# On Risk Charges and Shadow Account Options in Pension Funds

Peter Løchte Jørgensen*
Department of Economics and Business
Aarhus University
Fuglesangs Allé 4
DK-8210 Aarhus V
DENMARK
e-mail: plj@asb.dk

Nadine Gatzert
Chair for Insurance Economics and Risk Management
Friedrich-Alexander-University Erlangen-Nürnberg (FAU)
Lange Gasse 20
D-90403 Nürnberg
GERMANY
e-mail: nadine.gatzert@fau.de

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**Abstract**

This paper studies the economic implications of regulatory systems which allow equityholders of pension companies to not only charge a specific premium to compensate them for their higher risk (compared to policyholders), but also to accumulate these risk charges in a so-called shadow account in years when they are not immediately payable due to e.g. poor investment results. When surpluses are subsequently re-established, clearance of the shadow account balance takes priority over bonus/participation transfers to policyholders.

We see such a regulatory accounting rule as a valuable option to equityholders and our paper develops a model in which the influence of risk charges and shadow account options on stakeholders' value can be quantified and studied. Our numerical results show that the value of shadow account options can be significant and thus come at the risk of expropriating policyholder wealth. However, our analysis also shows that this risk can be remedied if proper attention is given to the specific contract design and to the fixing of fair contract parameters at the outset.

# 1 Introduction

When annual surpluses of life and pension (L&P) companies are distributed between the companies' stakeholders, it is common that equityholders charge a risk premium in return for providing the capital to support the companies' business and to compensate them for the risk they bear by ensuring policyholders' claims. The procedure for determining the annual risk premium is often formulated as a simple mathematical rule. It can for example be a function of current surplus, the degree of leverage, and/or the value of liabilities. Whatever the case, the actual premium paid to equity will vary from year to year as the financial situation of the company varies stochastically over time. It may be zero in years when surplus is negative or simply insufficient to reward equity, and it may be correspondingly larger when ample surpluses are re-established, so that over the long term, equity is fairly compensated for its risk.

The present paper is concerned with analyzing some important aspects of surplus distribution schemes in L&P companies stemming from the fact that in some countries, like for example Denmark, the regulatory system allows equity to determine its risk charge[1] according to a scheme which yields a strictly positive result in all years. This will be the case, for example, if the risk charge is specified as a flat percentage of the nominal value of liabilities. Under such a regime, the calculated risk charge may, however, not always be payable in the current year. In years when company surplus is insufficient to allow for immediate payment of equity's (positive) risk charge, the regulatory framework then permits company management to park the year's calculated risk charge in a so-called "shadow account", the balance of which must be disclosed in a note in the annual financial report.[2] The idea with the shadow account is that it can be cleared by payment of the amount "owed" to equityholders when surpluses are re-established at some later point in time. In a legal sense, the shadow account is thus not a liability (it has no claim in case of default), but economically it of course is, and the amounts involved can be significant. In Denmark, the risk charge is the main profit source of commercial L&P companies, but shadow accounts and risk charges are

[1] We shall refer to equity's risk premium as the "risk charge" throughout this paper. An alternative term which is sometimes used is "risk allowance". This term emphasizes the fact that the charge is "allowed" by the regulator.

[2] See the Appendix for an overview of the legal foundation for the risk charging and shadow account schemes of Danish L&P companies.

also in operation in mutual companies.[3] A typical annual risk charge is in the order of 1% of liabilities, and shadow account balances in some companies exceed DKK 5bn (app. EUR 670mn). At the end of the year 2011, Danish life and pension companies' shadow account balances totalled almost DKK 25bn (app. EUR 3.4bn) (Andersen and Dyrekilde (2012b)). As pension systems and regulatory actions in Scandinavian countries are often considered role models for other countries[4], a detailed analysis of shadow account options is of high relevance for regulatory authorities when assessing whether surplus is fairly distributed and whether stakeholders are adequately compensated for the risk they bear. The inclusion of this option may also have severe implications regarding the attractiveness of private annuities which already exhibit an insufficient demand in some countries, cf. for example Mitchell et al. (1999) and Brown (2001).

The purpose of this paper is to explain and analyze the consequences of a regulatory system that permits the operation of shadow accounts as introduced above. We show that permission to operate with a shadow account (or multiple shadow accounts in case of seggregated pools of policyholders) is really an option to equityholders which makes equity more valuable and less risky than without it. The flip side of the coin is that shadow account options may seriously expropriate policyholder wealth.

The remainder of the paper is organized as follows: In Section 2, we set up a simple and illustrative model to explain the nature of the shadow account option and to be able to analyze the effects of risk charges and shadow account options on equity and policyholder values. Section 3 briefly introduces and discusses additional assumptions necessary when using our model for valuation of the different components of the stakeholder claims including the shadow account option. Section 4 implements our model and presents a variety of illustrations and numerical results. By realistic parameterization of the model we confirm that shadow account options are potentially very valuable to equityholders. Allowing equity to operate with a shadow account will therefore, *ceteris paribus*, transfer value from policyholders to equity. However, as our numerical analysis of fair contracts

[3] The operation of shadow accounts enables mutual companies to build up sufficient solvency buffers over time and they also serve a purpose in determining a fair distribution of surpluses to different policyholder risk groups (e.g. with different levels of guaranteed returns) who will each have their own designated shadow accounts.

[4] See for example Mercer (2012) where in a global comparison of pension systems, Denmark comes out at the very top "achieving the first A-grade result in the history of this research". Interestingly, and in close relation to the subject studied in this paper, the Mercer-report also concludes that "the overall index value (grade) for the Danish system could be increased by providing greater protection of members' accrued benefits in the case of fraud, mismanagement or provider insolvency."

shows, the situation may be remedied with proper attention given to the design of the surplus distribution mechanism and in particular to the details of the risk charging scheme and the shadow account option design. Section 5 concludes.

# 2 Model

The analysis of risk charges and shadow account options and their implications for valuation will be performed within a model of a pension company having issued a family of identical traditional guaranteed with-profits policies, i.e. policies with a guaranteed annual minimum return and with a right to receive bonus. To be able to focus clearly on the object of interest here – risk charges and shadow account options – we make several simplifying assumptions regarding for example the pension contracts, the regulatory environment, and the investment universe. The model and its assumptions are described below.

We consider a pension company formed at time 0 by the infusion of capital by a group of pension savers and by a group of owners, i.e. equityholders. It is assumed that there are no further contributions from pension savers – i.e. we consider single premium contracts – and that there are no withdrawals (dividends) to equity prior to maturity and liquidation at time $T$. Considering only the pension savers' accumulation phase – i.e. the period from contract initiation up until the beginning of the retirement and payout phase – we refrain from modeling mortality and explicit life insurance elements of the pension contract.

The sum of the initial investments provided by the two investor groups forms the company's initial asset base. In return for their investments, each investor group acquires a claim on the company which expires and pays off at time $T$, and which is to be described in further detail below. We shall henceforth refer to the pension savers and to the managers/owners as liabilityholders and equityholders and to their initial investments as $L_0$ and $E_0$, respectively. The initial balance sheet looks as shown in Figure 1, where $\alpha = \frac{E_0}{A_0}$ and $\lambda = \frac{L_0}{E_0} = \frac{1-\alpha}{\alpha}$ are defined as the initial equity and leverage ratios, respectively.[5]

[5] In practice, equity ratios, $\frac{E_t}{A_t}$, are controlled to satisfy regulators' solvency capital requirements (SCRs), which may in turn be affected by e.g. investment portfolio risk and by the types of policies sold by the pension company. SCRs are higher for companies which have issued policies with guaranteed minimum returns of the type we aim to model in this paper. In Denmark, actual equity ratios average

**Figure 1: Initial balance sheet**

| Assets | Liabilities |
|---|---|
| $A_0$ | $L_0$ |
| | $E_0$ |
| $A_0$ | $L_0 + E_0 = A_0$ |

Upon formation of the pension fund at time 0 the assets will be invested in a well-defined (primarily wrt. volatility) reference portfolio of financial assets, and at the end of each year the (book) value of the entries on the liability side of the balance sheet will be updated according to well-specified rules that will depend on investment results as well as on specific contract details and guarantees. We will refer to the liability entry in the balance sheet as the policyholders' account since pension savers in with-profits pension funds actually have such an account where the balance gets updated annually. However, prior to time $T$ where it is paid out, it cannot be withdrawn at face value. Hence, the account balance does not necessarily equal the market value of the policyholder's claim prior to maturity (see also Guillen, Jørgensen, and Nielsen (2006)). It is therefore emphasized that while a market value of the investment portfolio is easily obtained at all times (it is simply observed), the same does not hold for liabilities and equity. The market values of these claims must be determined by treating them as contingent claims and by pricing them via appropriate and consistent valuation methods. But before we can do this, the two claims and the rules that govern how investment surpluses (or deficits) are distributed between them period by period until maturity must be described.

Since we are modeling traditional guaranteed with-profits policies, the mathematical description of liabilityholders' claim must reflect the fact that they have been promised a minimum return in each period. The discretely compounded constant guaranteed (annual) rate of return is denoted $r_G$ and it is assumed positive; i.e. $r_G \geq 0$. This claim must be honored before anything else, and we might say that in this sense policyholders have first priority on the company's assets. This is similar to senior debt in a standard corporate capital structure. In addition to their guaranteed return, policyholders are entitled to receive a share of the company's investment surplus when funds are adequate

around 10–15% with enormous variation across companies. We have set $\alpha = 20\%$ in the base case in our later numerical examples.

and the solvency situation allows this. The rate by which policyholders participate in the "upside" is called the participation rate and it is denoted by $\delta$, $0 \leq \delta \leq 1$. This right to participation is sometimes referred to – particularly by financial economists – as policyholders' bonus option, see e.g. Briys and de Varenne (1997).[6] It may be noted here that some newer actuarial literature essentially shares this view of policyholders' right to receive bonus as an option. Norberg (2001) is a good example. Having defined technical surplus as the difference between the second order retrospective reserve (based in part on *experienced* investment returns) and the first order prospective reserve (based in part on the promised technical or *guaranteed* return), and having also noted that technical surplus belongs to the insured, he goes on to analyze various ways to calculate and distribute this surplus – if positive – as bonus. So although our terminologies and mathematical models differ, our approaches are in fact very closely related.

We note that bonus which has been credited to policyholders' account is guaranteed in the sense that such amounts are also entitled to receive a minimum return of $r_G$ in subsequent periods. This feature is sometimes referred to as a ratchet- or cliquet-style guarantee. The implications of such a ratcheting mechanism are studied in Grosen and Jørgensen (2000) and in many later papers. See also Jørgensen (2004).

Equity is modeled as a residual claim – and as such with second priority status – as standard equity in usual corporate finance sense. However, here we shall explicitly model not only the feature concerning equity's right to charge a periodic risk premium and thus to withhold a part of the "upside" before bonus is distributed, but also its option to keep a shadow account where non-payable risk charges can be carried forward for later payment. The decisive feature here is that payment of equity's risk charge and the clearance or reduction of the balance in the shadow account take priority over bonus payments to pension savers. This feature of equity's risk charge can be seen as a parallel to the case of cumulative preference shares (see any corporate finance text such as e.g. Grinblatt and Titman (2002)) for which any unpaid preferred dividends from past periods must be paid in full before any dividends can be paid to common equity.

[6]Policyholders' right to share in pension insurance companies' profits – i.e. to receive bonus – is typically a statutory right. In Denmark, for example, the Ministerial Order no. 358 on "The Principle of Contribution" specifies rules for calculating and distributing the realized actuarial surplus of pension companies. In Germany, a similar Profit Sharing Act (MindZV) specifies, for example, that at least 90% (our $\delta$) of investment surplus must be shared with policyholders, see e.g. Table 3 and the accompanying text in Maurer, Rogalla, and Siegelin (2013).

In our model, equity's risk charge in period $t$ is calculated as a constant fraction $\theta \geq 0$ of liabilities in the beginning of the period, i.e. $L_{t-1}$. This corresponds to the standard practice in Danish life and pension companies (Pensionsmarkedsrådet (2004)). Note that a risk charge scheme can be operated with or without the option to keep a shadow account. When we include the shadow account option, the time $t$ balance in the shadow account is denoted $D_t$. We stress that this is an off-balance sheet entry (liability) which does not affect the accounting identity

$$A_t = L_t + E_t, \tag{1}$$

which must hold at all updating times $t \in [0, T] \cap \mathbb{N}$, where $\mathbb{N}$ refers to the set of natural numbers (including zero). It is one of the main points of this paper, however, to explain that shadow accounts should be thought of as economic/financial liabilities, although from an accountant's point of view they are not. The extended time $t$ balance sheet in Figure 2, where we have added the shadow account, $D_t$, as a shaded entry (which does not enter in the sum of liabilities) is meant to serve as an illustration of this important insight.

**Figure 2: Time $t$ balance sheet and off-balance sheet shadow account entry (shaded)**

| Assets | Liabilities |
|---|---|
| $A_t$ | $L_t$ |
| | $E_t$ |
| | $D_t$ |
| $A_t$ | $L_t + E_t = A_t$ |

To keep the model simple, we will assume that a negative book value of equity is allowable before maturity. This corresponds to the assumption that regulatory authorities will not step in and force liquidation of an insolvent fund prior to maturity. It is furthermore assumed that if equity is negative at the maturity date $T$, then equityholders will cover this loss with an infusion of additional capital so that liabilityholders' claim is in fact fully guaranteed at maturity.[7] The assumption of no limited liability of equity – and thus essentially of default protection of policyholders – is more realistic than it may seem at first glance. In fact, it is a quite natural one when the insurer is a subsidiary of a larger, financially sound company – say a large bank – which in practice and for example for reasons of reputational protection would always stand behind its life and pension insurance arm. The assumption could also be seen as a proxy for a certain type of regulation. We shall return to this discussion later in the paper when our numerical results are presented.

We now turn to describing the rules for the dynamic updating of the various accounts. We describe the general case in which both a non-trivial risk-charge scheme and a shadow account are in operation. Regimes without the shadow account option – or without the risk charge altogether – crystallize as special cases of the general case with appropriate parameters set equal to zero. The various cases will be further discussed and analyzed in the paper's numerical section.

[7] These assumptions are easily relaxed. One might equip equityholders with the put option to default at maturity (Briys and de Varenne (1997)). Alternatively, one could impose a dynamic barrier on the asset value that would trigger premature liquidation should assets drop below a given, possibly time dependent, value as in e.g. Grosen and Jørgensen (2002). A third possibility would be to introduce a third-party guarantor of liabilities' maturity claim as discussed in e.g. Gatzert and Kling (2007). Different assumptions regarding the default structure of the model would naturally affect fair values of the various balance sheet components, but they would not affect the qualitative results regarding the implications of risk charges and shadow account options focused on here.

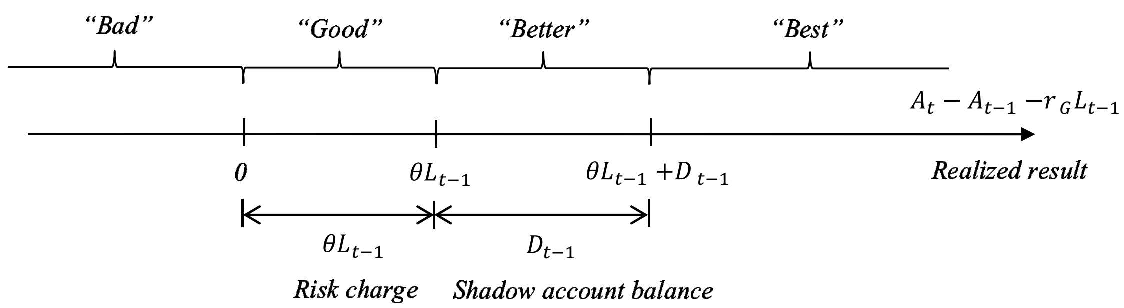
## 2.1 Periodic updating of account balances

In period (year) $t$ the company's *investment return* is given by $A_t - A_{t-1}$, and as explained above, liabilities must be credited with a rate of return of at least $r_G \geq 0$ in each period (year). After this transfer of the guaranteed return to liabilities, the remaining surplus for year $t$ is

$$A_t - A_{t-1} - r_G L_{t-1}. \tag{2}$$

This is called the year's *realized result*.[8] We now turn to describing the rules for distributing the realized result (even if negative) among stakeholders. The state of the world at time $t$ is divided into four cases according to the size of the realized result. We identify these situations as "Bad", "Good", "Better", and "Best", cf. Figure 3 below.

**Figure 3**




### 2.1.1 The Bad case

A Bad year is characterized by

$$A_t - A_{t-1} - r_G L_{t-1} < 0. \tag{3}$$

So in Bad years the company's realized result is negative. This means that no risk charge can be taken by equityholders and that the book value of equity will decrease precisely by the amount on the lefthand side of (3) since liabilities' guaranteed return must always be credited.[9] Since no risk charge can be paid, the shadow account balance will increase

[8] The term "realized result" is defined in Danish insurance legislation as the financial year's surplus or deficit after policyholders' accounts have been credited with the guaranteed return, and after deduction of insurance coverage expenses and costs as assumed in the company's "technical basis" (which must be filed with the Financial Supervisory Authority). The realized result is, in other words, the difference between the "actual" and the "presumed" development in the company's accounts. In our simplified model without insurance risks and costs, this difference is given simply as in relation (2).

[9] It may be noted that $\Delta E_t = \Delta A_t - \Delta L_t = A_t - A_{t-1} - (L_t - L_{t-1}) = A_t - A_{t-1} - (L_{t-1}(1 + r_G) - L_{t-1}) = A_t - A_{t-1} - r_G L_{t-1}$.

by $\theta L_{t-1}$ in period $t$. In the Bad case, account balances are therefore updated as follows:

$$\begin{aligned} L_t &= L_{t-1}(1+r_G) \\ E_t &= A_t - L_t \\ D_t &= D_{t-1} + \theta L_{t-1}. \end{aligned} \tag{4}$$

As can be seen from the last relation in (4) we have assumed – for simplicity – that the shadow account balance does not carry interest from time $t-1$ to $t$.[10]

□

The union of the sets describing the three remaining cases (Good, Better, and Best) form the complement to the Bad case, i.e. (3), cf. again Figure 3. These cases are, in other words, all (partly) characterized by

$$A_t - A_{t-1} - r_G L_{t-1} \geq 0, \tag{5}$$

meaning of course that in all of the remaining cases the realized result is positive. The characterization is further refined as follows.

### 2.1.2 The Good case

In the Good case,

$$A_t - A_{t-1} - r_G L_{t-1} < \theta L_{t-1}. \tag{6}$$

Combining this with condition (5) we have

$$0 \leq A_t - A_{t-1} - r_G L_{t-1} < \theta L_{t-1}. \tag{7}$$

The interpretation is straightforward: In the Good case, the realized result is positive (the investment return is large enough to cover liabilities guaranteed return) but not large enough to allow for full payment of equity's risk charge. Consequently, the shadow account balance increases in this case and there is no bonus. Hence, account balances are updated as follows at time $t$:

$$\begin{aligned} L_t &= L_{t-1}(1+r_G) \\ E_t &= A_t - L_t \\ D_t &= D_{t-1} + \big[\theta L_{t-1} - (A_t - A_{t-1} - r_G L_{t-1})\big], \end{aligned} \tag{8}$$

[10]In practice (in Denmark) the regulator allows for crediting of the shadow account balance with periodic interest as long as the particular scheme applied is disclosed.

where the increase in the shadow account balance (the term in square brackets in (8)) is computed as the permissible risk charge minus the (smaller) realized result, which can actually be transferred to equityholders' account.
In both the Better and the Best cases it holds that

$$A_t - A_{t-1} - r_G L_{t-1} \geq \theta L_{t-1}. \tag{9}$$

This means that the realized result is large enough that the risk charge can be credited in full to equity's account in these cases. What distinguishes the Better and Best cases is the status of the shadow account balance and thus the company's ability to pay out bonus in the present year.

#### 2.1.3 The Better case

The Better case is characterized by

$$A_t - A_{t-1} - r_G L_{t-1} \leq \theta L_{t-1} + D_{t-1}. \tag{10}$$

Combining with condition (9), one obtains

$$\theta L_{t-1} \leq A_t - A_{t-1} - r_G L_{t-1} \leq \theta L_{t-1} + D_{t-1}, \tag{11}$$

so the Better predicate covers years where the investment return is large enough to cover liabilities' guaranteed return as well as equity's risk charge. It may even be that the shadow account balance can be partly reduced (and even cleared), but in any case there will not be funds left for bonus distribution.[11] In the Better case, account balances are updated as follows:

$$\begin{aligned} L_t &= L_{t-1}(1 + r_G) \\ E_t &= A_t - L_t \\ D_t &= D_{t-1} - \big[(A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1})\big]. \end{aligned} \tag{12}$$

In this case, the term in square brackets (in (12)) is the positive amount by which the shadow account balance can be reduced after full payment of the risk charge out of the realized return.

□

[11] Note the following limiting cases: If the leftmost inequality in (11) is binding, i.e. if $A_t - A_{t-1} = r_G L_{t-1} + \theta L_{t-1}$, then $D_t = D_{t-1}$. The shadow account balance remains unchanged. If the rightmost inequality is binding, then $A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1} = D_{t-1}$ and $D_t = D_{t-1} - D_{t-1} = 0$, and the funds are just sufficient to fully clear the shadow account balance.

#### 2.1.4 The Best case

The Best case is a situation characterized by

$$A_t - A_{t-1} - r_G L_{t-1} > \theta L_{t-1} + D_{t-1}, \tag{13}$$

which means that in the years where relation (13) holds, the investment return has been adequate to cover not only liabilities' guaranteed return, equity's risk charge, and any balance in the shadow account. There will also be funds available for bonus distribution. As mentioned earlier, bonus is distributed with a share of $\delta > 0$ to liabilityholders. Consequently, the remainder is deposited with equityholders' account. Thus, in the Best case, accounts are updated as follows:

$$\begin{aligned} L_t &= L_{t-1}(1 + r_G) + \delta\,(A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1} - D_{t-1}) \\ E_t &= A_t - L_t \\ D_t &= 0. \end{aligned} \tag{14}$$

When looking at the increase in the (book) value of equity implied by the Best case system in (14),

$$\begin{aligned} \Delta E_t &= \Delta A_t - \Delta L_t \\ &= A_t - A_{t-1} - \big(L_t - L_{t-1}\big) \\ &= A_t - A_{t-1} - \big(L_{t-1}(1 + r_G) + \delta\,(A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1} - D_{t-1}) - L_{t-1}\big) \\ &= \theta L_{t-1} + D_{t-1} + (1 - \delta)\,(A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1} - D_{t-1})\,, \end{aligned} \tag{15}$$

we can observe that the (accounting) return of equity in the Best case can be decomposed into a risk charge, a transfer equal to the full, previous balance in the shadow account, and a $(1 - \delta)$-share of the surplus amount available for bonus.

□

At this point, note that one can simply set $D_t \equiv 0 \forall t$ to model a situation where risk charging by equityholders is allowed and practiced ($\theta > 0$) but where the maintainance/keeping of a shadow account is not. In this case there is no distinction between the Better and the Best case, cf. Figure 1. An even simpler model – corresponding to a situation in which risk charging is not allowed and where equity is solely compensated through the participation rate – is obtained by setting $\theta = 0$. Along with the assumption that $D_0 = 0$, this will ensure $D_t = 0, \forall t$, and it will correspond to the case where the Good, Better, and Best cases are combined.

Returning to the general model, we finally observe that the period-by-period updating rules for liabilities, equity, and the shadow account that we have described in eqs. (4), (8), (12), and (14) above, can be described in a more compact form which, to a certain extent, reflects the option elements embedded in the different claims. In particular, the development from time $t-1$ to time $t$ in the accounting (book) value of the liabilityholders' claim is given by

$$L_t = L_{t-1}(1+r_G) + \delta\Big[A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1} - D_{t-1}\Big]^+. \qquad (16)$$

Hence, for equity we have the relation

$$E_t = E_{t-1} + A_t - A_{t-1} - r_G L_{t-1} - \delta\Big[A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1} - D_{t-1}\Big]^+. \qquad (17)$$

Finally, the development in the shadow account balance is governed by

$$\begin{aligned} D_t &= D_{t-1} + \theta L_{t-1} \\ &\quad - \Big[A_t - A_{t-1} - r_G L_{t-1}\Big]^+ + \Big[A_t - A_{t-1} - r_G L_{t-1} - \theta L_{t-1} - D_{t-1}\Big]^+. \end{aligned} \qquad (18)$$

The "payoff functions" in (16)–(18) are visualized in Figures 4–6 below where the time $t$ accounting value of liabilities ($L_t$), equity ($E_t$), and the shadow account balance ($D_t$) conditional on ($D_{t-1},\ L_{t-1}$) and on the fixed parameters are plotted as a function of the realized result in period $t$, i.e. $A_t - A_{t-1}$.

**Figure 4: Development of liabilities from time $t-1$ to time $t$**

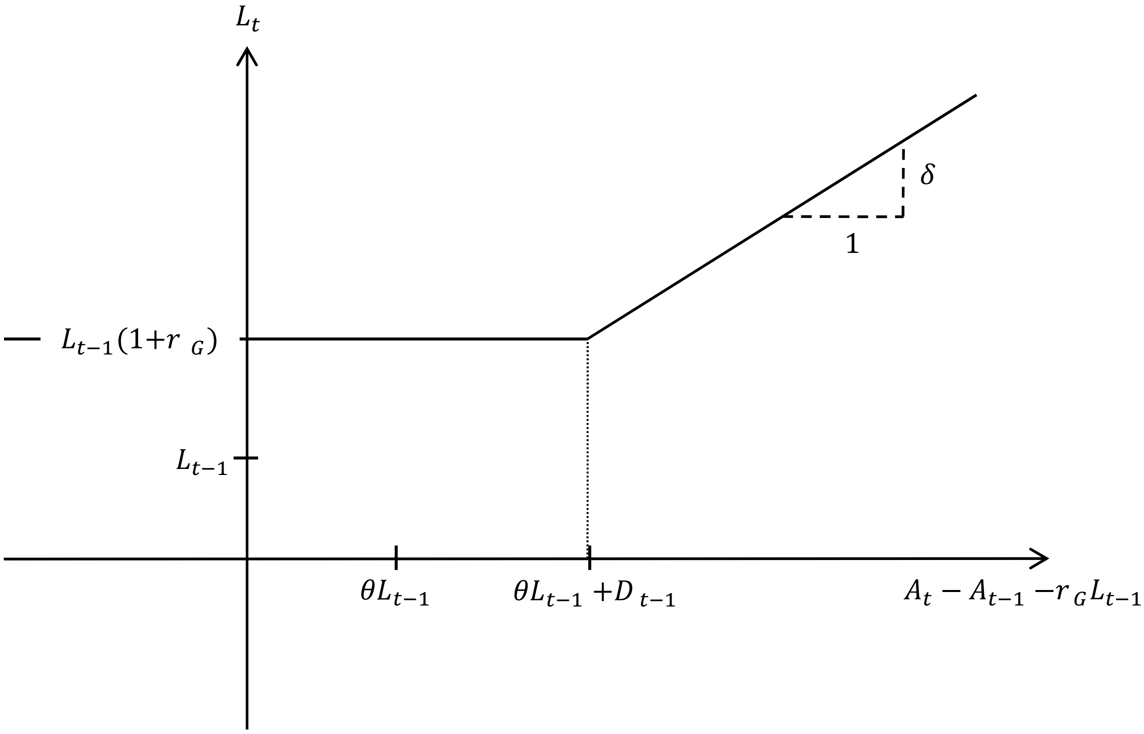
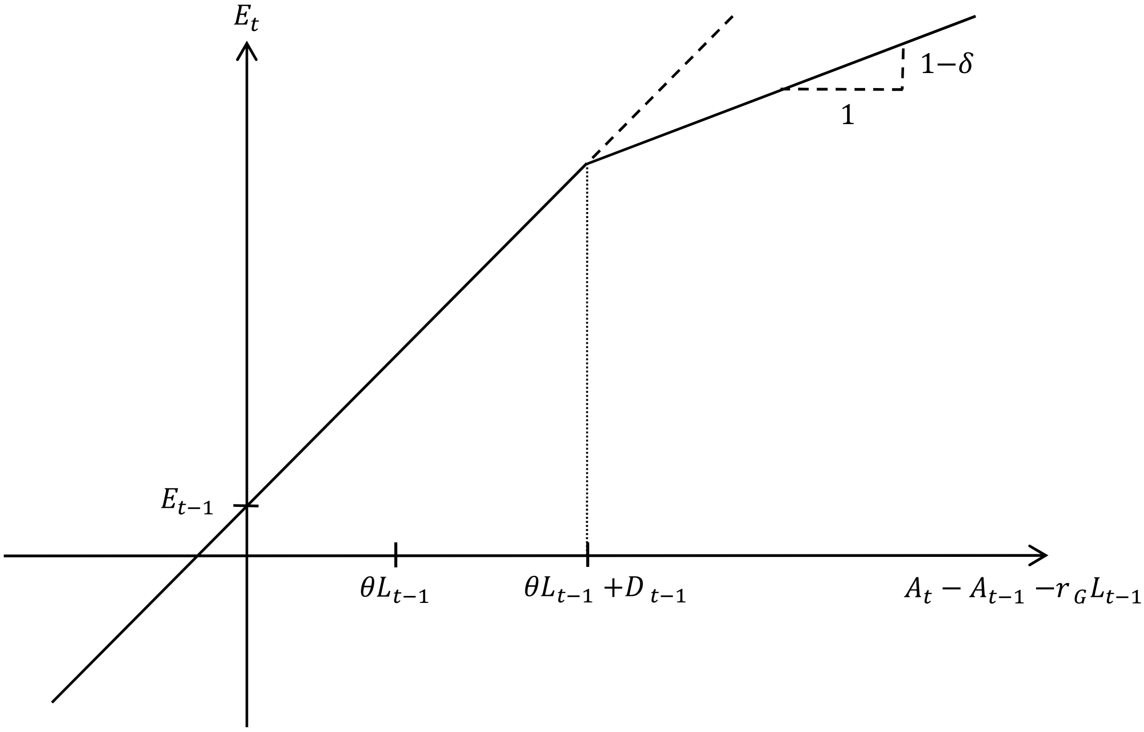
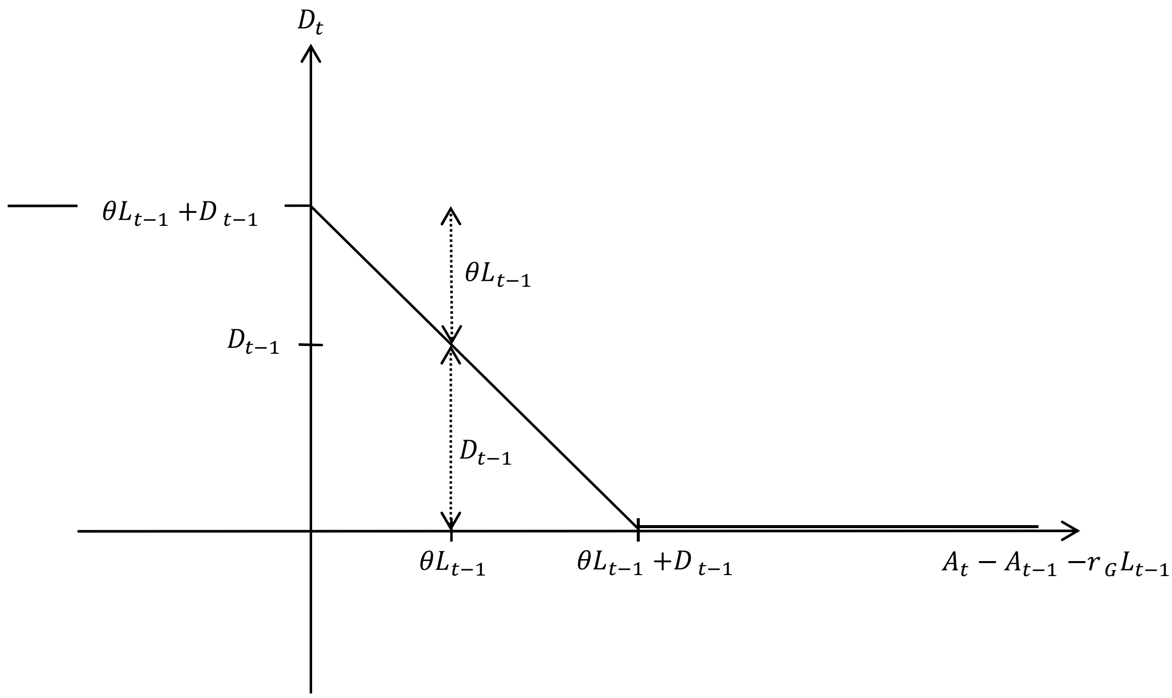



**Figure 5: Development of equity from time $t-1$ to time $t$**



It is important to realize that equations (16)–(18) and the accompanying figures merely represent a partial, one-period view on the development in the various accounts. It is not until multiperiod and cumulative payoffs are studied that the operation of a shadow account becomes meaningful and that the consequences of risk charges and shadow account transfers become fully visible.

**Figure 6: Development of shadow account balance from time $t-1$ to time $t$**




# 3 Valuation

Having fully described the various claims and the mechanisms for determining their maturity payoffs, we now focus on issues regarding valuation. Since final payoffs to liabilities and to equity are fully determined by the account updating mechanisms, by the model parameters, and by the path of asset values from time 0 to time $T$, we assume that the investment portfolio is an asset that trades freely in a perfect and frictionless market.[12] This means that we can price contracts as replicable European-style financial contingent claims using standard risk neutral valuation techniques. Assuming a constant (and continuously compounded) riskless rate of interest, $r_f$, the initial value of liabilities

[12]To be more precise about the path-dependence, it is only the set of asset values sampled at the annual updating points that matter for final payoffs, not the *entire* path.

is given by

$$V_0^L(A_0, D_0; T, r_f, r_G, \sigma, \alpha, \theta, \delta) = e^{-r_f T}\mathbf{E}_0^Q\left\{\tilde{L}_T\right\}. \tag{19}$$

Similarly, the initial value of equity can be represented as

$$V_0^E(A_0, D_0; T, r_f, r_G, \sigma, \alpha, \theta, \delta) = e^{-r_f T}\mathbf{E}_0^Q\left\{\tilde{E}_T\right\}. \tag{20}$$

In both of the above expressions, $\mathbf{E}_0^Q\{\cdot\}$ refers to risk neutral- or $Q$-expectations conditional on time 0 information. In addition to the initial asset value, $A_0$, the initial balance in the shadow account, $D_0$, is specified as an argument of the valuation functions. The purpose of this is to emphasize the significance of this variable in the valuation problem(s). This significance is illustrated and quantified in further detail in the later numerical study.

Before we can proceed with further analyses and evaluation of equations (19) and (20), the stochastic dynamics of the investment portfolio needs to be defined. To this end, we assume that the asset value dynamics is governed by a geometric Brownian motion (GBM) as, for example, in Black and Scholes (1973) and many other studies related to ours (e.g. Briys and de Varenne (1997), Grosen and Jørgensen (2002), and Gatzert and Kling (2007)). This choice is not made to facilitate the derivation of closed-form solutions for claim values. As already noted, our claims payoffs are highly path-dependent. This will prevent the derivation of such closed-form solutions irrespective of the choice of asset dynamics. So we must resort to numerical methods such as Monte Carlo simulation in order to evaluate the central relations (19) and (20). We prefer the geometric Brownian motion to more complex dynamic models in order to keep matters reasonably simple. This allows us to focus on other more important details, and if deemed necessary, the assumption concerning the GBM is easily relaxed. The GBM process governing the dynamics in the asset value is given by

$$dA_t = \mu A_t\, dt + \sigma A_t\, d\mathcal{W}_t^P, \tag{21}$$

where $\mu$ denotes the (continuously compounded) expected return, $\sigma$ is the constant asset return volatility, and $\mathcal{W}^P$ is a standard Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$ on the finite time interval $[0, T]$. The GBM process implies normal distributed log returns, see e.g. Björk (2009).

For purposes of valuation – pertinent to the later Monte Carlo simulation work – the risk neutralized parallel to (21) is needed. It is given by (see again Bjørk (2009))

$$dA_t = r_f A_t \, dt + \sigma A_t \, d\mathcal{W}_t^Q, \tag{22}$$

where $\mathcal{W}^Q$ is a standard Brownian motion under the equivalent risk-neutral probability measure $Q$ and $r_f$ is the constant riskless rate of interest.

# 4 Numerical results and illustrations

In this section, a range of numerical results are presented to illustrate and to further clarify and quantify various aspects of our model. First, single, simulated scenarios are presented to emphasize some essential implications of operating with risk charges and shadow accounts. We next study the design of fair contracts. By fair we mean that computed initial fair values of equity and liabilities should equal the amounts initially invested by these stakeholders. Finally, a sensitivity analysis is performed to illustrate the effects of changes in key parameters on the different value components – including the value of the shadow account option.

## 4.1 Single illustrative scenarios

For a set of given parameters, Figure 7 illustrates the dynamic evolution of balance sheet entries for a single simulated scenario over a 20-year period. The point of reference is a simulated evolution of the market value of the underlying investment portfolio. The figure then contains plots of the evolution in the book values of equity and liabilities resulting from this particular asset value development under three different assumptions regarding the risk charge (RC) and shadow account (SA) regime: a regime without risk charging and shadow accounts, a regime where only a risk charge is imposed, and a regime where both risk charging and shadow account operation is in effect.[13]

Figure 7 illustrates how the imposal of a risk charge and the operation of a shadow account – *ceteris paribus* – benefit equityholders and hurt liabilityholders. This is seen by noting that the (book) value of equity is always higher when a risk charge is imposed than when not. Furthermore, the equity value is further increased if a shadow account

[13] In the current example, the parameter $\delta$ has been calibrated so that the contract, which includes both a risk charge and the operation of a shadow account, is initially fair, cf. the next subsection on fair contracts.

**Figure 7: Dynamic evolution of balance sheet entries for a single simulated scenario**

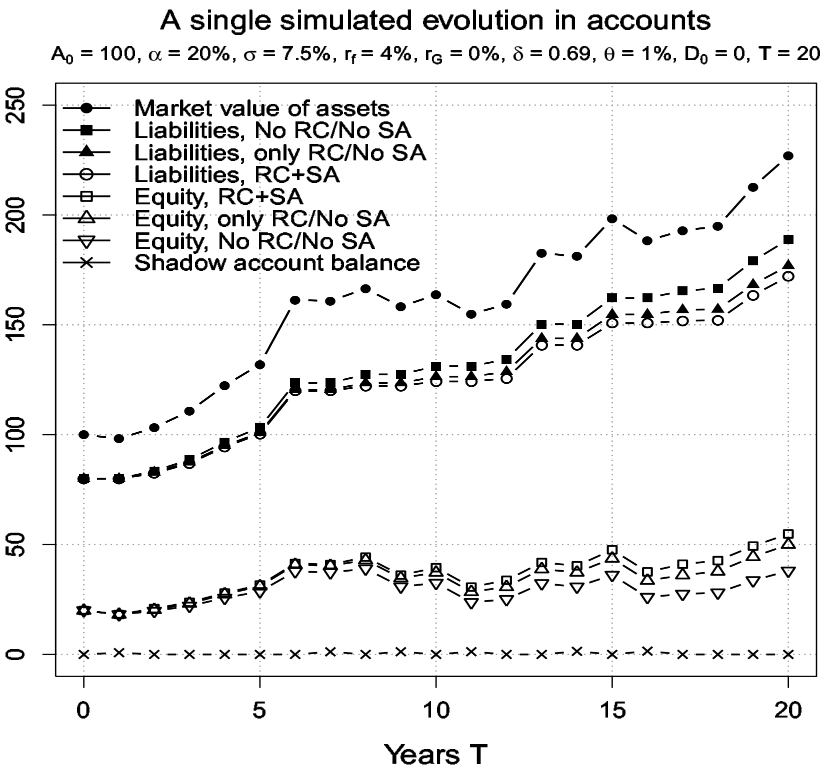
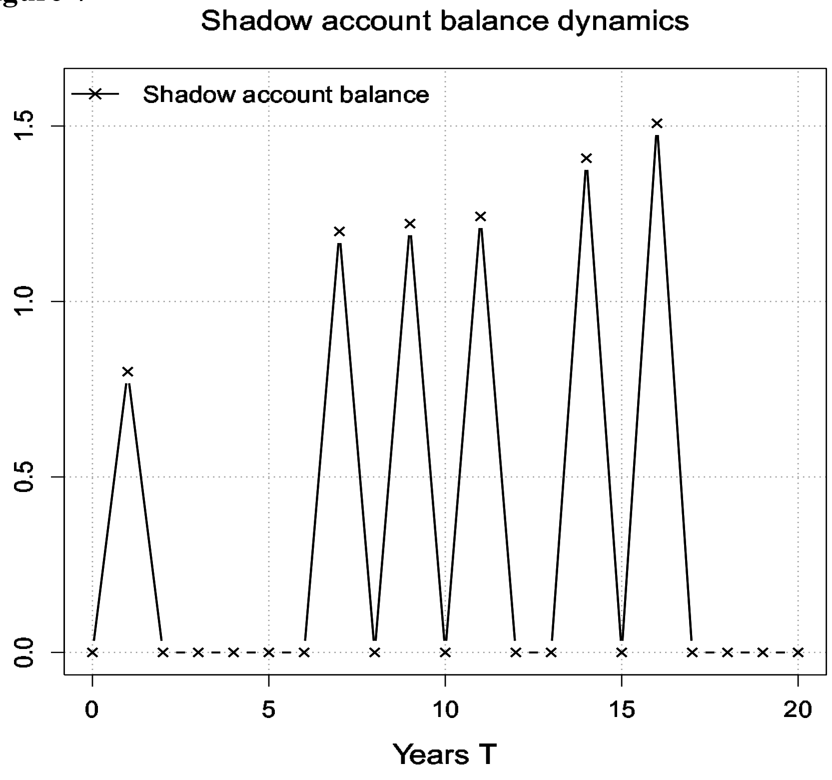


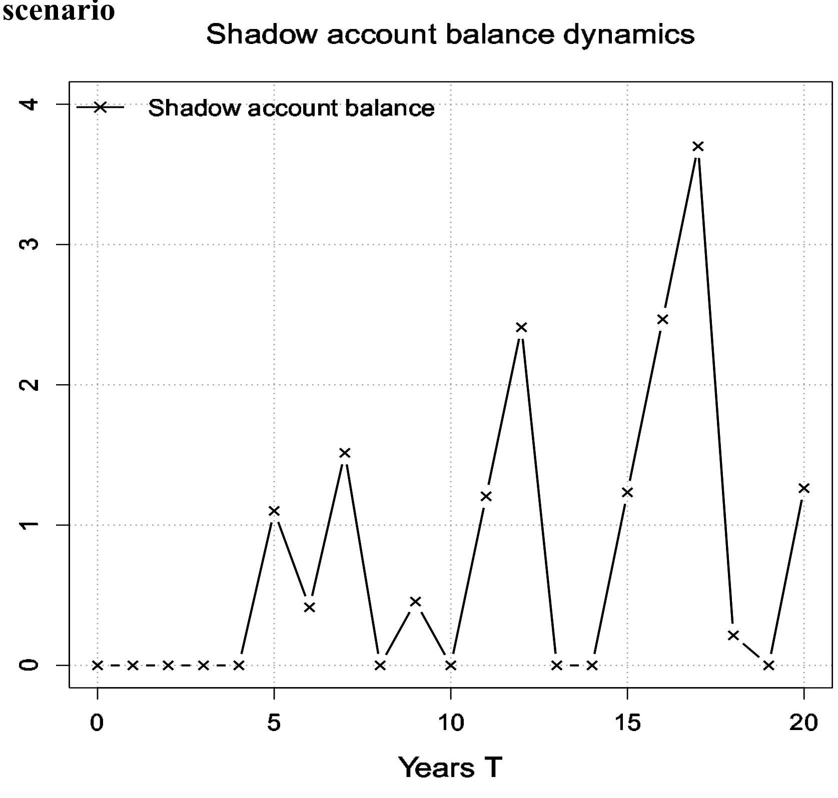

**Figure 8: Dynamic evolution of shadow account balance for single simulated scenario in Figure 7**



also is in operation. Naturally, it is vice versa for liabilities. In the plotted scenario, the maturity value of equity is about 38 for a pure participating contract, it is 50 when a 1% risk charge is applied, and it is roughly 55 when a shadow account is also in operation. The corresponding maturity values of liabilities are 189, 177, and 172. In all three cases, equity and liability values add up to the market value of assets at maturity, which is about 227 in this scenario.

The development in the balance of the shadow account – which is barely noticeable in Figure 7 – has been separated out and enlarged in Figure 8 for clarity. Note in this case how a positive shadow account balance is always brought back down to zero in the subsequent period. It also expires at zero. Not all cases are like this. It may take several periods to clear the shadow account, and the shadow account may expire with a positive balance. Figure 9 shows an alternative scenario for the shadow account development. It is emphasized that the plots in Figures 7–9 are merely randomly generated examples of the dynamics of the various balance sheet entries and the shadow account balance in our model.

**Figure 9: Dynamic evolution of shadow account balance for an alternative single simulated scenario**



## 4.2 Fair contracts

To some extent the previous section has already illustrated how the values of the stakeholders' ownership shares of the fund are affected by the specification of the risk charging and shadow account regime. In this section, we dig a bit deeper into the question of how parameter specification affects value components and explore and illustrate how parameters must be set in order to ensure that contracts are fair at initiation. By "fair" we mean that parameters and contract characteristics are such that stakeholders' initially invested amounts equal the computed arbitrage free initial value of their acquired contingent claim. In mathematical terms, this section will provide a host of examples of contract specifications and parameter combinations which ensure that

$$E_0 \equiv \alpha A_0 = V_0^E \left( A_0, D_0; T, r_f, r_G, \sigma, \alpha, \theta, \delta \right), \tag{23}$$

and therefore also

$$L_0 \equiv (1-\alpha) A_0 = V_0^L \left( A_0, D_0; T, r_f, r_G, \sigma, \alpha, \theta, \delta \right). \tag{24}$$

In Figures 10a–10f below we work from a base case where the fund operates with a shadow account and where $A_0 = 100, \alpha = 0.20, D_0 = 0, r_f = 4.0\%, r_G = 0.0\%, \theta = 1.0\%$, and $\sigma = 7.5\%$. With these parameters the fair participation rate, $\delta$, equals $0.688$.

[Insert Figures 10a–10f]

Figures 10a–10f are produced by varying the asset volatility and another key parameter, and by then solving (by iterated Monte Carlo simulation with 10 million paths) for the participation rate that ensures that the new parameter combination is fair to both sides of the contract. This procedure generates a family of fair contract curves which can be studied in the figures.

The first thing to notice from these figures is the general negative relation between asset volatility and fair participation rates. This is as expected in the present setting without default during the contract term, since increasing asset volatility increases the value of the liabilityholders' call option on the "upside" of realized periodic returns, and the participation rate should therefore be lowered when volatility is increased – and vice versa – to re-establish a fair contract. A parallel view would be to think of the guarantee-issuing equity as having sold a put option on assets to liabilityholders. The value of such a put option also increases in volatility.[14]

It can also be noted that the negative relation between asset volatility and fair participation rates means that value is transferred from liabilityholders to equity if asset volatility is lowered without a corresponding increase in participation rates. This theoretical property of the model is quite consistent with empirical observations from recent years where managers of some pension funds with significant interest rate guarantees have lowered the level of risk in their investment portfolios – or have ”threatened” to do so unless liabilityholders would agree to renegotiate their interest rate guarantees or to give them up entirely.[15]

A final general observation from Figures 10a–10f is that an asset volatility below a certain threshold will in some cases require participation rates above 100% in order for the contract to be fair. It is hard to imagine such a contract being effectuated in practice in the pension market. However, an interesting parallel worth mentioning is the retail market for structured investment products in which participation rates above 100% are quite standard, see e.g. Baubonis, Gastineau, and Purcell (1993) and Chen and Wu (2007).

Looking at the individual Figures 10a–10f, a further number of interesting observations can be made from studying the displacement of the fair $(\sigma, \delta)$-curve as a third parameter is varied. Starting from the top-left Figure, 10a, it is seen that higher guaranteed rates and lower participation rates go hand-in-hand in fair contracts. Moreover, guaranteed rates above the riskless rate are not possible (if contracts are to be valued at par). When policyholders are guaranteed a return equal to the riskless rate they cannot also participate in the ”upside” and $\delta$ drops to zero as seen in the figure. The next Figure, 10b (top-right), shows that as equity's risk charge increases, policyholders participation rate must increase as well in order for the contracts to remain fair.

The middle-left Figure, 10c, compares situations in which the initial shadow account balances differ. In accordance with intuition, policyholders would, ceteris paribus, prefer to join a fund where policyholders are not already indebted to equity, i.e. they prefer

[14] Note that there is a natural connection here to the Put-Call parity, although the link is not simple since we are effectively dealing with a *sequence* on interrelated options on realized periodic returns.

[15] Woolner (2010) describes how, in a controversial move, Danish pension company Sampension in 2010 redefined their guarantees from fixed to ”intentional”. Sampension was subsequently sued by policyholders.

an initial shadow account balance, which is as low as possible. Hence, the higher the initial shadow account balance, the higher the participation rate must be in order to fairly compensate policyholders at the outset. In addition, Figure 10d (middle-right) shows that policyholders should also require higher participation rates for higher initial leverage of the fund. Again this is as expected.

The bottom-left Figure, 10e, illustrates fair participation and asset volatility pairs for different risk charge/shadow account regimes. Compared to a situation in which equity does not charge for risk, policyholders would require higher participation rates when equity *does* impose a risk charge, and an even higher particpation rate when a shadow account is also in operation.

Finally, Figure 10f at the bottom right shows the effect of varying time to maturity. Time to maturity turns out to have negligible effect on the fair $(\sigma, \delta)$-relationship when the initial shadow account balance is zero. The figure is therefore constructed with a positive initial shadow account balance ($D_0 = 25$) where results are non-trivial. One can observe that when $T$ is increased – and there is thus more time to earn an investment return and to clear the given shadow account balance – policyholders can accept a lower participation rate. This is also a quite intuitive result.

## 4.3 Contract valuation and sensitivity analysis

Having considered in the previous section fair contract designs in some detail, we now move on to looking more directly at contract values and their sensitivities to parameter changes in our model setup. Since the market value of equity and liabilities always add up to the total value of assets (which is fixed to 100), a table of equity values also implies a table of liability values and vice versa. In Tables 1 and 2, we nevertheless take different perspectives – equity's and liabilities', respectively – in order to focus on changes to parameters that may seem more directly relevant to, or to some extent actually controlled by, one type of stakeholder.

The point of departure of both tables is a particular set of parameters which lead to a fair contract and which are circled in the tables. From this ”anchor point”, key parameters are then changed, and the resulting contract values are reported in the table. All other contracts in the tables are thus not stricly fair, but the exercise will give us a clear idea of which stakeholder(s) stands to gain or lose when parameters change, for example because of altered market conditions. Similarly, the tables will be informative about the strength of the incentive a stakeholder may have to attempt to manipulate a parameter (to the extent that this is possible).

As before, all contract values in Tables 1 and 2 are obtained by Monte Carlo simulation. An average Monte Carlo error is given in the bottom line of the tables.

Table 1 focuses on equity value from an initial point where asset volatility is 7.5% and where neither a risk charge nor a shadow account is in operation. With other parameters fixed as shown in the table's header, such a contract is fair with $\delta = 0.5918$. From the rest of the table, one can see the effects of changing asset volatility and of imposing a risk charge of increasing magnitude. In addition, the impact of introducting a shadow account is exhibited. To a certain extent all of these variables are under equityholders' control or influence.

Table 1 re-confirms that equity value is decreasing in asset volatility. The sensitivity of equity value to asset value volatility reflects our assumption of no limited liability for equityholders. Given this assumption, equity may have a strong incentive to reduce risk in the fund's investment portfolio. From the initial point, for example, equity value can be increased by approximately 33% by lowering volatility from 7.5% to 5.0%. This property of the model is consistent with the observed practice of many pension funds having switched their investments to less risky assets as interest rates have dropped further in recent years. The tendency is often stronger the stronger the pension funds' exposure to interest rate guarantees, and we are aware of companies which used to have significant stock investments but which are now almost 100% invested in short-term, low-duration (Northern European) government bonds.[16]

[16]This tendency is also a consequence of the advance of Solvency II-related risk-based capital requirements.

**Table 1:**

Equity value's dependence on asset volatility ($\sigma$), risk charge ($\theta$), and shadow account (SA) operation

$A_0 = 100, \alpha = 0.20, D_0 = 0, T = 20, r_f = 4\%, r_G = 0\%, \delta = 0.5918$

Based on $10^7$ simulations, $\delta$ calibrated so $V_0^E = 20$ in base case (circled)

| | | Asset volatility, $\sigma$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | 2.5% | 5.0% | 7.5% | 10.0% | 12.5% | 15.0% |
| $\theta = 0.0\%$ and no SA | | 30.99 | 26.57 | 20.00 | 12.70 | 5.10 | -2.67 |
| $\theta = 0.5\%$ | No SA | 33.67 | 28.86 | 22.15 | 14.80 | 7.19 | -0.58 |
| | SA | 33.84 | 29.48 | 23.02 | 15.85 | 8.40 | 0.77 |
| $\theta = 1.0\%$ | No SA | 36.17 | 31.01 | 24.15 | 16.78 | 9.15 | 1.44 |
| | SA | 36.54 | 32.23 | 25.89 | 18.86 | 11.53 | 4.04 |
| $\theta = 1.5\%$ | No SA | 38.51 | 33.01 | 26.04 | 18.65 | 11.05 | 3.34 |
| | SA | 39.11 | 34.85 | 28.60 | 21.71 | 14.50 | 7.16 |
| $\theta = 2.0\%$ | No SA | 40.68 | 34.89 | 27.84 | 20.43 | 12.82 | 5.17 |
| | SA | 41.55 | 37.33 | 31.19 | 24.40 | 17.31 | 10.09 |
| | | Average Monte Carlo error: 0.0062 | | | | | |

Similarly, Table 1 shows that equity value is positively affected by imposing a risk charge and further so if also a shadow account is being operated. We see the value of the shadow account option as the difference between the "SA" and "No SA" values in the table. For example, by imposing a risk charge of 1%, which is in line with risk charges observed in practice, and by activating a shadow account, the market value of equity increases by approximately 30% (from 20.00 to 25.89) in our example. The shadow account option value is equal to about 1.75 in this case. In general the shadow account option value increases in volatility.

We finally note that the negative equity values in the top-right corner of Table 1 are not errors. They are a consequence of an increased volatility (and no risk charge or shadow account) in combination with our base assumption of no limited liability of equity. As was noted earlier in the paper, the assumption of no limited liability for equity is not as strange as it may seem. Life and pension companies are often subsidiaries of large financial conglomorates and/or banks that would only as an absolute last resort walk away from an insolvent L&P subsidiary and let it default. The assumption of no limited liability is therefore quite realistic, and in fact, one also regularly sees L&P companies trade at negative prices, i.e. other financial institutions or companies sometimes need to be *paid* to take over the L&P business of a bank, say, that does no longer want to embrace and support this type of business. This phenomenon is particularly pronounced for companies that are burdened by high level guarantees. The assumption of default protection of policyholders may alternatively be seen as a proxy for regulation designed to protect the pension benefits of policyholders in virtually all circumstances. This would then include situations where the regulator could force L&P company owners to supply additional equity capital when necessary. Having said that, an assumption of limited liability of equity may be more appropriate and correct in certain situations, and equityholders are of course in general not legally obliged to cover a possible default. We have therefore analyzed this case as well, and with such an assumption, negative equity values naturally cannot occur, but our qualitative results regarding risk charges and shadow account options are not materially affected.[17]

Turning to Table 2, where the perspective of liabilityholders is taken, the basis of comparisons is a case with a realistic risk charge of 1%, with a shadow account with zero initial balance in operation, and with fairly high riskless and guaranteed interest rates of 8% and 4%, respectively. Liabilities are fairly valued at 80.00 in this case with $\delta = 0.6181$ and other parameters as given in the table header. The choice of these fairly high interest rate parameters for the base case is made to reflect a situation with contracts that were initiated (fairly, presumably) in a past where conditions, and interest rates in particular, were different (higher) than today.

[17] Numerical results for the limited liability case are available from the authors upon request.

**Table 2:**

Liability value's dependence on riskless interest rate ($r_f$)
and initial shadow account balance ($D_0$).
$A_0 = 100, \alpha = 0.20, T = 20, \theta = 1\%,$
$\sigma = 7.5\%, \delta = 0.6181.$ Results based on $10^7$ simulations.
$\delta$ calibrated so $V_0^L = 80$ in base case (circled)

$r_G = 4.0\%$

| | Initial shadow account balance, $D_0$ | | | | | |
|---|---|---|---|---|---|---|
| $r_f$ | 0.0 | 10.0 | 20.0 | 30.0 | 40.0 | 50.0 |
| 8.0% | 80.00 | 78.34 | 76.73 | 75.15 | 73.60 | 72.08 |
| 6.0% | 90.37 | 87.79 | 85.32 | 82.96 | 80.72 | 78.56 |
| 4.0% | 107.88 | 103.93 | 100.39 | 97.27 | 94.53 | 92.13 |
| 2.0% | 137.80 | 132.24 | 128.20 | 125.28 | 123.16 | 121.63 |
| 0.0% | 188.29 | 181.92 | 178.81 | 177.23 | 176.39 | 175.93 |
| | Average Monte Carlo error: 0.0053 | | | | | |

$r_f = 4\%$

| | Initial shadow account balance, $D_0$ | | | | | |
|---|---|---|---|---|---|---|
| $r_G$ | 0.0 | 10.0 | 20.0 | 30.0 | 40.0 | 50.0 |
| 4.0% | 107.89 | 103.91 | 100.39 | 97.27 | 94.52 | 92.13 |
| 3.0% | 97.62 | 94.11 | 90.80 | 87.71 | 84.85 | 82.24 |
| 2.0% | 89.13 | 85.99 | 82.94 | 80.01 | 77.17 | 74.49 |
| 1.0% | 81.91 | 79.15 | 76.35 | 73.60 | 70.88 | 68.22 |
| 0.0% | 75.74 | 73.24 | 70.71 | 68.16 | 65.59 | 63.03 |
| | Average Monte Carlo error: 0.0061 | | | | | |

The first panel of the table then allows us to study what has happened to the value of the base contract as market riskless interest rates have dropped and as shadow account balances may have increased as a result of poor investment results. The effect of falling interest rates is of course significantly positive to liabilityholders. This is mainly because their guarantees have become more valuable, but we also see that the increase is partially reversed to the extent that shadow account balances are increased. The fact that positive shadow account balances hurt liabilityholder value is one that new pension savers should be particularly aware of – if it is not compensated for somehow – before they choose the company with which to trust their pension savings. To facilitate such comparisions by potential customers, in Denmark regulatory authorities specifically require pension companies to disclose their shadow account balances as well as their historically applied risk charging scheme.

In the second panel of Table 2, the riskless rate has been fixed at 4%. We then vary $D_0$ as before, as well as the guaranteed interest rate, $r_G$, which is lowered from the original value of 4% (which is a rate applied in many actual contracts from some years back) in steps of 1% down to 0%. At first sight it may seem unnatural to experiment with varying the guaranteed rate, which is supposed to be fixed for the entire life-span of a contract. However, in recent years, practice has seen some L&P companies lowering the guaranteed rates not only for new contracts but also for older, in-progress contracts. In some cases the reduction of the guaranteed rates have been negotiated with policyholders, but in other cases it has simply been dictated.[18] In any case, the effect of lowering $r_G$ is that policyholders' value is expropriated, and with accompanying increases in the shadow account balances, the effect to policyholder value can be detrimental as the table illustrates. Consider for example a contract as in the base case. If the riskless rate drops to 4%, then the immediate effect on the contract value is an increase from 80.00 to 107.88 (and equity becomes negative). But if the guaranteed rate is then lowered from 4% to 2% (as for example recently dictated by the Danish government, cf. Footnote 18), then the value of liabilities drops to 89.13. If, in addition, the shadow account balance has increased to 30, then all benefits from the significant fall in riskless interest rate is lost, and the contract value is back almost precisely at 80 where it started.

[18]In Denmark, L&P companies' beginning practice of lowering of guaranteed rates was officially (and temporarily) sanctioned on June 12, 2012 when the Ministry of Business and Growth and the Danish Insurance Association signed an agreement that prevented L&P companies from paying dividends to equity and from crediting pensions savers' accounts with returns exceeding 2%, irrespective of the level of their guaranteed rates, for the years 2012 and 2013. The agreement was part of a string of initiatives meant to ensure financial stability and to prepare the L&P sector for the upcoming Solvency II regulatory requirements. It has been criticized that the agreement did not prevent or regulate equity's clearance of any positive shadow account balances during the period thus making the agreement a "Gift worth billions" to pension fund owners (see e.g. Andersen and Dyrekilde (2012a)). A press release and the full text of the agreement are available at `www.evm.dk`. See also Footnote 15.

# 5 Conclusion

This paper has analyzed risk charges and shadow account options in life and pension companies. We have explained that in combination with a simple proportional risk charging scheme, the permission to operate a shadow account is really an option to equityholders that can be very valuable. An important implication of this result is that if a shadow account option is granted to equity of an L&P company (e.g. by a country's financial regulator) without a corresponding compensation to the company's policyholders, then the wealth of the latter group can be seriously expropriated. An alternative – and perhaps more positive – way of stating this main conclusion of our paper would be to say that our research has shown that the presence of a shadow account option means that the *fair* risk premium that equity should require as compensation for the risk that it bears by providing the company's equity buffer is *lower* than it would otherwise be. Regulators – and to a certain extent also policyholders – might see this as a positive thing, and it is certainly more comfortable to imagine this being the reason for the introduction of the shadow account option in the first place, rather than the fact that this instrument can be used to expropriate policyholders' wealth. In any case, regulators should be aware of the potential impact of this option and they should work to ensure that the attractiveness of private pensions is still given for policyholders as well as for equityholders providing the capital to back the guarantees offered to the former group. This is also of high social relevance for most industrialized countries due to the prevailing problems encountered in public pension and social security systems.

There are a number of directions in which to extend our work in future research. Firstly, it would be relevant and interesting to refine the default structure of our model, e.g. by allowing for premature default or restructuring, or by considering in more detail the alternative default assumptions that were only briefly discussed during our analysis. One could also extend the model with stochastic interest rates and (a) more advanced asset value process(es) – perhaps with a more realistic feedback mechanism from company solvency to the volatility of the investment portfolio. The inclusion of periodic premiums, surplus withdrawals (dividends) to equity, and/or heterogeneous policyholder risk groups and multiple shadow accounts are additional issues that could be analyzed and which could make the model setup more realistic. A final suggestion for future research would be to perform a study of how risk charges and shadow account options affect shortfall probabilities and the likelihood of default in the model.

# Appendix

In brief, the legal foundation of risk charges and shadow account operation by Danish L&P companies is the following. The Danish Financial Business Act ("Lov om finansiel virksomhed") requires L&P companies to file their "technical basis" with the Danish Financial Supervisory Authority (DFSA). The technical basis should explain, among many other things, the company's rules for calculating and distributing the realized result (see also footnote 8) between company stakeholders. These rules must be "precise, clear, and fair".

The quite general guidelines of the Financial Business Act are clarified in the Ministerial Order no. 358 on the Principle of Contribution ("Bekendtgørelse om kontributionsprincippet", see also footnote 6). The Order specifies that equity's total return must be decomposed into its share of investment asset returns and a risk charge, where the latter must be justified by the risk that equity assumes by ensuring policyholder claims. The risk charging scheme must be disclosed via the DFSA. The same Order states that if equity in a previous year has not received its calculated risk charge in full, then the lacking amount can be charged in later years' positive realized results.

Finally, the DFSA's "Guide to the Ministerial Order on the Principle of Contribution" ("Vejledning om bekendtgørelse om kontributionsprincippet") specifically refers to the construct where equity can park its unpaid risk charge receivable as a "shadow account".

The above-mentioned legal documents are in Danish. They are available for example from www.retsinformation.dk.

# References


Andersen, L. and B. Dyrekilde (August 20, 2012a): "Milliardgave til pensionskassers ejere (Gift worth billions to pension fund owners)," *Jyllands-Posten, Erhverv & Økonomi, p. 1*. In Danish.

——— (August 20, 2012b): "Pensionskunder skylder ejerne milliarder (Pension savers owe billions to owners)," *Jyllands-Posten, Erhverv & Økonomi, p. 8*. In Danish.

Baubonis, C., G. L. Gastineau, and D. Purcell (1993): "The Banker's Guide to Equity-Linked Certificates of Deposit," *Journal of Derivatives*, 1(2):87–95.

Björk, T. (2009): *Arbitrage Theory in Continuous Time,* Oxford University Press, 3rd edition.

Black, F. and M. Scholes (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81(3):637–654.

Briys, E. and F. de Varenne (1997): "On the Risk of Life Insurance Liabilities: Debunking Some Common Pitfalls," *Journal of Risk and Insurance*, 64(4):673–694.

Brown, J. R. (2001): "Private Pensions, Mortality Risk and the Decision to Annuitize," *Journal of Public Economics*, 82(1):29–62.

Chen, K. and L. Wu (2007): "An Anatomy of Bullish Underlying Linked Securities," *Global Finance Journal*, 18(1):34–46.

Gatzert, N. and A. Kling (2007): "Analysis of Participating Life Insurance Contracts: A Unification Approach," *Journal of Risk and Insurance*, 74(3):547–570.

Grinblatt, M. and S. Titman (2002): *Financial Markets and Corporate Strategy,* McGraw-Hill, 2nd, international edition.

Grosen, A. and P. L. Jørgensen (2000): "Fair Valuation of Life Insurance Liabilities: The Impact of Interest Rate Guarantees, Surrender Options, and Bonus Policies," *Insurance: Mathematics and Economics*, 26(1):37–57.

——— (2002): "Life Insurance Liabilities at Market Value: An Analysis of Insolvency Risk, Bonus Policy, and Regulatory Intervention Rules in a Barrier Option Framework," *Journal of Risk and Insurance*, 69(1):63–91.

Guillen, M., P. L. Jørgensen, and J. P. Nielsen (2006): "Return Smoothing Mechanisms in Life and Pension Insurance: Path-dependent Contingent Claims," *Insurance: Mathematics and Economics*, 38(2):229–252.

Jørgensen, P. L. (2004): "On Accounting Standards and Fair Valuation of Life Insurance and Pension Liabilities," *Scandinavian Actuarial Journal*, 104(5):372–394.

Maurer, R., R. Rogalla, and I. Siegelin (2013): "Participating Payout Life Annuities: Lessons from Germany," *ASTIN Bulletin*, 43:159–187.

Mercer (2012): "Melbourne Mercer Global Pension Index," Melbourne, Australian Centre for Financial Studies.

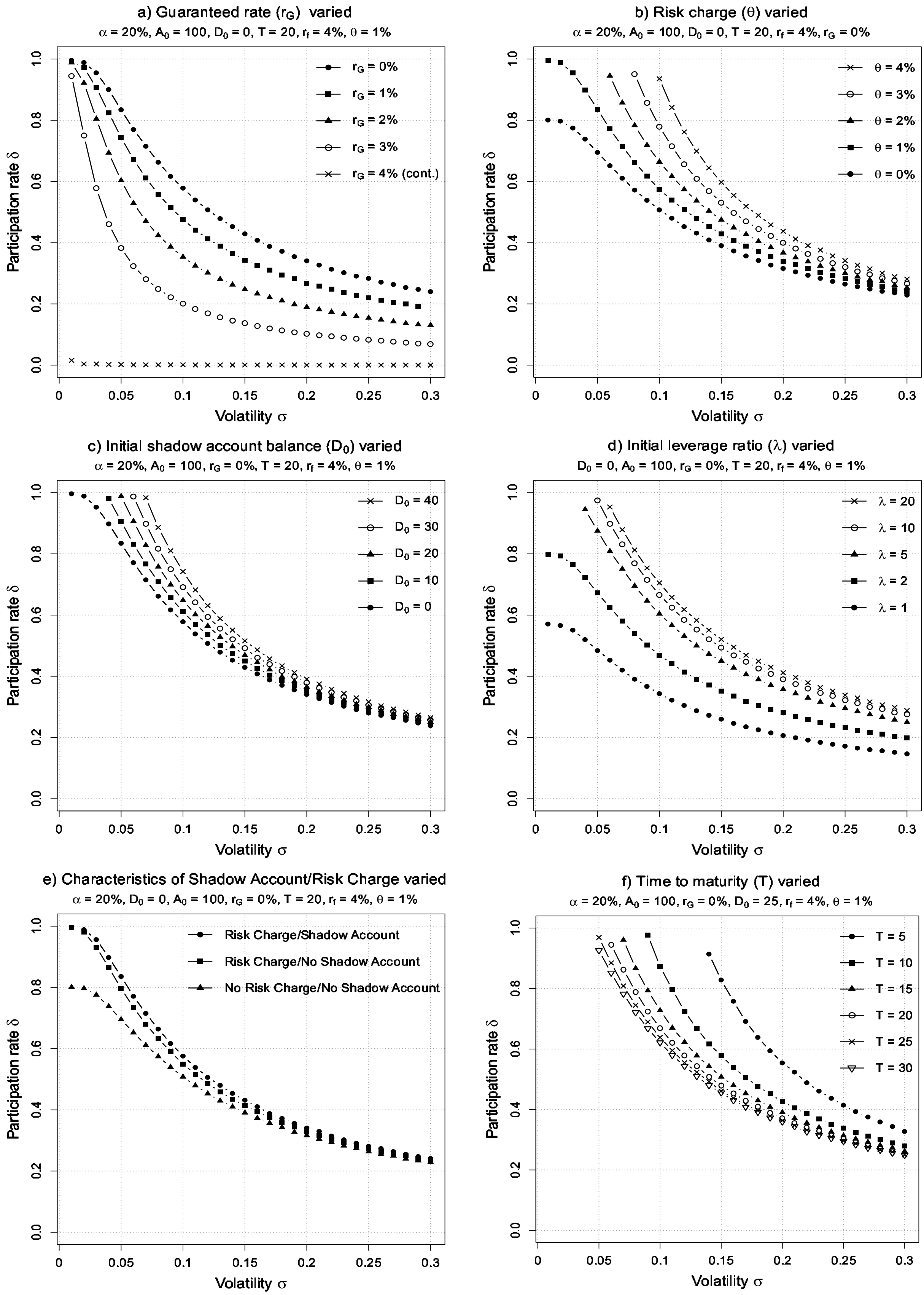
Mitchell, O. S., J. M. Poterba, M. Warshawsky, and J. R. Brown (1999): "New Evidence on the Money's Worth of Individual Annuities," *American Economic Review*, 89(5):1299–1318.

Norberg, R. (2001): "On Bonus and Bonus Prognoses in Life Insurance," *Scandinavian Actuarial Journal*, 2:126–147.

Pensionsmarkedsrådet (2004): "Pensionsmarkedsrådets redegørelse for driftsherretillæg (eng.: The Pension Market Council's Explanation of Risk Charges)," Paper dated July 2nd, 2004, Pensionsmarkedsrådet.

Woolner, A. (2010): "Sampension moves to intentional guarantees," *risk.net*. Published July 9, 2010 at `http://www.risk.net/insurance-risk`.

## Figure 10



a) Guaranteed rate (r_G) varied
α = 20%, A_0 = 100, D_0 = 0, T = 20, r_f = 4%, θ = 1%
r_G = 0%
r_G = 1%
r_G = 2%
r_G = 3%
r_G = 4% (cont.)
Participation rate δ
Volatility σ
b) Risk charge (θ) varied
α = 20%, A_0 = 100, D_0 = 0, T = 20, r_f = 4%, r_G = 0%
θ = 4%
θ = 3%
θ = 2%
θ = 1%
θ = 0%
Participation rate δ
Volatility σ
c) Initial shadow account balance (D_0) varied
α = 20%, A_0 = 100, r_G = 0%, T = 20, r_f = 4%, θ = 1%
D_0 = 40
D_0 = 30
D_0 = 20
D_0 = 10
D_0 = 0
Participation rate δ
Volatility σ
d) Initial leverage ratio (λ) varied
D_0 = 0, A_0 = 100, r_G = 0%, T = 20, r_f = 4%, θ = 1%
λ = 20
λ = 10
λ = 5
λ = 2
λ = 1
Participation rate δ
Volatility σ
e) Characteristics of Shadow Account/Risk Charge varied
α = 20%, D_0 = 0, A_0 = 100, r_G = 0%, T = 20, r_f = 4%, θ = 1%
Risk Charge/Shadow Account
Risk Charge/No Shadow Account
No Risk Charge/No Shadow Account
Participation rate δ
Volatility σ
f) Time to maturity (T) varied
α = 20%, A_0 = 100, r_G = 0%, D_0 = 25, r_f = 4%, θ = 1%
T = 5
T = 10
T = 15
T = 20
T = 25
T = 30
Participation rate δ
Volatility σ