risk-return profiles based on the terminal payoff distributions for different unit- and equity-linked products with and without guarantees, thereby also comparing the case of annual and single premiums. They point out that this approach is preferable to sample illustrations and historical backtesting when providing information for potential customers. Gatzert, Huber, and Schmeiser (2010) take a behavioral insurance perspective and examine customer's willingness to pay for investment guarantees in unit-linked life insurance product based on an empirical survey, while Huber, Gatzert, and Schmeiser (2011) investigate in an experimental study whether different forms of price presentations will influence consumers' choice to purchase an investment guarantee in a unit-linked life insurance contract. Their results indicate that – contrary to typical consumer goods – different price presentation

[1] For instance, the share of unit-linked products in total European life insurance premium volume increased from 36% in 2009 (drop from 42% following the financial crisis) to about 40% in 2010 (CEA, 2012, p. 3).

does not show statistically significant effects, but that other factors such as consumers' experience with insurance or investment products or consumers' price perception of the product were significant predictors for explaining the relationship between the price presentation and consumer evaluation. However, in their analysis, focus is not laid on presenting or analyzing risk-return profiles of these product variants.

Hence, in this paper we extend previous work by specifically focusing on the impact of different premium payment schemes with respect to savings premiums and guarantee costs[2] on risk and return of a mutual fund with *otherwise given* contract characteristics such as the underlying fund strategy and the investment guarantee, as the premium scheme itself can already have a considerable impact on the terminal payoff distribution and thus risk-return profiles. In addition, such an analysis can provide important information for consumers and providers in designing and choosing attractive products by simply adjusting the premium scheme (if possible) instead of or in addition to changing other product features. Therefore, the aim of this paper is to investigate this issue in more depth by comparing the impact of annual versus upfront savings premiums invested in the mutual fund as well as the effect of constant upfront guarantee costs that have to be paid in addition to the savings premium versus the guarantee price as an annual percentage fee subtracted from the fund value, which is often offered in practice. To ensure comparability between the different cases considered, all product variants have identical present values of premium payments and the same input parameters.

In a simulation analysis, we first calibrate guarantee levels to imply the same costs for the different premium payment strategies. Second, the performance of the product variants with different premium payment strategies is contrasted, measured with the Sharpe ratio, the Omega, and the Sortino ratio. This way, insight for providers and customers with different risk-return preferences regarding the terminal payoff distribution is provided. We further investigate the impact of the guarantee level and the contract term. The analysis shows that even though the value of premiums paid into the contract is the same for all product variations, the type of savings premium (upfront versus annual) as well as the guarantee cost presentations (upfront versus annual fee) already has a considerable impact on the performance.

[2] The notion "savings premium" refers to the part of the total premium paid by the customer invested in the mutual fund, whereas the "guarantee costs" refer to the part of the total premium that has to be paid in addition to the "savings premium" and is used by the provider to secure the guarantee.

Thus, in contrast to, e.g., Gatzert and Schmeiser (2009) or Graf, Kling, and Russ (2011), we specifically focus on the premium payment method not only with respect to the savings premium, but also in regard to guarantee costs and thereby ensure comparability by fixing the present value of premium payments. We further look at performance measures that account for different risk-return preferences. Our results emphasize that in regard to risk-return profiles and customer preferences, it is not only the underlying fund strategy or the type of guarantee included that has an impact on the terminal payoff distribution, but that for given contract characteristics, strong differences in performance and risk already arise only due to the type of premium payment. This is true even if the present value of premiums and value of the guarantee from the provider's perspective remains unchanged. Therefore, the premium payment scheme may make a considerable difference in the attractiveness of the product from a customer's perspective, and providers may increase the attractiveness of a product by individually adjusting the type of premium payment for each consumer depending on his or her risk-return and premium payment preferences.

The remainder of the paper is organized as follows. Section 2 introduces the model of a mutual fund product with investment guarantee including pricing and performance measurement. Numerical results are presented in Section 3 and Section 4 summarizes the main results.

2. MODELING THE MUTUAL FUND WITH INVESTMENT GUARANTEES

The underlying mutual fund

In regard to the underlying mutual fund, we model the unit price S_t at time t assuming a geometric Brownian motion (GBM).[3] We only focus on the financial part and do not consider early surrender or death, whereby the latter can be insured against by purchasing an additional term life insurance. The geometric Brownian motion can be described by the following stochastic differential equation under the objective measure $\mathbb{P}$,

$$dS_t = S_t\left(\mu dt + \sigma dW_t\right),$$

---

3 As an alternative, the Heston (1993) model with stochastic variance can be implemented, for instance.

with $S_0 = S(0)$, a constant drift μ, volatility σ, and a standard $\mathbb{P}$-Brownian motion (W_t) with $0 \leq t \leq T$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $(\mathcal{F}_t)$, $0 \leq t \leq T$, denotes the filtration generated by the Brownian motion. The solution of the stochastic differential equation is given by (see, e.g., Björk, 2004)

$$S_t = S_{t-1} \cdot e^{\left(\mu-\sigma^2/2\right)+\sigma\left(W_t-W_{t-1}\right)} = S_{t-1} \cdot e^{\left(\mu-\sigma^2/2\right)+\sigma Z_t} = S_{t-1} \cdot R_t,$$

where Z_t are independent standard normally distributed random variables. Hence, the continuous one-period return $r_t = \ln\left(R_t\right)$ is normally distributed with an expected value of $m = \mu - \sigma^2/2$ and standard deviation σ.

Assumptions on savings premium payments

In the following analysis, we distinguish the type of premium paid into the mutual fund in the beginning of each year, and start with a given constant annual savings premium P_S^{annual}. We then compare this case to a single upfront savings premium $P_S^{upfront}$ by calculating the sum of premium payments, discounted with the riskless interest rate r,

$$P_S^{upfront} = \sum_{t=0}^{T-1} e^{-r \cdot t} \cdot P_S^{annual},$$

which is hence equivalent to the annual premium payments in terms of the present value. The calculation thus assumes that both, the annual and the upfront premium scheme, imply the same maturity payoff if they are invested risk-free, i.e. if premiums are compounded with the riskless interest rate until maturity T, which is then compared to the case when investing the capital in the risky mutual fund. In particular, depending on the type of premium payment, the total fund value develops according to

$$F_t^i = \left(F_{t-1}^i + P_{S,t-1}^i\right) \cdot \frac{S_t}{S_{t-1}}, i = annual, upfront, \tag{1}$$

where $P_{S,t}^{annual} = P_S^{annual}, t = 0, \ldots, T-1$, and $P_{S,0}^{upfront} = P_S^{upfront}, P_{S,t}^{upfront} = 0, t \geq 1$. Thus, at time T, the fund value amounts to

$$F_T^{annual} = P_S^{annual} \cdot \sum_{t=0}^{T-1} \frac{S_T}{S_t} \quad \text{and} \quad F_T^{upfront} = P_S^{upfront} \cdot \frac{S_T}{S_0}.$$

The terminal fund value depends on the development over time and on future conditions in the financial market, but also on the type of premium payment method and assumptions with respect to the capital market model. Thus, the terminal value of the investment can fall below a critical value (e.g., the sum of gross premium payments).

Introducing and evaluating an investment guarantee

To prevent such a default situation for the customer, mutual funds and also unit-linked life insurance contracts are often equipped with a guarantee providing a minimum payoff G_T of the investment at maturity T (see, e.g., Gatzert and Schmeiser, 2009). In the presence of an investment guarantee, the customer's terminal payoff L_T consists of the maximum of the value of the investment in the underlying fund and the fixed guaranteed payment G_T, i.e.,

$$L_T^i = \max\left(F_T^i, G_T^i\right) = F_T^i + \max\left(G_T^i - F_T^i, 0\right),\, i = annual, upfront\ savings\,, \tag{2}$$

and can thus be written as the value of the underlying assets plus a put option on this value with strike price G_T^i. Without guarantee, the terminal payoff is given by $L_T^i = F_T^i$.

To evaluate the guarantee, we use risk-neutral valuation. Under the risk-neutral pricing measure $\mathbb{Q}$ (see Harrison and Kreps, 1979), the drift of the unit price process changes to the riskless rate of return r. In case of the geometric Brownian motion, the process is thus given by

$$dS_t = S_t\left(r dt + \sigma_S dW_t^{\mathbb{Q}}\right),$$

where $W^{\mathbb{Q}}$ is a standard $\mathbb{Q}$-Brownian motion.

The value of the investment guarantee at time $t = 0$ is then given by the expected value of the payoffs under the risk-neutral measure $\mathbb{Q}$, discounted with the riskless interest rate r. The cost of the investment guarantee is the price of a European put option on the mutual fund value at maturity with strike price G_T (see Equation (2)). Thus, the single upfront premium for the guarantee $P_{G,t=0}^{i}$, which has to be paid in addition to the savings premium at time 0, is given by

$$P_{G,t=0}^{i} = E^{\mathbb{Q}}\left(e^{-rT} \cdot \max\left(G_T^i - F_T^i, 0\right)\right), i = upfront, annual\ savings \tag{3}$$

and depends on the type of savings premium. Closed-form solutions can be derived in special cases (e.g., single upfront savings premium and geometric Brownian motion). In general and due to path-dependencies, the value of the guarantee at time zero must typically be derived using, e.g., numerical approximations or simulation techniques.

Alternatively, guarantee costs can be charged by means of an annual percentage fee α of the fund value at the end of each contract year (for the following see also Huber, Gatzert, and Schmeiser, 2011). To make the cases of a constant upfront guarantee cost and the annual percentage fee comparable, the same total annual premium (and upfront premium, respectively) is assumed to be paid by the customer as in the case where guarantee costs are charged separately and in addition to the savings premium. Thus, in case of upfront savings premiums, we set the new upfront savings premium to be invested in the mutual fund to

$$P_S^{upfront,\alpha} = P_S^{upfront} + P_{G,t=0}^{upfront}.$$

In the case of annual savings premiums, we calculate the annual guarantee costs based on the single upfront guarantee costs $P_{G,t=0}^{annnual}$ in Equation (3) as

$$\tilde{P}_G^{annual} = P_{G,t=0}^{annual} / \sum_{t=0}^{T-1} e^{-r\cdot t}.$$

The adjusted annual savings premium to be invested in the mutual fund is then given by

$$P_S^{annual,\alpha} = P_S^{annual} + \tilde{P}_G^{annual}.$$

This way, the annual premium payments for the contract with guarantee when subtracting a percentage fee are the same as when using constant guarantee premiums (which in contrast to the former case are not invested in the mutual fund but paid in addition to the savings premium, such that the provider can purchase adequate risk management instruments).

To calculate and calibrate the annual percentage fee for the guarantee costs, let $F_{t,-}^{i,\alpha}$ denote the value of the investment fund for i = upfront and annual savings premiums at the end of the t-th year before subtracting the fee and $F_{t,+}^{i,\alpha}$ the value of the investment fund after subtracting the fee, i.e.,

$$F_{t,+}^{i,\alpha} = F_{t,-}^{i,\alpha} \cdot \left(1-\alpha^{i}\right),\ t = 1,...,T.$$

Thus, in case of annual premium payments, for instance, the development of the fund is described analogously to Equation (1) by

$$F_{t,-}^{annual,\alpha} = \left(F_{t-1,+}^{annual,\alpha} + P_S^{annual,\alpha}\right) \cdot \frac{S_t}{S_{t-1}} = \left(F_{t-1,-}^{annual,\alpha} \cdot \left(1-\alpha^{annual}\right) + P_S^{annual,\alpha}\right) \cdot \frac{S_t}{S_{t-1}},\ F_{0,+}^{annual,\alpha} = 0,$$

and analogously for upfront savings. Due to the annual subtraction of the percentage fee, the fund value is reduced, which in turn has an impact on the value of the investment guarantee (still fixed at G_T). From the insurer's perspective, α has to be calibrated such that the value of the fee income

$$I_G^{i,\alpha} = E^{\mathbb{Q}}\left(\sum_{t=1}^{T} \alpha^{i} \cdot F_{t,-}^{i,\alpha} \cdot e^{-r \cdot t}\right) = \sum_{t=1}^{T} \alpha^{i} \cdot e^{-r \cdot t} \cdot E^{\mathbb{Q}}\left(F_{t,-}^{i,\alpha}\right),\ i = upfront, annual\ savings$$

equals the actual value of the guarantee at time t = 0, i.e.,

$$P_{G,t=0}^{i,\alpha} = E^{\mathbb{Q}}\left(e^{-r \cdot T} \cdot \max\left(G_T^{i} - F_{T,+}^{i,\alpha}, 0\right)\right),$$

such that

$$P_G^{i,\alpha} \overset{!}{=} I_G^{i,\alpha},\ i = upfront, annual$$

holds for the calibrated value of α (see Huber, Gatzert, and Schmeiser, 2011). In summary, for both guarantee cost payment methods (absolute and percentage fee), the customer pays the same total premium. This way, it is ensured that only the *price presentation* differs. Table 1 provides an overview of the different premium payment methods for savings and guarantee costs that are compared in the following analysis.

Table 1: Overview of considered premium payment methods for savings premium and guarantee costs in case of a mutual fund with or without guarantee

	Annual savings		*Upfront savings*	
	Savings premium	*Guarantee costs*	*Savings premium*	*Guarantee costs*
Without guarantee	P_S^{annual}	0	$P_S^{upfront} = \sum_{t=0}^{T-1} e^{-r\cdot t} P_S^{annual}$	0
With guarantee G_T and constant guarantee costs • Guarantee costs have to be paid at t = 0 in addition to the savings premium and are not invested in the fund (i.e. used to purchase risk management instruments)	P_S^{annual}	$P_{G,t=0}^{annual}$	$P_S^{upfront}$	$P_{G,t=0}^{upfront}$
With guarantee G_T and annual percentage fee α instead of constant guarantee cost • Ensure comparability: assume that total premium payment is the same as in the case of constant guarantee costs • Increases the savings premium (invested in the fund)	$P_S^{annual,\alpha}$ $= P_S^{annual} + \tilde{P}_G^{annual}$ with $\tilde{P}_G^{annual} =$ $P_{G,t=0}^{annual} / \sum_{t=0}^{T-1} e^{-r\cdot t}$	0, but α^{annual}	$P_S^{upfront,\alpha}$ $= P_S^{upfront} + P_{G,t=0}^{upfront}$	0, but $\alpha^{upfront}$

Measuring shortfall risk and performance

To assess the effect of the different premium calculation schemes as laid out in Table 1, the shortfall probability (defined as the probability that the fund value at maturity falls below the guarantee level) and the performance is calculated, where the former is defined as

$$SP^i = P\left(F_T^i < G_T^i\right), i = upfront, annual.$$

The performance can be assessed based on risk-return models of the payoff distribution at maturity, L_T, which depends on whether a guarantee is included or not (in the latter case, the final payoff simply corresponds to the fund value at maturity, $L_T^i = F_T^i, i = upfront, annual$) as well as on the premium payment method.

To analyze the maturity payoff L_T (the superscript i is omitted in the following for simplification), we follow Gatzert and Schmeiser (2009) and calculate its expected value $E\left(L_T\right)$ and standard deviation $\sigma\left(L_T\right)$ under the objective measure $\mathbb{P}$. Furthermore, these figures can be used for performance measurement by way of a version of the Sharpe ratio (see Sharpe, 1966), which relates risk and return and in the following is defined as the difference between the contract's expected payoff $E\left(L_T\right)$ and the value of the premium payments compounded to maturity

$$B_T \left(= P \cdot \sum_{t=0}^{T-1} e^{r(T-t)} \right),$$

divided by the standard deviation of the maturity payoff $\sigma\left(L_T\right)$:

$$Sharpe\ ratio\left(L_T\right) = \frac{E\left(L_T\right) - B_T}{\sigma\left(L_T\right)}.$$

In addition to the Sharpe ratio, the Omega and the Sortino ratio are employed using the same assumptions. In this case, the relevant risk measures are lower partial moments, which belong to the class of downside-risk measures that describe the lower part of a density function. Thus, only negative deviations are taken into account (see, for example, Fishburn (1977), Sortino and van der Meer (1991)). The lower partial moment of order k is given as

$$LPM_k\left(L_T, B_T\right) = E\left(\max\left(B_T - L_T, 0\right)^k\right).$$

For decision making, the degree of risk aversion can be controlled by varying the power k, where in general, k = 0, 1, 2 are consistent with maximization of expected utility for investment decisions, where an increasing power k in principle represents a higher risk aversion. The form of utility functions and assumptions that makes a decision based on the Sharpe ratio, Omega or the Sortino ratio consistent with the concept of expected utility maximization is discussed in, e.g., Fishburn (1977), Sarin and Weber (1993), and Farinelli and Tibiletti (2008). For k = 0, only the number of shortfall occurrences is counted; for k = 1, all deviations are weighted equally. Hence, the Omega (see Shadwick and Keating, 2002) and the Sortino measures (see Sortino and van der Meer, 1991) can be obtained by

$$Omega\left(L_T\right)=\frac{E\left(\max\left(L_T-B_T,0\right)\right)}{LPM_1\left(L_T,B_T\right)},$$

$$Sortino\,ratio\left(L_T\right)=\frac{E\left(\max\left(L_T-B_T,0\right)\right)}{\sqrt{LPM_2\left(L_T,B_T\right)}}.$$

Thus, in the following, the Sortino ratio is considered as an example for a performance measure for a more risk-averse decision-maker, followed by the Omega, while the Sharpe ratio represents results for a less risk-averse customer.

3. NUMERICAL ANALYSIS

Input parameters

The following numerical analysis is intended to provide insight with respect to central effects of premium payment methods (regarding savings and guarantee costs) of mutual funds on the terminal payoff distribution and performance from the customer perspective. Therefore, the following input parameters were chosen for illustration purposes and were subject to sensitivity analyses.

The contract duration is set to T = 10 years and the annual savings premium is P = 120. Regarding the evolution of the underlying mutual fund, as a starting point, we use estimates as in Bohnert and Gatzert (2011) that are based on the historical performance of two representative German total return indices from 1988 until 2009. The estimation for a portfolio of stocks is based on monthly data for the German stock market index DAX and results in an expected one-period return $m = \mu - 0.5\sigma^2 = 8.00\%$ and a volatility $\sigma = 21.95\%$. The riskless rate of return is set to 2.5%. To generate the geometric Brownian motion, Monte-Carlo simulation with 500,000 paths is used. The same set of random numbers is used in the respective analyses. Results were examined with respect to robustness for different sets of random numbers.

The base case

We first consider a portfolio with a 100% investment in stocks. Results for the terminal payoff distribution including descriptive statistics and performance measures are displayed in Table 2 for annual and upfront savings premiums as well as different assumptions with respect to the guarantee costs (see Table 1 for an overview of the premium payment schemes). To ensure comparability, the guarantee level G_T in case of the annual and upfront savings premium is calibrated to obtain the same guarantee costs according to Equation (3) (see also Gatzert and Schmeiser, 2009). For G_T = 1,200 in case of annual premiums, which corresponds to the minimum payment of the sum of savings premiums paid into the contract, the upfront guarantee cost results in $P_{G,t=0}^{annual} = 112$. To achieve the same cost in case of upfront premiums, G_T must be calibrated to 950.

This procedure implies that the value of compounded premium payments at maturity is the same in all cases with guarantee with B_T = 1,525 (assuming a riskless interest rate as stated above). This also holds true for the case where the guarantee costs are subtracted by means of an annual percentage of the fund value at the end of each year, because the savings premium is increased such that the total annual (and single) premium paid into the contract is the same as in the case with constant guarantee costs (see Table 1). Thus, the upfront guarantee costs of 112 are transformed to an annual premium of 13, implying a new annual savings premium of 120+13=133. In case of upfront savings, the new premium amounts to 1,283. This also impacts the development of the mutual fund, while from the customer perspective the total premium payment remains the same in both cases (annual percentage fee and constant additional premium).

Table 2 shows that α is lower in the case of the upfront savings premium as compared to the annual savings (1.00% versus 1.68%), which is also due to the fact that annual premiums in principle imply a higher guarantee value as compared to the single upfront financing case (see also Graf, Kling, and Russ (2011)). However, due to the calibration of the annual and single upfront premium, the annual percentage fee can as well change and be higher in case of the upfront savings premiums.

Table 2: Terminal payoff distribution of a mutual fund for annual and upfront savings premiums as well as with or without guarantee in the case of geometric Brownian motion

	Annual savings			*Upfront savings*		
	Without guarantee	*With guarantee (upfront costs)*	*With guarantee (annual fee)*	*Without guarantee*	*With guarantee (upfront costs)*	*With guarantee (annual fee)*
Savings premium P_S^i	120	120	133	1,075	1,075	1,283
Guarantee G_T	0	1,200	1,200	0	950	950
Price of guarantee at t=0 P_G^i *and* α	0	112	1.68%	0	112	1.00%
Premiums at time T B_T	1,380	1,525	1,525	1,380	1,525	1,525
Shortfall probability $P(L_T<G_T)$	13.96%	-	-	9.22%	-	-
$E(L_T)$	2,223 (3)	2,252 (1)	2,235 (2)	3,044 (3)	3,063 (1)	3,059 (2)
$E(L_T) - B_T$	843 (1)	727 (2)	711 (3)	1,664 (1)	1,538 (2)	1,534 (3)
$\sigma(L_T)$	1,150 (1)	1,117 (2)	1,090 (3)	2,402 (1)	2,381 (2)	2,378 (3)
Sharpe ratio	0.73 (1)	0.65 (2)	0.65 (2)	0.69 (1)	0.65 (2)	0.65 (2)
Omega	14.36 (1)	11.47 (2)	11.31 (3)	20.02 (1)	16.46 (2)	16.38 (3)
Sortino ratio	5.54 (3)	5.66 (1)	5.56 (2)	7.74 (1)	7.48 (2)	7.46 (3)

Even though the *present value* of premium payments is the same for the different premium payment methods (annual versus single savings; constant upfront guarantee costs versus annual percentage fee), the *type* of payment scheme certainly has a considerable impact on the terminal payoff distribution and performance, both in case of savings premium and the guarantee costs.

When comparing annual versus upfront savings premiums, for instance, the numerical examples exhibit a considerably higher expected terminal payoff by one third (and a standard deviation twice as high) in the case of upfront savings, as the premiums are paid entirely at inception and can thus be compounded for a longer time with the achieved risky rate of return, which on average is higher than the riskless rate (equity premium). This is consistent with literature on the (controversially discussed) cost-average effect in savings products (see, e.g., Langer and

Neuhauser, 2003, pp. 3f) or dollar-cost averaging (Constantinides, 1979), respectively. In particular, the annual savings premium scheme implies a lower risk in the sense of the standard deviation, as it in principle represents a mixed strategy of risky and riskless investment and thus clearly differs from the upfront payment scheme despite implying the same present value. Overall, in the present setting, the tradeoff between risk and return leads to higher performance figures for upfront savings except for the Sharpe ratio.

Table 2 further shows that the expected payoff $E(L_T)$ is higher for the constant guarantee costs (that are paid in addition to the savings premium and are not invested in the mutual fund) compared to the case of the annual percentage fee, even when taking into account the premium payments, i.e. $E(L_T) - B_T$. Furthermore, in the latter case, the expected payoff minus the premium value is highest if no guarantee is included at all, and, at the same time, the volatility is highest, as guarantees reduce the upside potential, while at the same time providing downside protection.

With respect to the performance of the different product variants, the ranking generally depends on the type of performance measure used and on the type of savings premium payment. While according to the Sharpe ratio and the Omega, the case *without* guarantee implies the highest performance in the present setting despite the higher volatility and downside risk, the Sortino ratio, which accounts for downside risk based on the lower partial moment of order two, ranks the contract *with* guarantee and constant guarantee costs paid in addition at time zero highest in case of annual savings. However, in case of upfront savings premiums, all three performance measures rank best the case without guarantee, since the higher expected terminal payoff outweighs the higher risk associated with the contracts, which can also be seen by the lower shortfall probability. The comparison of different risk-return models thus shows the relevance of choosing the adequate measure depending on individual preferences.[4]

---

[4] In addition, further analyses regarding the impact of the underlying model of the mutual fund using the Heston (1993) model with stochastic variance instead of a geometric Brownian motion indicate that the model choice can play an important role regarding the performance and thus a possible order of preferences due to different fund and risk-return characteristics. For instance, when using a comparable long-term volatility level of 22% for the Heston model, one can observe an increasing discrepancy between the performance of annual and upfront savings premiums, as the guarantee level in case of the upfront premium must be considerably reduced due to a higher default risk as compared to the geometric Brownian motion. Under these assumptions and due to the lower expected payoff, the performance figures would lead to the same result in all cases, ranking the product without guarantee highest. Hence, while the general results regarding the relevance of the premium payment scheme

Thus, in general, financial performance figures are higher if no guarantee is included in the mutual fund, as these guarantees reduce the upside potential due to costs. However, at the same time, guarantees provide protection against downside risk. Thus, when comparing the cases with and without guarantee, customers need to decide which risk and performance measure best reflects their risk aversion, as higher risk aversion may imply a preference for the case with guarantee as shown in case of the Sortino ratio. However, more analysis is needed to obtain a more comprehensive picture.

The impact of the guarantee level on performance

Hence, we next examine the effect of different guarantee levels on the performance of mutual funds for varying premium payment methods. As exhibited in Figure 1, for fixed given guarantee levels in all four product variants, the premium scheme has a considerable impact on the performance of the products, depending on the guarantee level (which is thus not calibrated to ensure the same guarantee costs as is done in Table 2).

In particular, as already seen in Table 2, the downside risk appears to have a great impact on the results. The Sortino ratio is generally increasing for higher guarantee levels, while the other two performance measures exhibit a decrease. Furthermore, the discrepancy between the performance figures for the case of constant guarantee costs and the annual percentage fee variant increases substantially for higher guarantee levels for all three performance measures, especially in the case of the Sortino ratio, which takes into account the lower partial moment of order 2, thus reflecting a stronger degree of risk aversion, and the Omega, which at least takes into account the extent of the shortfall. In particular, the product variant with constant guarantee costs implies the highest performance figures, both for upfront and annual savings, respectively. However, while the performance is higher for the upfront savings premiums in most cases when considering the Omega and the Sortino ratio, this is not the case for the Sharpe ratio, which only takes into account the expected value and standard deviation of the terminal payoff distribution, thus weighting positive and negative deviations around the expected value equally.

---

remains, the model choice can have a strong impact for the preference of a product and should be taken into account.

Figure 1: Performance measures for different guarantee levels ("alpha" refers to the guarantee costs as a constant annual percentage of the fund value)

a) Sharpe ratio

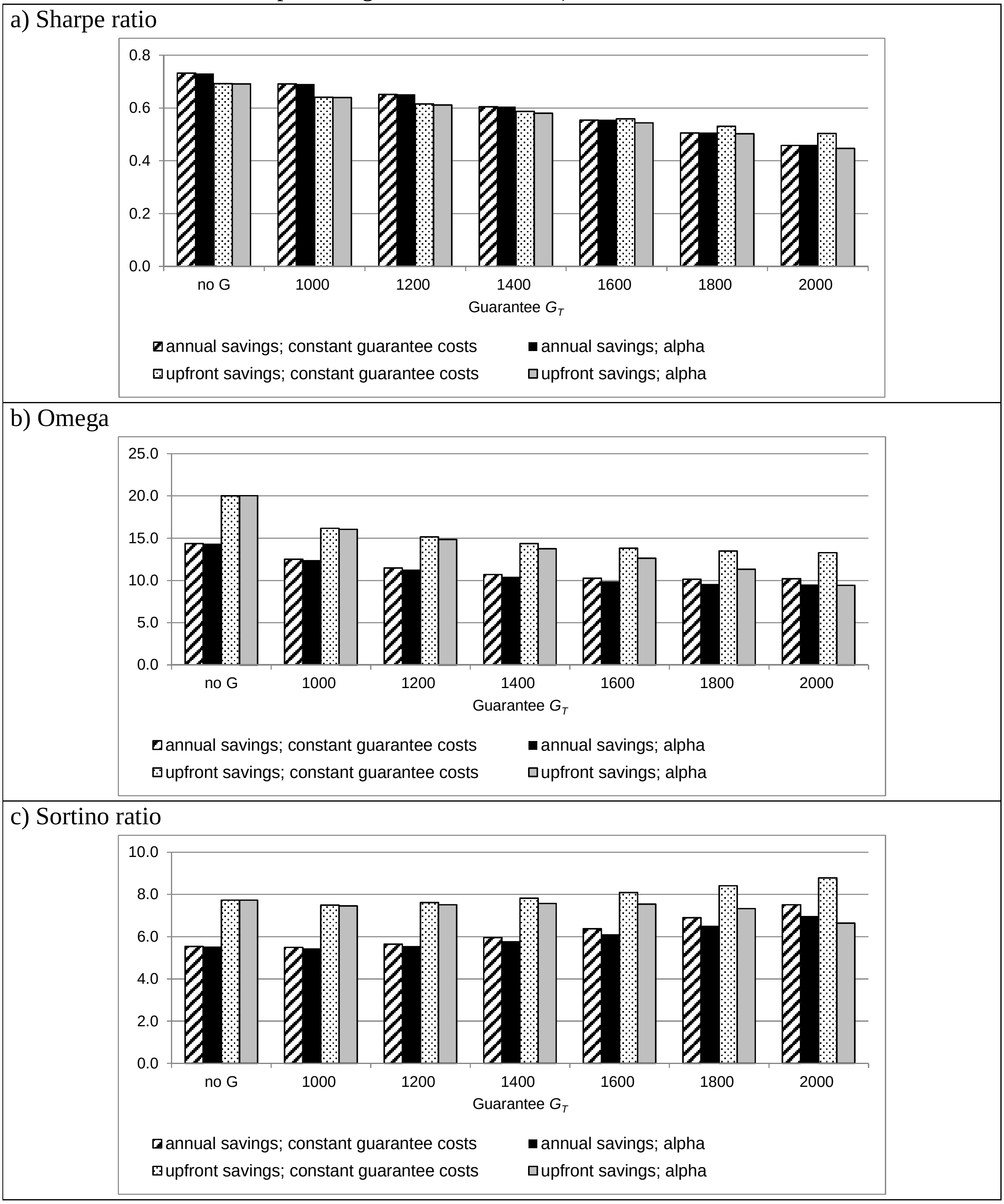
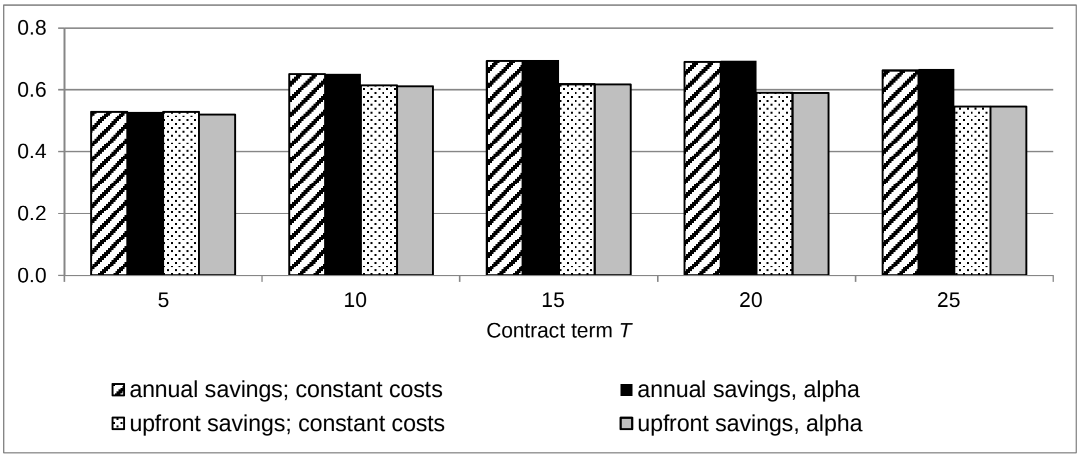
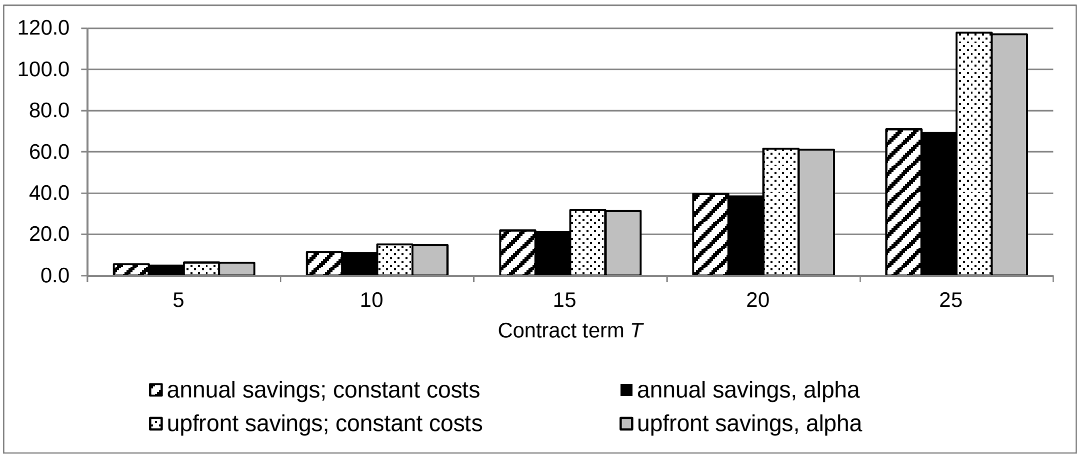


Figure 2: Performance measurement for different contract terms (guarantee levels are adjusted to ensure comparability, see Table 2) ("alpha" refers to the guarantee costs as a constant annual percentage of the fund value)

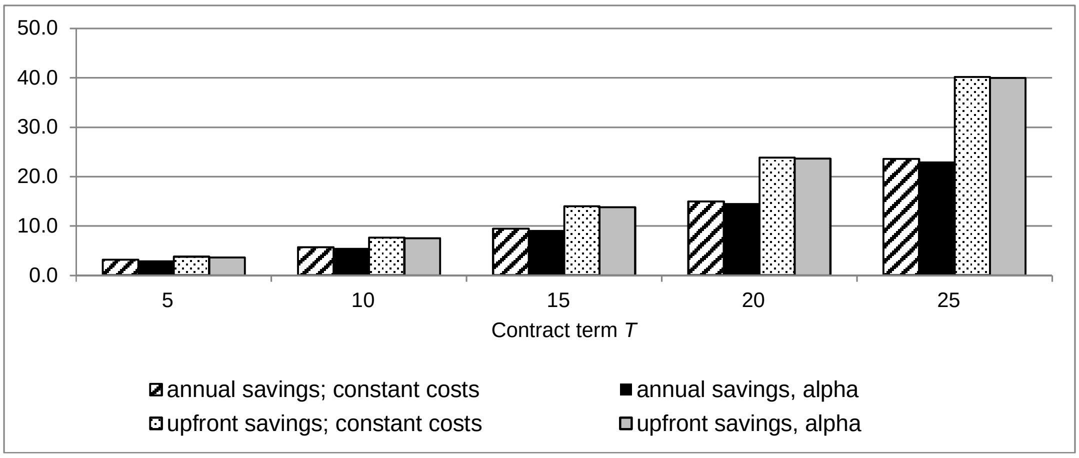
a) Sharpe ratio



b) Omega



c) Sortino ratio



Therefore, the degree of risk aversion as reflected in the three performance measures has an important impact on the evaluation of a product, as more risk averse decision makers using the Sortino ratio in the assumed manner, for instance, might prefer a higher guarantee level and in the present setting further prefer constant guarantee costs as opposed to an annual percentage fee. In addition, upfront savings feature a more favorable tradeoff between downside risk and return, as the risk of the fund falling below the fixed guarantee level is lower than in case of the annual premiums. Upfront savings are thus generally preferred in the presence of risk aversion (see Sortino and Omega), which is not the case for decision-makers that use the standard deviation as the relevant risk measure.

The impact of contract term in the case of a geometric Brownian motion

Further differences in the performance measures can be observed when considering the impact of the contract term as illustrated in Figure 2. Here, the guarantee level is adjusted in each case to ensure comparability as done in Table 2 and is thus increasing for higher contract terms.

While the two performance measures Omega and Sortino ratio, which are based on downside risk measures and thus reflect higher risk aversion, exhibit an increase for higher contract terms, the Sharpe ratio first shows an increase and then decrease, which can be related to the tradeoff between the higher expected terminal payoff and the higher value of premium payments as well as the standard deviation, which is increasing for higher contract terms. In addition, in case of the Sharpe ratio, annual savings premiums imply a higher performance as compared to the upfront savings, which is exactly the opposite in case of the Sortino ratio and the Omega, where upfront savings clearly imply better results, which is similar to the results for higher guarantee levels as exhibited in Figure 1. Furthermore, the difference increases for higher contract terms in the considered example. Finally, in contrast to increasing the guarantee level, the discrepancy between constant guarantee costs and annual percentage fee for a given savings premium payment scheme remains comparably small for both annual and upfront savings.

4. SUMMARY AND DISCUSSION

This paper investigates the performance of a mutual fund with and without investment guarantees. In contrast to previous literature, focus is specifically laid on the impact of the premium payment method on risk and return regarding the payoff distribution at maturity for otherwise fixed product characteristics, information of great relevance to potential customers having different risk-return preferences. In the analysis, the case of annual and upfront savings premiums invested in the mutual fund were compared as well as the case of constant guarantee costs that are paid upfront in addition to the savings premium and an annual percentage fee that is annually subtracted from the fund value. To make the different cases comparable, the guarantee level and savings premiums were first calibrated to yield the same total value of premium payments at maturity using the risk-free rate for compounding. Results were derived in a simulation analysis for different guarantee levels and contract terms.

The findings emphasized that it is not only the underlying mutual fund or the type of investment guarantee that have an impact on the performance, but that the type of premium payment scheme itself can already have a considerable impact on shortfall risk and performance of mutual fund products for otherwise fixed parameter. In particular, if an investment guarantee is included, the type of guarantee cost payment also heavily influences the terminal payoff distribution. Thus, even if customers pay the same value of premiums, the characteristic of the terminal payoff distribution differs tremendously. When looking at the performance figures, the product variants without guarantees and upfront savings generally implied the highest values in the considered examples. This is consistent with literature on the (controversially discussed) cost-average effect and is due to the fact that in case of upfront premiums, a higher expected terminal payoff and higher standard deviation result from a longer investment period in the risky asset (thus receiving the equity premium above the risk-free interest rate), while the annual premium payment scheme generally represents a mixed strategy of risky and riskless investment.

However, the analysis also showed that the ranking of the product variants strongly depends on the performance measure chosen and on the degree of risk aversion as reflected in the lower partial moments and risk preferences in general. For example, for increasing guarantee levels, the Sharpe ratio and Omega exhibited a decrease, while the Sortino ratio, where risk is measured using the lower partial moment of the second order and can thus be considered to reflect a higher degree of risk aversion, showed an increase. Similarly, when increasing the contract term (and the guarantee level accordingly), the Sortino ratio and the Omega both showed an increase, while the Sharpe ratio was decreasing. In regard to the premium payment method, for a given guarantee level, the annual savings premium (with constant guarantee costs) implied better results for the Sharpe ratio, while the Omega and Sortino ratio suggested that upfront savings (with constant guarantee costs) perform better. Thus, more risk-averse decision-makers that base their decision on the Sortino ratio, for instance, may prefer a higher guarantee level and upfront savings, along with constant guarantee costs (instead of an annual percentage fee), if the product characteristics and the present value of premium payments are otherwise equal.

Further aspects to be considered in practice concern costs and management fees that often differ considerably for different products and were thus ignored in the present analysis to ensure comparability and an isolated analysis of the impact of premium payment methods. In future research, risk-return profiles should be considered along with a comparison of different life insurance product types (traditional and innovative ones) with varying premium payment schemes in order to examine to what extent the premium payment scheme alone may already impact the ranking of different product types.

REFERENCES

Björk, T, (2004): Arbitrage Theory in Continuous Time, New York: Oxford University Press.

Bohnert, A., Gatzert, N. (2011): Analyzing Surplus Appropriation Schemes in Participating Life Insurance from the Insurer's and the Policyholder's Perspective, *Insurance: Mathematics and Economics*, 50(1): 64–78.

Boyle, P. P., Schwartz, E. S. (1977): Equilibrium Prices of Guarantees under Equity-Linked Contracts, *Journal of Risk and Insurance*, 44(4): 639–660.

Boyle P. P., Tian W. (2009): Optimal Design of Equity-Linked Products with a Probabilistic Constraint, *Scandinavian Actuarial Journal*, 2009(4): 253–280.

Brennan, M. J., Schwartz, E. S. (1976): The Pricing of Equity-linked Life Insurance Policies with an Asset Value Guarantee, *Journal of Financial Economics*, 3(3): 195–213.

CEA (2012): CEA Statistics N°45: The European Life Insurance Market in 2010 - Executive Summary, www.insuranceeurope.eu/facts-figures/statistical-publications/life (accessed 07/23/2013).

Constantinides, G. M. (1979): A Note on the Suboptimality of Dollar-Cost Averaging as an Investment Policy, *Journal of Financial and Quantitative Analysis*, 14(2): 443–450.

Farinelli, S., Tibiletti, L. (2008): Sharpe Thinking in Asset Ranking with One-side Measures, *European Journal of Operational Research*, 185(3): 1542–1547.

Fishburn, P. C. (1977): Mean-Risk Analysis with Risk Associated with Below-Target Returns, *American Economic Review*, 67(2): 116–126.

Gatzert, N., Holzmüller, I., Schmeiser, H. (2011): Creating Customer Value in Participating Life Insurance, *Journal of Risk and Insurance*, 79(3): 645–670.

Gatzert, N., Huber, C., Schmeiser, H. (2010): On the Valuation of Investment Guarantees in Unit-Linked Life Insurance: A Customer Perspective, *Geneva Papers on Risk and Insurance*, 36(1): 36–29.

Gatzert, N., Schmeiser, H, (2009): Pricing and Performance of Mutual Funds: Lookback versus Interest Rate Guarantees, *Journal of Risk*, 11(4): 887–908.

Gerber, H. U., Shiu, S. W. (2003): Pricing Perpetual Fund Protection with Withdrawal Option, *North American Actuarial Journal*, 7(2): 60–77.

Graf, S., Kling, A., Russ, J. (2011): Financial Planning and Risk-Return Profiles, Working Paper, University of Ulm.

Harrison, J. M., Kreps, D. M. (1979): Martingales and Arbitrage in Multiperiod Securities Markets, *Journal of Economic Theory*, 20(3): 381–408.

Heston S. L. (1993): A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies*, 6(2): 327–343.

Huber, C., Gatzert, N., Schmeiser, H. (2011): How Do Price Presentation Effects Influence Consumer Choice? The Case of Life Insurance Products, Working Paper, Friedrich-Alexander-University of Erlangen-Nuremberg and University of St. Gallen.

Kling, A., J. Ruß, Schmeiser, H. (2006): Analysis of Embedded Options in Individual Pension Schemes in Germany, *Geneva Risk and Insurance Review*, 31(1): 43–60.

Lachance, E.-M., Mitchell, O. S. (2003): Understanding Individual Account Guarantees, *American Economic Review*, 93(2): 257–260,

Langer, T., Neuhauser, N. (2003): Gibt es einen Cost-Average-Effekt? Working Paper, University of Münster.

Mayers, D., Smith, Jr., C. W. (1983): The Interdependence of Individual Portfolio Decisions and the Demand for Insurance, *Journal of Political Economy*, 91(2): 304–311.

Sarin, R. K., Weber, M. (1993): Risk-Value Models, *European Journal of Operational Research*, 70(2): 135–149.

Shadwick, W. F., Keating, C. (2002): A Universal Performance Measure, *Journal of Performance Measurement*, 6(3): 59–84.

Sharpe, W. F. (1966): Mutual Fund Performance, *Journal of Business*, 39(1): 119–138.

Sortino, F. A., van der Meer, R. (1991): Downside Risk, *Journal of Portfolio Management*, 17(4): 27–31.