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Analyzing Surplus Appropriation Schemes in Participating Life Insurance from the Insurer's and the Policyholder's Perspective

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ANALYZING SURPLUS APPROPRIATION SCHEMES IN PARTICIPATING LIFE INSURANCE FROM THE INSURER'S AND THE POLICYHOLDER'S PERSPECTIVE

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ABSTRACT

This paper examines the impact of three surplus appropriation schemes often inherent in participating life insurance contracts on the insurer's shortfall risk and the net present value from an insured's viewpoint. 1) In case of the *bonus system,* surplus is used to increase the guaranteed death and survival benefit, leading to higher reserves; 2) the *interest-bearing accumulation* increases only the survival benefit by accumulating the surplus on a separate account; and 3) surplus can also be used to *shorten the contract term,* which results in an earlier payment of the survival benefit and a reduced sum of premium payments. The pool of participating life insurance contracts with death and survival benefit is modeled actuarially with annual premium payments; mortality rates are generated based on an extension of the Lee-Carter (1992) model, and the asset process follows a geometric Brownian motion. In a simulation analysis, we then compare the influence of different asset portfolios and shocks to mortality on the insurer's risk situation and the policyholder's net present value for the three surplus schemes. Our findings demonstrate that, even though the surplus distribution and thus the amount of surplus is calculated the same way, the type of surplus appropriation scheme has a substantial impact on the insurer's risk exposure and the policyholder's net present value.

1. INTRODUCTION

Participating life insurance contracts are an important product design in the German insurance market and comprise various mechanisms of how surplus is distributed to the policyholders. Previous work has shown that different surplus distribution schemes can significantly impact

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the insurer's risk exposure. In this context, an important issue has not been comprehensively analyzed to date, which is the concrete appropriation of distributed surplus. In particular, in Germany, policies may feature different appropriation schemes.[1] Surplus appropriation refers to the way earned surplus, determined via a given surplus distribution mechanism, is actually credited to the individual policyholder. In the case of the bonus system, surplus is used to increase the guaranteed death and survival benefit. In contrast to this, the interest-bearing accumulation emphasizes the survival benefit, which is increased by the surplus, while the death benefit is kept constant. The third alternative uses the surplus to shorten the contract term, which results in an earlier payment of the survival benefit and a reduced sum of premium payments. These three schemes have not been comparatively examined, even though their impact on the insurer's risk situation and the policyholders' expected payoff can differ considerably. The aim of this paper is to fill this gap and to analyze this issue in depth.

In the literature, participating life insurance, along with its surplus distribution mechanisms and interest rate guarantees, have attracted widespread attention. Research on the risk-neutral valuation of participating life insurance contracts includes, for example, Briys and de Varenne (1997), who study the fair value of a point-to-point guarantee, where the company guarantees only a maturity payment and an optional participation in the terminal surplus at maturity, and determine a closed-form solution based on contingent claims theory. Grosen and Jørgensen (2002) extend this framework by including a regulatory intervention rule, which reduces the insolvency probability and can be priced similar to barrier options. In Grosen and Jørgensen (2000), a cliquet-style interest rate guarantee is modeled, where surplus is annually credited to the policy reserves based on a reserve-dependent surplus distribution mechanism to smooth market returns. Once the surplus is credited to the reserves, it becomes part of the guarantee and is then annually at least compounded with the guaranteed interest rate, thus implying cliquet-style effects. Besides the bonus option and the minimum interest rate guarantee, the authors also include and evaluate a surrender option by means of American-style derivatives pricing. Based on the model by Grosen and Jørgensen (2000), Jensen, Jørgensen, and Grosen (2001) develop and apply a finite difference algorithm in order to numerically evaluate the contracts and further integrate mortality risk. Different annual surplus smoothing schemes are also examined in Hansen and Miltersen (2002) for the Danish case and in Ballotta, Haberman, and Wang (2006), where specific focus is laid on a comparison and tradeoff of fair contract parameters. A comparison of different surplus distribution models of participating life insurance with respect to model risk can also be found in Zemp (2011).

[1] See, e.g., Schradin, Pohl, and Koch (2006, p. 17).

Other work that focuses on the surrender option embedded in participating life insurance contracts includes Albizzati and Geman (1994), who also account for stochastic interest rates, as well as Bacinello (2003) for an Italian-style contract. Siu (2005) treats the surrender option by means of a regime-switching model for economic states, including interest rates, expected growth rates and volatility of risky assets, and also presents approximation methods for participating American-style contracts. Schmeiser and Wagner (2010) compute fair values of options to early exercise, including the paid-up option, the resumption option, and the classical surrender option.

Furthermore, several papers have focused on combining risk pricing and risk measurement. Barbarin and Devolder (2005) propose a model to first assess the risk of a point-to-point guarantee and, second, calibrate the terminal bonus participation parameter to obtain fair contracts. Graf, Kling, and Russ (2011) extend the approach used by Barbarin and Devolder (2005) and generalize previous results by proving that the combination of actuarial and financial approaches can always be conducted as long as the insurance contracts do no introduce arbitrage opportunities. Gatzert and Kling (2007) determine the real-world risk implied by fair contracts with the same market value, and Gatzert (2008) further integrates different asset management and surplus distribution strategies in the analysis of participating life insurance contracts with the aim to assess their impact on the contracts' fair value, while keeping the default put option value constant. Kleinow and Willder (2007) study hedging strategies and calculate fair values for maturity guarantees, where the surplus participation depends on the insurer's management decisions regarding the investment portfolio.

With respect to surplus distribution schemes and risk measurement, Gerstner et al. (2008) provide a general asset-liability management framework for life insurance, which incorporates, *inter alia*, a reserve-dependent bonus distribution mechanism based on Grosen and Jørgensen (2000). As an application of their model, they study the impact of different parameter settings and exemplary products on the insurer's shortfall risk. Based on a single premium term-fix insurance and thus focusing purely on financial risks, Kling, Richter, and Russ (2007a) analyze the risk exposure of an insurer offering cliquet-style interest rate guarantees for different contract characteristics, including the initial reserve situation, asset allocation, and the actual surplus distribution. Kling, Richter, and Russ (2007b) extend this framework and consider the financial risk inherent in three surplus distribution systems, including surplus appropriation. The first system incorporates a cliquet-style interest rate guarantee, where the guaranteed rate also has to be paid on surplus, the second mechanism represents an interest-bearing accumulation, where surplus cannot be reduced once it has been credited to the policyholder's account (but without cliquet-effects), and third, a surplus model, where the insurer can reduce surplus to keep the insurance company in business and to avoid insolvencies. As in Kling, Richter, and Russ (2007a), mortality effects are not included.

Hence, what remains open is a holistic analysis of the financial and mortality risk of surplus appropriation schemes on the basis of a typical life insurance product, which is modeled actuarially by considering death and survival benefits. The explicit combination of actuarial pricing and reserving, as well as financial approaches, with respect to shortfall risk and valuation in the analysis of surplus appropriation schemes has not been done to date and is intended to offer insight into the impact of the type and characteristics of surplus schemes on an insurance company's risk exposure and the policyholder's net present value. Furthermore, the system of shortening the contract term has not yet been examined.

The aim of this paper thus is to fill this gap by analyzing the impact of surplus appropriation schemes on a life insurer's risk exposure. In addition and apart from this perspective on risk, we further study the policyholders' net present value, i.e., the difference between the expected discounted death or survival benefit and the sum of premium payments. The model of the life insurance company is based on a participating life insurance contract[2] with annual premiums, where mortality rates are modeled using an extension of the Lee-Carter (1992) model proposed by Brouhns, Denuit, and Vermunt (2002), and the asset base follows a geometric Brownian motion. In contrast to previous literature, insurance liabilities for a pool of policies with death and survival benefits are calculated using actuarial reserving rules, which depend on the surplus mechanism. In particular, based on the smoothing surplus distribution scheme of Grosen and Jørgensen (2000), we analyze and compare three companies with different appropriation schemes, including the bonus system,[3] the interest-bearing accumulation,[4] and shortening the contract term. In a numerical simulation analysis, we study the influence of different asset portfolios and shocks to mortality on the insurer's risk situation and the policyholder's net present value. Our findings demonstrate that, even though the surplus distribution and thus the amount of surplus is calculated the same way, the type of surplus appropriation

[2] In this paper, we use the expression "participating life insurance" analogously to an endowment contract.

[3] The bonus system accounts for cliquet-style effects, and, by including mortality, surplus leads to higher payments to the policyholders during the contract term due to the increased death benefits.

[4] The interest-bearing accumulation has been studied in a similar form in Kling, Richter, and Russ (2007b) but without death benefits or explicit actuarial reserving rules.

scheme substantially impacts the insurer's risk exposure and the policyholder's net present value. In addition, the effect of the choice of the asset portfolio as well as shocks to mortality differ considerably with respect to the insurer's risk level depending on the respective surplus appropriation scheme, which should be taken into account in the context of underwriting activities and in asset management.

The paper is structured as follows. Section 2 introduces the model framework of the insurance company and the three surplus appropriation schemes under consideration as well as the asset and mortality model. Numerical results are presented in Section 3, and Section 4 concludes.

2. MODEL FRAMEWORK

2.1 Overview of the insurance companies

We consider three life insurance companies that differ only in their surplus appropriation scheme, i.e., the way the surplus distributed to the policyholders is actually appropriated to their accounts. The schemes are present in the German insurance market, but may as well be extended to similar schemes in other countries. In the case of the bonus system, surplus is used to increase the guaranteed death benefit as well as the survival benefit and thus increases the policy reserves. In contrast to this, the interest-bearing accumulation emphasizes the survival benefit, which is increased by the surplus (and guaranteed until maturity), while the death benefit is kept constant. The third alternative uses the surplus to shorten the contract term, which results in an earlier payment of the survival benefit and a reduced sum of premium payments. The corresponding balance sheet for all three types of companies is exhibited in Table 1.

Table 1: Balance sheet of a life insurance company at time *t*

Assets	Liabilities
A_t	E_t
	PR_t
	IA_t
	RD_t
	B_t
A_t	A_t

The model is constructed in discrete time, where time zero indicates the inception of the contract, and $t = T$ (in years) maturity. We further assume that the policy period coincides with the accounting year. Here, A_t represents the market value of assets, and E_t is the book value of equity, which, similar to the model in Kling, Richter, and Russ (2007a, 2007b), is assumed to be constant over time. Furthermore, PR_t denotes the book value of the policy reserves, IA_t is the book value of the interest-bearing accumulation account, RD_t is the book value of the surplus account for the reduction of contract duration, and B_t denotes the buffer account, which is determined residually by subtracting E_t, PR_t, IA_t, RD_t, and dividends paid to equityholders from the market value of the asset base, A_t. To gain insight into general effects of different surplus distribution schemes, we assume a run-off scenario without new business.

2.2 Modeling the liability side

The participating life insurance contracts

In the following analysis, we consider a pool of traditional participating life insurance contracts with a contract term of n years, which are actuarially priced based on a mortality table. Hence, the constant annual (net) premium for an x-year old policyholder (for all three surplus schemes) is given by

$$P = S_1 \cdot \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}},$$

where S_1 denotes the initial guaranteed sum insured in case of death or survival, paid in arrear, and $A_{x:\overline{n}|}$ and $\ddot{a}_{x:\overline{n}|}$ represent present values of an endowment insurance for n years and a temporary annuity for n years on the life of x, respectively, given by

$$A_{x:\overline{n}|} = \sum_{t=0}^{n-1} v^{t+1} \cdot {}_t p_x \cdot q_{x+t} + v^n \cdot {}_n p_x \text{ and } \ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1} v^t \cdot {}_t p_x \,, \tag{1}$$

where $v = \left(1 + r^G\right)^{-1}$, and r^G is the one-year calculatory actuarial interest rate. The probability of an x-year old insured to survive t years is denoted by ${}_t p_x$, whereas q_{x+t} is the probability of an $x+t$-year old insured to die within the subsequent year.

Figure 1 illustrates the development of cash flows resulting from the insurance product over time, thereby distinguishing between December 31st of year $t-1$ and January 1st of year t, denoted by '–' and '+', respectively.

Figure 1: Development of cash flows from the insurance product over time ('–' denotes December 31st, and '+' denotes January 1st for each year)

- 0 +		- 1 +		…	- t +		…	- n-1 +		- n +		time
x		x + 1			x + t			x + n - 1		x+n		age
0	0	S_1	0		S_t	0		S_{n-1}	0	S_n	0	sum insured
0	$P_0 = P$	0	$P_1 = P$		0	$P_t = P$		0	$P_{n-1} = P$	0	0	premium
0	0	0	D_1		0	D_t		0	D_{n-1}		D_n	dividend

As displayed in Figure 1, while the premium payment is constant during the contract term, the benefit payment varies depending on the surplus appropriation scheme of the respective company. The dividend payments are based upon the development of assets and death benefits and thus also change over time. In the following, we consider a cohort of policies with maturity $n = T$.

Modeling mortality probabilities

Regarding the mortality probabilities, we distinguish two cases. For actuarial pricing, the mortality table by the German Actuarial Association ("DAV 2008 T") is used. However, when determining the actual number of deaths during the contract term (relevant for valuation and shortfall risk as well as the determination of actual policy reserves), we use a further development of the Lee-Carter (1992) model, which consists of a demographic part and a time series part. The central death rate or force of mortality $\mu_x(\tau)$ is modeled through

$$\ln\left[\mu_x(\tau)\right] = a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau} \iff \mu_x(\tau) = e^{a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau}},$$

where a_x and b_x are time constant parameters indicating the general shape of mortality over age and the sensitivity of the mortality rate at age x to changes in k_τ, respectively, where k_τ is a time varying index that reflects the general development of mortality over time, and $\varepsilon_{x,\tau}$ is an error term with mean 0 and constant variance. Brouhns, Denuit, and Vermunt (BDV) (2002) propose a modification to the model by modeling the realized number of deaths at age x and time τ, $D_{x,\tau}$, as

$$D_{x,\tau} \sim Poisson\left(E_{x,\tau} \cdot \mu_x(\tau)\right) \text{ with } \mu_x(\tau) = e^{a_x + b_x \cdot k_\tau}$$

where $E_{x,\tau}$ is the risk exposure at age x and time τ. The advantages of the BDV (2002) model are that the restrictive assumption of homoscedastic errors made in the Lee-Carter (1992) model is given up and that the resulting Poisson distribution is well suited for a counting variable, such as the number of deaths. The model can be calibrated via the Maximum-Likelihood approach using a uni-dimensional Newton method as proposed by Goodman (1979).[5] Lee and Carter (1992) propose to fit an appropriate ARIMA process on the estimated time series of k_τ, using Box-Jenkins time series analysis techniques to forecast k_τ,

$$k_\tau = \phi + \alpha_1 \cdot k_{\tau-1} + \alpha_2 \cdot k_{\tau-2} + \ldots + \alpha_p \cdot k_{\tau-p} + \delta_1 \cdot \varepsilon_{\tau-1} + \delta_2 \cdot \varepsilon_{\tau-2} + \ldots + \delta_q \cdot \varepsilon_{\tau-q} + \varepsilon_\tau$$

where p and q are chosen using Box-Jenkins time series analysis techniques and ϕ is the drift term. To model shocks to mortality, k_τ is multiplied by a factor δ. Values of δ less than one result in mortality rates greater than estimated, and for δ greater than one, mortality rates are smaller than estimated.

Policy reserves

The actuarial reserve for the considered endowment insurance for an $x+t$-year old insured at time t (and conditioned on the existence of the contract) is denoted by ${}_tV_x$, and its prospective calculation is given by

$${}_tV_x = S_{t+1} \cdot A_{x+t:\overline{n-t}|} - P \cdot \ddot{a}_{x+t:\overline{n-t}|}, \tag{2}$$

where S_{t+1} is the current guaranteed sum insured in case of death or survival payable at the end of year t, and P denotes the constant level premium. As before, the present values are calculated actuarially as defined in Equation (1) based on the mortality table and the calculatory (guaranteed) interest rate. Hence, the total portfolio policy reserve at the end of year t is determined by

$$PR_{t^-} = \left(N - \sum_{i=1}^{t} d_i \right) \cdot {}_tV_x, \tag{3}$$

[5] Standard Maximum-Likelihood methods are not feasible due to the presence of the bilinear term $b_x k_t$.

where N is the initial number of contracts sold and d_i is the actual number of deaths that occurred during year i, determined based on the BDV (2002) model. Thus, to obtain the policy reserve in the portfolio, the number of policies still in force is multiplied by the actuarial reserve for one contract.

Buffer account

As described in Table 1, the buffer account at the end of year t for all three companies, i.e. for all three surplus appropriation schemes under consideration, is given residually by

$$B_{t^-} = A_{t^-} - PR_{t^-} - IA_{t^-} - RD_{t^-} - E_t, \tag{4}$$

where IA_{t^-} is set to zero in case of the bonus system and in the case of shortening the contract term. The account RD_{t^-} is used only in the case of shortening the contract term and therefore set to zero in the other two cases. Furthermore, equity capital is kept at a constant level as assumed in Kling, Richter, and Russ (2007b), i.e. $E_t = E_{t-1}$.

At the end of the last year, i.e. in T^-, the buffer account is paid out to the policyholders in the sense of a terminal bonus after subtracting dividends. The terminal bonus (TB_T) cannot become negative and is given by the residual of the remaining assets and the policyholder accounts as well as dividends and equity capital, resulting in

$$TB_T = \max\left(B_{T^-} - D_T, 0\right) = \max\left(A_{T^-} - PR_{T^-} - IA_{T^-} - RD_{T^-} - E_T - D_T, 0\right).$$

Since the buffer account has been filled by the excess premiums of the policyholders, this procedure supports the comparability of the three companies with the different surplus appropriation schemes.[6]

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[6] We do not consider effects resulting from reserves, which are passed on to the next generation of policyholders. Here, we refer to Døskeland and Nordahl (2008) and Faust, Schmeiser, and Zemp (2011).

2.3 Modeling the asset side

Development of the asset base

The total initial capital A_0 consists of equity capital and the first premium payments, which are invested in a portfolio consisting of stocks and bonds that is assumed to follow a geometric Brownian motion

$$dA_t = \mu \cdot A_t \cdot dt + \sigma \cdot A_t \cdot dW_t^P ,$$

where μ is the drift of the assets, σ the asset volatility, and W^P a standard Brownian motion under the real-world measure P on the probability space $(\Omega, \mathcal{F}, P)$, where $\mathcal{F}$ is the filtration generated by the Brownian motion. The solution of the stochastic differential equation is given by (see Björk, 2009)

$$A_t = A_{(t-1)} \cdot e^{\left(\mu - \sigma^2/2 + \sigma \cdot \varepsilon\right)} = A_{(t-1)} \cdot e^{r_t} ,$$

with ε being a standard normally distributed random variable and r_t being the continuous one-period return of the portfolio with expected return $E\left(r_t\right) = m = \mu - 0.5 \cdot \sigma^2$ and a standard deviation of σ. Under the risk-neutral pricing measure Q, the drift of the process changes to the risk-free rate r_f. Different (μ,σ)-combinations representing different portfolio compositions are generated by assuming that

$$r_t = a \cdot r_S + \left(1 - a\right) \cdot r_B ,$$

where r_B and r_S stand for the continuous one-period returns of bonds and stocks, respectively, which follow a normal distribution with expected values of $E(r_B) = m_B$ and $E(r_S) = m_S$, standard deviations of σ_B and σ_S, and a coefficient of correlation of ρ.

To account for decrements in the portfolio of policyholders due to death, one needs to distinguish between the end and the beginning of a year with respect to the evolution of the net assets, i.e. assets invested in the capital market minus payments for deaths during year t. The term A_{t^-} thus describes the value of assets at the end of year t, which is given by

$$A_{t^-} = A_{(t-1)^+} \cdot e^{r_t} - S_t \cdot d_t , \; with \; A_{0^-} = 0, A_{0^+} = P \cdot N + E_0 , \tag{5}$$

where S_t is the sum insured prevailing in year t (that depends on the surplus schemes), and d_t is the number of deaths between time t-1 and t, N is the number of contracts sold, and P is the constant premium for each individual contract (same for all surplus schemes).

Furthermore, equityholders receive annual dividend payments D_t that depend on the development of assets and death benefit payments:

$$D_t = \beta \cdot \max\left(A_{t^-} - A_{(t-1)^+}, 0\right),$$

where β denotes the fraction of the increase in assets that is paid out as dividends.

The insurer is solvent if the buffer account plus equity capital is positive, $B_{t^-} + E_t \geq 0$, implying that assets are sufficient to cover the liabilities, i.e., $A_{t^-} \geq PR_{t^-} + IA_{t^-} + RD_{t^-}$. In this case, the insurer pays out dividends D_t to the equityholders only if $B_{t^-} \geq D_t$, leading to

$$B_{t^+} = B_{t^-} - D_t, \; if \; B_{t^-} \geq D_t.$$

The dividend payment is set to zero if the insurer is solvent but does not have enough reserves to pay the dividends. Hence, if $B_{t^-} + E_t \geq 0$, but $B_{t^-} < D_t$, then $D_t = 0$.[7] At the beginning of the subsequent year, premiums are due, which results in an asset development (see also Equation (5)) given by

$$A_{t^+} = A_{t^-} - D_t + P \cdot \left(N - \sum_{i=1}^{t} d_i \right) = A_{(t-1)^+} \cdot e^{r_t} - S_t \cdot d_t - D_t + P \cdot \left(N - \sum_{i=1}^{t} d_i \right).$$

If the insurer is insolvent, i.e., $B_{t^-} + E_t < 0$ and thus $A_{t^-} < PR_{t^-} + IA_{t^-} + RD_{t^-}$, the equity capital is not sufficient to cover the losses, the company is closed down, and the remaining funds $A_{(t-1)^+} \cdot e^{r_t} \cdot (1-c)$ are distributed to the remaining policyholders in the portfolio, reduced by the costs of insolvency c. Note that death benefits are not fully paid out, but the beneficiaries receive a remaining fraction of the assets.

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7 The insurer also remains solvent if the buffer account becomes negative, but equity capital is sufficient to cover the losses in this period, i.e., $B_{t^-} < 0, \textit{but } B_{t^-} + E_t \geq 0$. In this case, equity capital is reduced by the amount of the loss and $B_{t^-} = 0$. In the next period, we assume that the amount of equity capital is increased again to the original amount by using gains from the next period (see Equation (4)).

2.4 Surplus appropriation schemes

After having defined the general development of assets and liabilities, we now further specify and distinguish the three companies under consideration that differ only in their surplus appropriation scheme. Let r^G denote the calculatory and guaranteed interest rate, and r_t^P be the actual policy interest rate credited to the policy reserves for period t-1 to t, which is to be determined at time t-1.[8] To smooth market returns and to obtain less volatile and more stable returns, the surplus distribution approach is based on Grosen and Jørgensen (2000). Here, a reserve-based system is used with

$$r_t^P = \max\left\{ r^G, \alpha \cdot \left(\frac{B_{(t-1)^+}}{PR_{(t-1)^-} + IA_{(t-1)^-} + RD_{(t-1)^-}} - \gamma \right) \right\}, \tag{6}$$

where γ indicates a required proportion of the buffer account divided by the policyholder's accounts, which constitute the guaranteed liabilities,[9] i.e. γ represents the target buffer ratio. The second adjusting parameter for distributing surplus to the policyholders is the surplus distribution ratio α. It controls the fraction of the excess amount of the target buffer ratio, which is actually credited to the policyholders.[10]

The policy reserves earn at least the guaranteed interest rate r^G and serve as the basis for determining the surplus to be credited to the policyholders. The absolute amount of surplus generated in the t-th year is thus given by the difference between the policy interest rate and the actuarial interest rate, multiplied by the policy reserve:

$$PR_{(t-1)^-} \cdot \left(r_t^P - r^G \right). \tag{7}$$

While the amount of surplus is calculated the same way for all three companies (and, hence, the surplus distribution approach is the same), the appropriation scheme and thus the way the surplus is distributed to policyholders differs and plays an important role regarding the insur-

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8 This is in line with the declaration in advance, see Schradin, Pohl, and Koch (2006, p. 14).

9 Since the buffer account should provide a cushion to absorb losses with respect to the guarantees on the balance sheet's liabilities side, all three policyholder accounts have to be considered in the denominator.

10 Usually, regulations, such as those in Germany, specify a maximum period of time for the surplus to be kept in the buffer account and buffer, respectively, until it has to be credited to the insureds (see, e.g., Schradin, Pohl, and Koch (2006, p. 14)).

ance payouts and thus also for the evolution of the asset base (see Figure 1). The different schemes under consideration are presented in detail in the following subsections.

Company 1: Bonus system

In the case of the bonus system, the surplus is used to increase the initially guaranteed sum insured S_1 (death and survival benefit) by calculating a new insurance with the same time to maturity (and the same type) like the original insurance, using the surplus as a single premium for the new contract. By using the actuarial equivalence principle, the surplus per insured results in an additional sum insured ΔS_t of

$$\Delta S_t = \frac{PR_{(t-1)^-} \cdot \left(r_t^P - r^G\right) \Big/ \left(N - \sum_{i=1}^{t} d_i\right)}{A_{x+t:\overline{T-t|}}}$$

which leads to an increased sum insured of

$$S_{t+1} = S_t + \Delta S_t .$$

In this setting, the surplus insurance also participates in future surplus and thus involves cliquet-style interest rate effects. In particular, the increased sum insured impacts the development of the policy reserves (see Equations (2) and (3)) as well as the amount of surplus that can be distributed to the policyholders (see Equation (7)), thus inducing cliquet-style effects.

Company 2: Interest-bearing accumulation

In the case of the interest-bearing accumulation, the sum insured is kept constant, i.e., we set $S_t = S_1, \forall t = 1,\dots,T$. Hence, surplus is not used to increase the sum insured, but instead is accumulated on a separate account IA_t. Once funds are credited to this account, they belong to the policyholders and cannot be withdrawn anymore. This implies an interest rate guarantee of at least zero percent. The account is paid out at maturity in case of survival. If the policyholder dies during the contract period, only the constant sum insured is paid out, and the remainder is kept by the insurer and – in the form of the (non-guaranteed and used for smoothing) buffer account – is later eventually paid out to the remaining policyholders that are still alive at maturity as an optional bonus. Hence, this surplus appropriation scheme emphasizes the survival benefit as compared to death and survival benefit in the case of the bonus system. The recursive forward projection of the interest-bearing account is given by

$$IA_{t^-} = IA_{t-1^-} \cdot \left(1 + r^{IA}\right) \cdot \left(1 - d_t \Big/ \left(N - \sum_{i=1}^{t-1} d_i\right)\right) + PR_{(t-1)^-} \cdot \left(r_t^P - r^G\right),\ IA_0 = 0 .$$

At time *t*, the account value is calculated based on its value in the previous period and an interest rate r^{IA} and is adjusted for deaths, i.e. funds that belonged to policyholders who died within the *t*-th year are passed on to the collectivity of policyholders. Finally, new surplus is added to the account.

Company 3: Shortening the contract term

In the third case, the surplus is used to decrement the remaining years to maturity, thus resulting in earlier benefit payments to the policyholders, where S_t is kept constant. Hence, the total contract term $n(t)$ is considered as a function of time *t*, i.e. *n* can be reduced from each period to the next, starting with $n(0) = T$.

Since the insurer operates in discrete time with one year representing one period, the contract term is not reduced until the total surplus earned is sufficient to finance the gap between the actuarial reserve for $n(t-1)$ and the actuarial reserve for a reduced contract term of at least one year. If the surplus is not sufficient to reduce the contract term for one year or surplus remains after reducing the contract term, the remaining surplus amount RD_t is transferred to the next year. Analogously to the case of the interest-bearing accumulation, the fraction of deaths and an interest rate r^{RD} is accounted for:

$$RD_{t^-} = RD_{(t-1)^+} \cdot \left(1 + r^{RD}\right) \cdot \left(1 - d_t \Big/ \left(N - \sum_{i=1}^{t-1} d_i\right)\right) + PR_{(t-1)^-} \cdot \left(r_t^P - r^G\right),\ RD_0 = 0 .$$

Hence, we first determine the value of the actuarial reserve for an unchanged contract period at time *t*, i.e. for $n(t-1)$, denoted by ${}_tV_x\left(n(t-1)\right)$ and determined analogously to Equation (2), where *n* must be replaced by $n(t-1)$. Next, we add to this the surplus RD_{t^-} per policyholder, which results in

$${}_tV_x^{surplus}\left(n(t-1)\right) = {}_tV_x\left(n(t-1)\right) + RD_{t^-} \Big/ \left(N - \sum_{i=1}^{t} d_i\right).$$

Third, we calculate the actuarial reserves for a contract period decremented by *h* years, where *h* starts with zero and is incremented successively to the total remaining contract term at time *t*, i.e. $n(t-1) - t$. This is given by

$$h_{\max}(t) = \max_{h \in H(t)} \left\{ h : {}_tV_x^{surplus}\left(n(t-1)\right) - {}_tV_x\left(n(t-1)-h\right) \geq 0 \right\}$$

where ${}_tV_x\left(n(t-1)-h\right)$ is determined based on Equation (2) by replacing n with $n(t-1)-h$ and $H(t) = \{0,\ldots,n(t-1)-t\}$. By $h_{\max}(t)$, we indicate the maximum number of years by which the contract term can be reduced in year *t*. The new policy period, starting from year *t*, is given by

$$n(t) = n(t-1) - h_{\max}(t).$$

The policy reserve (for one individual contract) ${}_tV_x\left(n(t)\right)$ can then be calculated, and the surplus account is defined by

$$RD_{t^+} = \left({}_tV_x^{surplus}\left(n(t-1)\right) - {}_tV_x\left(n(t)\right) \right) \cdot \left(N - \sum_{i=1}^{t} d_i \right).$$

Finally, we can determine the actuarial reserve for the pool of contracts, i.e., PR_{t^-}, analogously to Equation (3) by replacing ${}_tV_x$ with ${}_tV_x\left(n(t)\right)$. At maturity, any remaining amount RD_T is paid out to the policyholders.

2.5 Evaluating the surplus appropriation schemes from different perspectives

To assess the impact of the three surplus appropriation schemes from perspectives of the insurer and the policyholder, we calculate the company's shortfall risk and the policyholder's net present value of the contract. Of course, these two figures are certainly relevant to both parties. For instance, the net present value from the policyholder's perspective can also be interpreted as the counter value of the contract to the insurer.[11] Overall, however, both numbers will be relevant to the insurer and the policyholder and are laid out in what follows.

A shortfall of the company occurs if the value of the assets A_{t^-} falls below the value of liabilities, $A_{t^-} < PR_{t^-} + IA_{t^-} + RD_{t^-}$ (or, equivalently, if $B_{t^-} + E_t < 0$). Hence, the shortfall probability is defined as

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[11] Furthermore, policyholders often evaluate their contracts based on individual preferences instead of assuming a risk-neutral valuation approach that implicitly assumes replicability of cash flows. However, the fair value expressed by the net present value is still a relevant figure for policyholders.

$$SP = P\left(T_s \le T\right),$$

where the time of default is defined as $T_s = \inf\left\{t : A_{t^-} < PR_{t^-} + IA_{t^-} + RD_{t^-}\right\},\ t = 1,...,T$.

The net present value (*NPV*) of the contract is calculated as the expected value under the risk-neutral probability measure Q of the difference between the discounted contract payoff and the discounted sum of premium payments taking into account the case of default. For an individual policyholder, the *NPV* is thus given by

$$\begin{aligned} NPV = {} & E^Q\left(\sum_{t=0}^{T-1}\left({}_t p'_x \cdot q'_{x+t} \cdot S_{t+1} \cdot e^{-r_f\cdot(t+1)} - {}_t p'_x \cdot P \cdot e^{-r_f\cdot t}\right)\cdot 1\left\{T_s > T\right\}\right) \\ & + E^Q\left({}_T p'_x \cdot \left(S_T + \left(IA_{T^-} + RD_{T^-} + TB_T\right)\cdot \frac{1}{N - \sum_{i=1}^{T} d_i}\right)\cdot e^{-T\cdot r_f} \cdot 1\left\{T_s > T\right\}\right) \\ & + E^Q\left(\sum_{t=0}^{T-1}\left({}_t p'_x \cdot \left(A_{t^+} \cdot e^{r_{t+1}}\right)\cdot(1-c)\cdot \frac{1}{N - \sum_{i=1}^{t} d_i} \cdot e^{-r_f\cdot(t+1)} - {}_t p'_x \cdot P \cdot e^{-r_f\cdot t}\right)\cdot 1\left\{T_s = t+1\right\}\right) \end{aligned}$$

where ${}_t p'_x$ and q'_{x+t} are the survival and death probabilities, respectively, derived through the BDV (2002) model. Mortality and market risks are assumed to be independent,[12] and the insurance company does not demand a risk premium for mortality risk.[13] In contrast to the actuarial pricing, which does not account for the surplus distribution or default, the possibility of default and the surplus distribution and appropriation is considered in the calculation of the "fair" net present value. Thus, the annual premium payment P is the same for all three surplus schemes, but the amount of surplus differs. If no shortfall occurs, i.e. if $T_s > T$, the policyholder receives a death benefit or a survival benefit, which also includes the terminal bonus. If default occurs during the contract term, the remaining assets are distributed among the policyholders still alive (and to the heirs of those who died within the year of default).

3. NUMERICAL ANALYSIS

In this section, numerical results are presented based on the model introduced in the previous section with respect to the insurer's risk exposure and the policyholder's net present value for

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12 See, e.g., Carriere (1999, p. 340), Gründl, Post, and Schulze (2006), Hanewald (2011).

13 See, e.g., Bacinello (2003, p. 468), Gründl, Post, and Schulze (2006).

each of the three surplus appropriation schemes.[14] After presenting the input parameters, we next study the extent to which differences in shortfall risk arise for the three surplus appropriation schemes with regard to varying asset portfolios and different shocks to mortality. Second, we extend this viewpoint and study the effects on the net present value. Numerical results are derived using Monte Carlo simulation methods based on the same set of 500,000 asset paths.

Input parameter

The underlying policies are participating life insurance contracts issued to $x = 35$ year old males with a contract term of $T = 30$ years. With an initial sum insured of $S_1 = 1$, the actuarial annual premium is given by $P = 0.0247$. A total number of $N = 100{,}000$ contracts are sold. Assumptions about the evolution of the assets are based on the historical performance (1988 until 2009) of two representative German total return indices. The estimation for the stocks, which is based on monthly data for the German stock market index DAX, results in an expected one-period return $m_S = 8.00\%$ and a volatility $\sigma_S = 21.95\%$. The estimation for the bonds, which is based on monthly data for the German bond market index REXP, leads to an expected one-period return of bonds $m_B = 6.02\%$ and a volatility of bonds $\sigma_B = 3.30\%$. The estimated correlation coefficient of returns of the two indices is $\rho = -0.1648$.[15] Furthermore, we assume the distribution ratio to be $\alpha = 70\%$ and the target buffer ratio to be $\gamma = 10\%$. Unless stated otherwise, we assume further relevant parameters to be those stated in Table 2.

Table 2: Parameters for the analysis

Expected one-period returns of stocks	m_S	8.00%
Volatility one-period returns of stocks	σ_S	21.95%
Expected one-period returns of bonds	m_B	6.02%
Volatility one-period returns of bonds	σ_B	3.30%
Correlation between stocks and bonds	ρ	-0.1648
Stock portion	a	10%
Guaranteed interest rate	r^G	2.25%
Rate of interest for the interest-bearing accumulation account	r^{IA}	0%
Rate of interest for the account RD_t	r^{RD}	0%

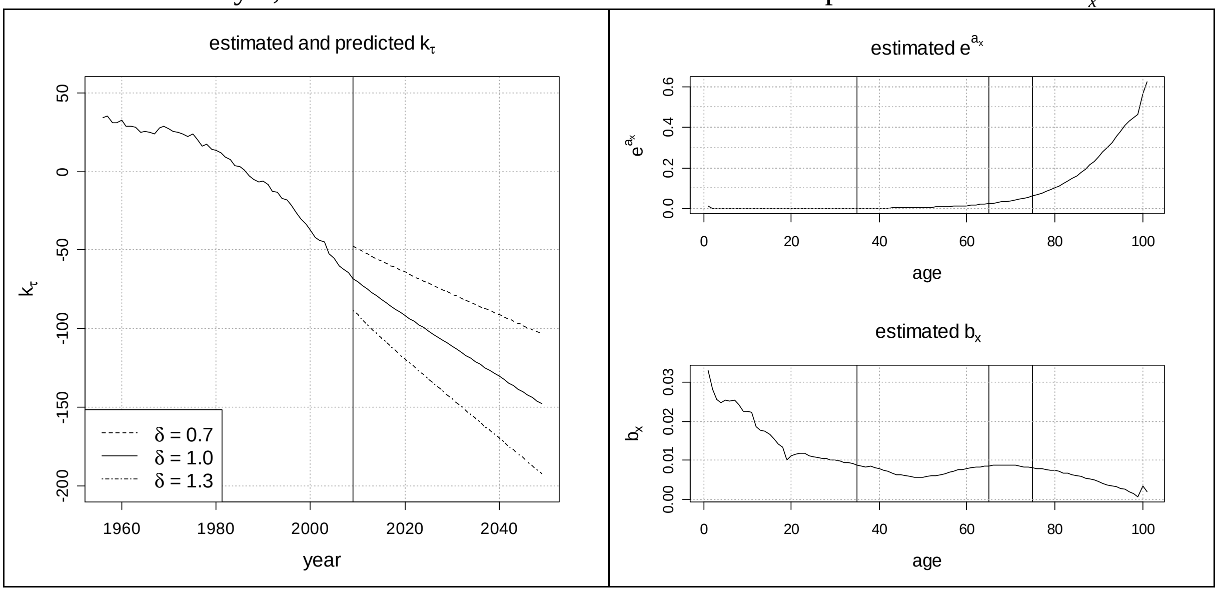
[14] For robustness, we also calculated the mean loss in addition to the shortfall probability and found that the general patterns of the results were similar.

[15] The correlation coefficient is significant at a level of 0.01.

Risk-free rate	r_f	3%
Number of contracts sold	N	100,000
Sum insured in t = 0	S_1	1
Level premium for T = 30	P	0.0247
Equity in t = 0	E_0	600
Contract term	T	30
Age of the policyholders in t = 0	x	35
Dividend payment ratio	β	5%
Distribution ratio	α	70%
Target buffer ratio	γ	10%
Reduction coefficient for costs of insolvency	c	20%
Shock to mortality	δ	1
Inception date	τ_0	2009

The estimation of the parameters for the BDV (2002) model is conducted on the basis of mortality data for Germany for the years 1956 until 2008. Numbers of deaths and exposure to risk are available through the Human Mortality Database. For the years 1956 to 1990, data for East and West Germany are combined, whereas from 1990 to 2008, data for the total of Germany is used. The estimated values of e^{a_x}, which can be interpreted as a mean central death rate at age x, and b_x are given in Figure 2.

Figure 2: Estimated values of the mortality index k_τ and predicted values of k_τ for different shocks to mortality δ, and estimated values of the time constant parameters e^{a_x} and b_x

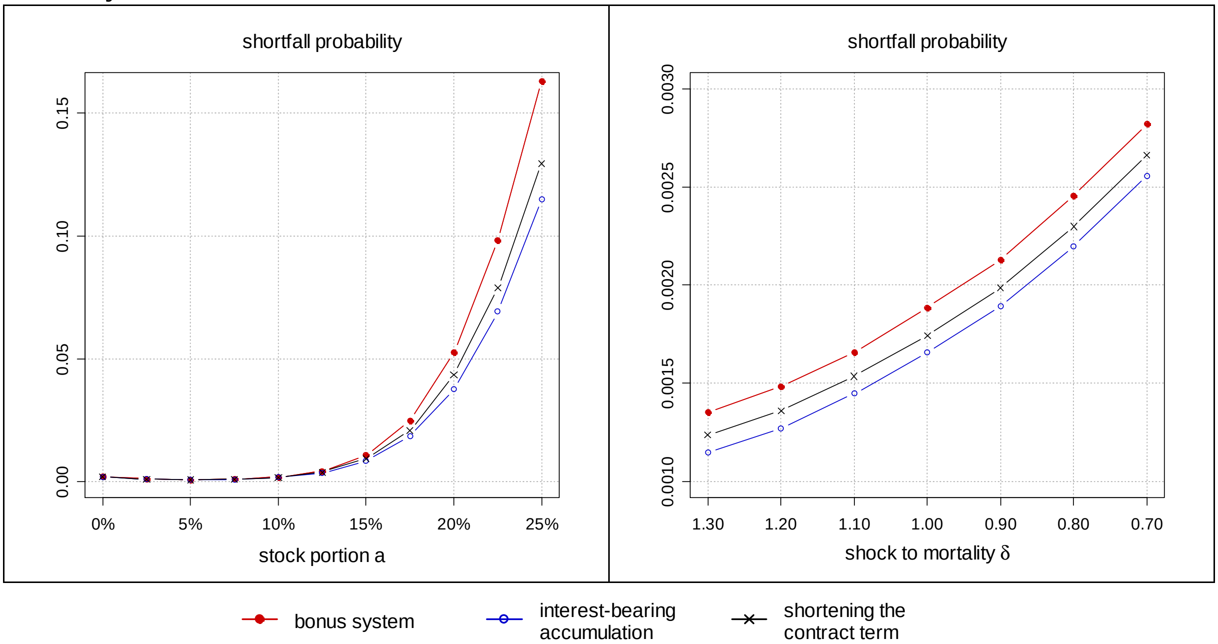


Furthermore, Figure 2 shows the estimated time series k_τ as well as its prediction by using Box-Jenkins time series analysis techniques, illustrating the effect of different shocks to mortality. Based on the Bayesian information criterion, an ARIMA (3,1,0) process is used. Its parameters are given as follows: drift ϕ = -1.9275 (0.5295), α_1 = -0.0224 (0.1229), α_2 = 0.1124 (0.1227), and α_3 = 0.4608 (0.1246), where the standard errors are given in parentheses. Residual autocorrelation can be excluded after applying the Box-Ljung test (Portmanteau test), ACF and PACF analyses.

The impact of surplus appropriation schemes on shortfall risk

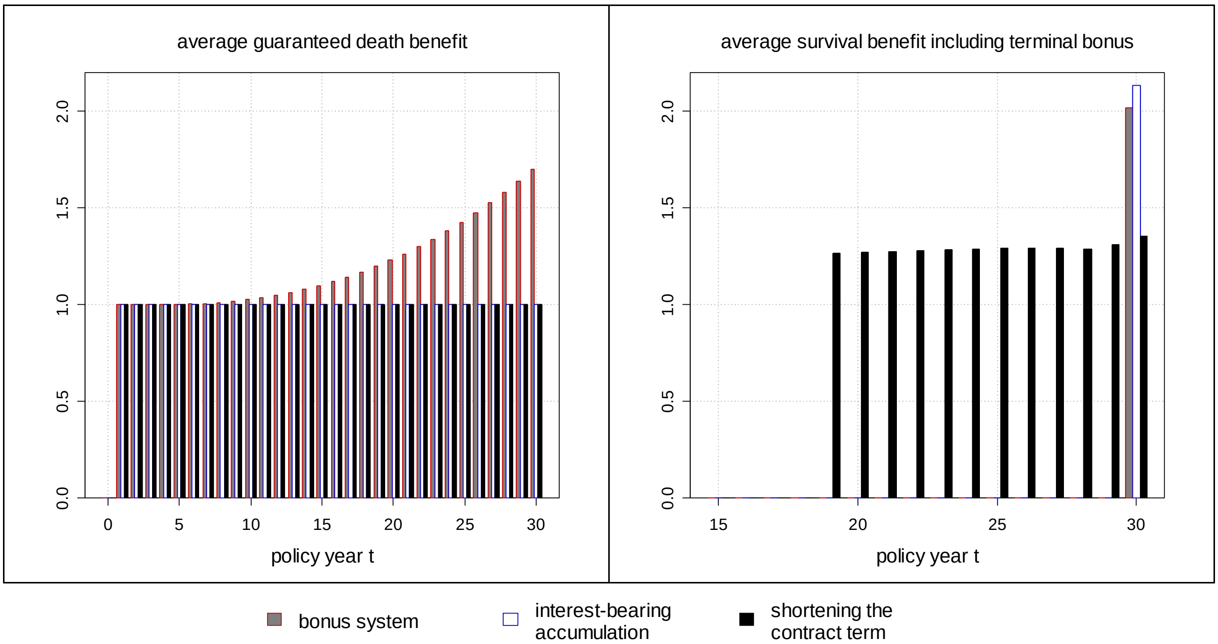
First, in Figure 3, we study the shortfall risk of the three companies for different asset allocations (left graph) and different levels of mortality (right graph). As expected, a riskier investment leads to substantially higher shortfall probabilities. One can see a rapid increase in the shortfall probability as the stock ratio grows. For instance, stock ratios between 0% and 5% result in a shortfall probability of around 0.001, while a stock ratio of 20% implies a shortfall probability of about 0.05, and for stock ratios of 20% upwards, the default risk increases exponentially. Thus, despite the fact that gains at the capital market are smoothed via the buffer account (see Equation (6)), once the surplus is credited to the insureds, it is transformed to guarantees, which have to be generated in subsequent periods.

Figure 3: Shortfall probability for varying investments in stocks and varying shocks to mortality



Furthermore, the results show that, even though the amount of surplus is calculated in the same way according to the smoothing scheme given in Equation (6), the specific type of appropriation scheme can have a very different impact on shortfall risk. In particular, the bonus system leads to the highest shortfall probability and thus dominates the system of shortening the contract term, which, in turn, dominates the interest-bearing accumulation scheme. This order is mainly due to the different types of guarantees implied by the considered schemes as illustrated in Figures 4 and 5, which is highest in the case of the bonus system, since, once the surplus is credited to the policyholder, the guaranteed death and survival benefit are raised, which in turn increases the policy reserves. This leads to cliquet-style effects, since a higher reserve resulting from a higher sum insured is subject to the effect of compound interest, i.e. the guaranteed interest rate r^G is also paid on the surplus. As illustrated in Figure 4 (left graph), this implies an increasing guaranteed death benefit starting from around the 10th policy year on, which at T = 30 reaches a value that is more than 70% higher than in case of the other two systems. In contrast to the bonus system, the guaranteed death benefit is constant and equal to one in case of the interest-bearing accumulation and shortening the contract term. The average guaranteed survival benefit at maturity T = 30 of the interest-bearing accumulation is slightly higher than the bonus system, but does not compensate for the considerably higher death benefits during the contract term (right graph in Figure 4).

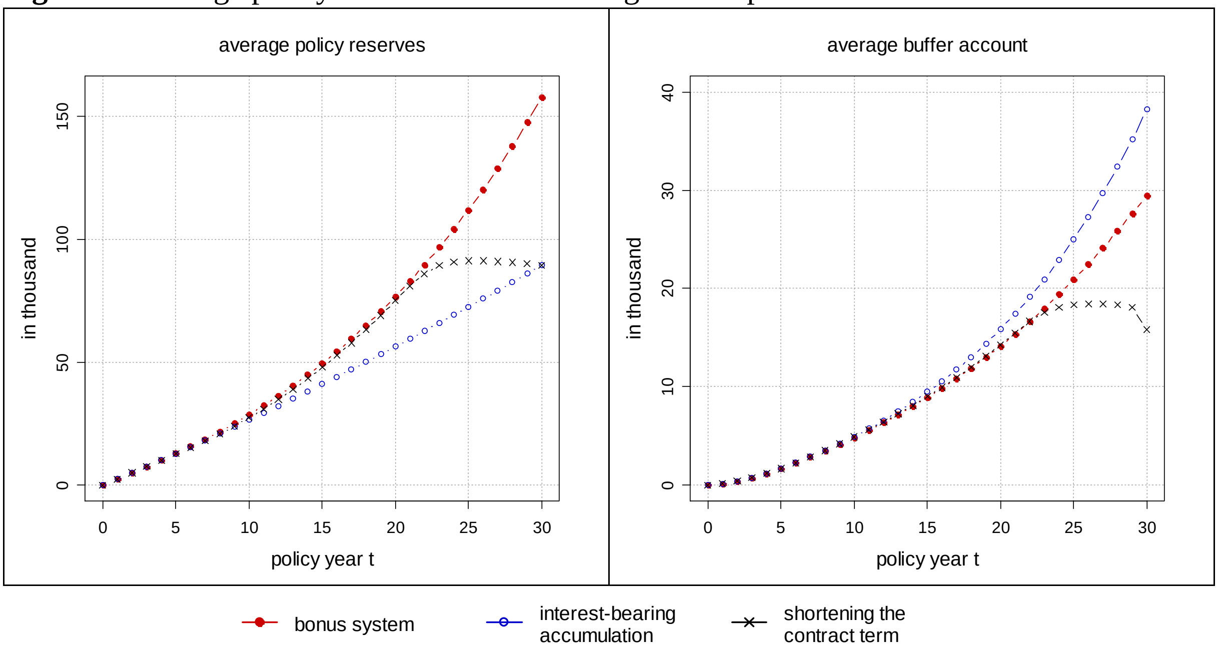
Figure 4: Average guaranteed sum insureds and average survival benefit including the terminal bonus



Notes: Average guaranteed death benefit$_t = E(S_t \mid T_s > t)$, *where* T_s *denotes the time of default; average survival benefit including terminal bonus*$_t = E\left(S_t + TB_t^{per\ insured} \mid T_s > t \wedge T_{adj} = t\right)$, *where* T_{adj} *denotes the time when the survival benefit is paid.*

The system with shorting the contract term is more difficult to compare to the other two systems as the survival benefit is paid out earlier between the 19th and the 30th policy year, on average. However, when considering the left graph in Figure 5, the development of the policy reserves shows that the bonus system implies the highest average policy reserves, followed by shortening the contract term and the interest-bearing accumulation. Furthermore, Figure 5 (right graph) shows how the buffer account is built up over time and that the interest-bearing accumulation features the highest value throughout the contract term, followed by the bonus system and shortening the contract term.[16] Thus, even though the comparability is still limited, the order of the three systems with respect to shortfall risk can be generally confirmed by analyzing the policy reserves, the buffer account, and guaranteed sums insured in case of death and survival, implying that the bonus system has the highest risk, followed by shortening the contract term and the interest-bearing accumulation.

Figure 5: Average policy reserves and the average development of the buffer account



Notes: Average policy reserves$_t = E\left(PR_{t^-} \mid T_s > t \wedge T_{adj} \geq t\right)$, *where* T_s *denotes the time of default and* T_{adj} *the time when the survival benefit is paid; average buffer account*$_t = E\left(B_{t^+} \mid T_s > t \wedge T_{adj} \geq t\right)$.

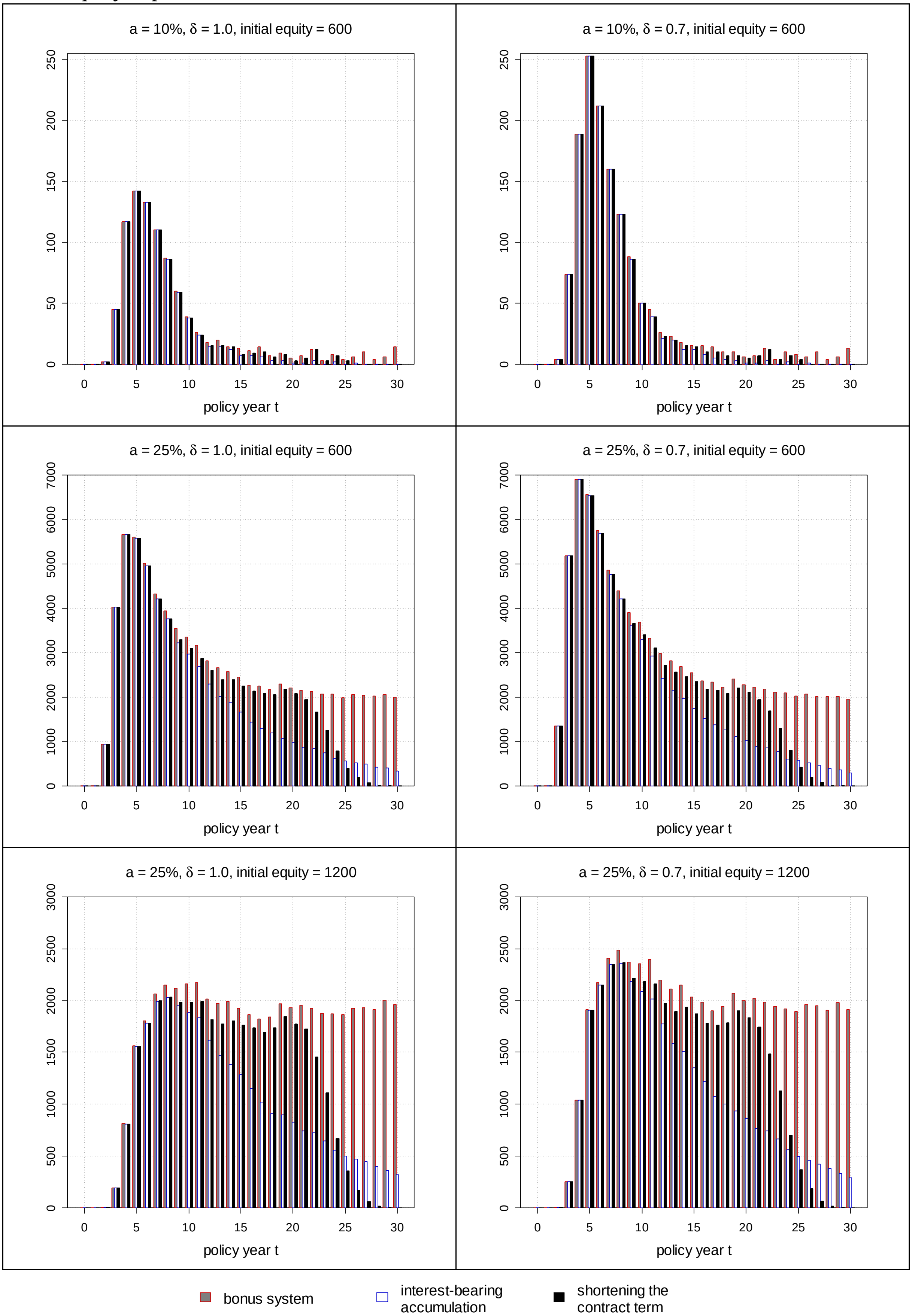
[16] Note that the values of the average buffer account for the system of shortening the contract term in the last two policy years 29 and 30 may vary when using different sets of random numbers due the low number of scenarios in which the survival benefit is paid in these last periods. In most cases, the survival benefit is paid out until the 28th year.

Thus, depending on the surplus appropriation scheme, the companies can afford a riskier asset base, implying a higher expected return while keeping the shortfall risk constant, while still achieving the same shortfall probability. For example, a stock portion of a = 19.8% in the case of Company 1 (bonus system), a = 20.5% for Company 3 (shortening the contract term), and a = 21.1% in the case of Company 2 (interest-bearing accumulation) lead to the same shortfall probability of 0.05.

Moreover, it becomes apparent that the gap, i.e., the absolute difference in shortfall risk associated with the three surplus appropriation schemes expands with an increasing stock portion, even though gains at the capital market are smoothed via the buffer account. This implies that it is considerably riskier for Company 1 (bonus system) to *ceteris paribus* hold an asset portfolio containing more high-risk assets than it is for Company 2 (interest-bearing accumulation) and Company 3 (shortening the contract term), as a more risky asset stratgey implies a higher surplus, which in turn emphasizes the cliquet-style effects and thus the difference between the three surplus appropriation systems. This behavior is also illustrated in Figure 6, where the number of shortfalls is displayed for different stock portions, shocks to mortality, and higher initial equity capital. In particular, a comparison of the case with a = 10% and a = 25% (for $\delta = 1$) shows that the bonus system and shortening the contract term exhibit a much higher number of shortfalls when increasing the stock portion, especially for higher contract years (starting from the 13th year), as compared to the interest-bearting accumulation.

Next, we focus on the shortfall risk resulting from a change in mortality as illustrated in the right graph of Figure 3. We thereby analyze the insurer's shortfall probability as a function of a shock to mortality (δ), for $\delta \in [0.7, 1.3]$, where $\delta = 0.7$ can be considered to represent a pandemic. Since these shocks to mortality are modeled by multiplying the negative time trend k_τ by δ, values of δ less than 1 increase the mortality rates, while values of $\delta > 1$ decrease the mortality rates (see also Figure 2, left graph). The results demonstrate that, even though the level of shortfall risk is reasonably small for a stock portion of 10% and for the given set of parameters, the relative changes in risk with respect to a shock to mortality are not negligible. For example, a change from $\delta = 1$ (no shock to mortality) to $\delta = 0.7$ results in about 25% more deaths within the considered contract period of $T = 30$ years, and, in the case of the bonus system, increases the insurer's risk by about 50%. In addition to this general effect of higher shortfall probability for an increasing shock to mortality, the order of the surplus appropriation schemes with respect to the level of shortfall risk remains similar to the case of varying the asset portfolio.

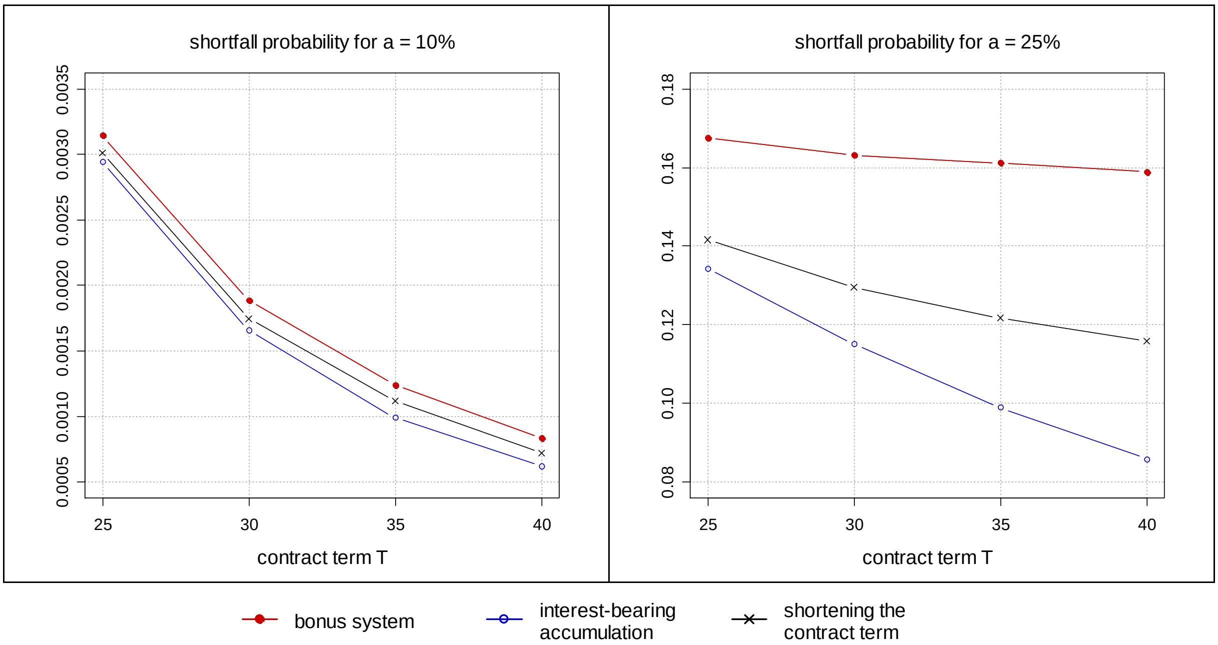
Figure 6: Number of shortfalls for different stock portions, shocks to mortality, and higher initial equity capital



However, in contrast to the asset base, for the considered range of parameters, the difference between the shortfall risk of the three companies remains fairly stable without showing a gap as in the case of high stock portions. Thus, in contrast to the stock portion, where particularly the bonus system exhibits an increasing gap compared to the two other schemes, all three companies are affected in a similar severe way by shocks to mortality. This can also be seen from Figure 6 when comparing the left column to the right column. In particular, an increase in mortality by $\delta = 0.7$ implies a considerably higher number of shortfalls during the first contract years, where the difference between the three surplus schemes is still negligible with respect to guaranteed death or survival benefit as well as policy reserves (see Figures 4 and 5), while towards the end of the contract term, the number of defaults remain overall stable, even though the number of deaths increase for higher ages (compensated by a higher buffer account towards the end of the contract term, see right graph of Figure 5). This holds true for a given stock portion, even when increased to 25% and when increasing the amount of initial equity from 600 to 1,200, which implies that more defaults occur towards the end of the contract period. In particular, the increase in E_0 from 600 to 1,200 (for a = 25% and $\delta = 1.0$, see second and third row in Figure 6, left graphs) implies a reduction in the shortfall probability from SP = 16.4% to SP = 10.3% in case of the bonus system, from SP = 13.0% to SP = 7.2% for shortening the contract term, and from SP = 11.5% to SP = 5.9% in case of the interest-bearing accumulation, which illustrates the importance of the initial buffer situation. Hence, while there is a considerable difference between the three schemes when comparing different stock portions (different rows in Figure 6), their reaction with respect to shocks to mortality is similar and the amount of the initial equity capital (and thus the initial buffer) has a considerable impact on shortfall risk.

Another important factor in life insurance is the contract duration. Figure 7 displays the shortfall risk for different contract terms and different stock portions (a =10%, 25%). All other parameters being unchanged, including the sum insured, the level premium has to be adjusted (e.g., in case of T = 40 to P = 0.0181). Here, three effects interact. First, due to the lower premium payments for a longer contract term, reserves build up more slowly, which defers the surplus distribution mechanism. Second, the predicted time varying index k_τ that reflects the general development of mortality over time is strictly monotonic decreasing as displayed in Figure 2. This enlarges the discrepancy in mortality between the projected mortality and the mortality given by the mortality table (premium calculation) for the additional 10 policy years. Third, mortality rates are higher for people aged 66 to 75 compared to 65 years and younger, which implies that the additional contract period pronounces the shortfall risk associated with guaranteed death benefits and, therefore, the company with the bonus system.

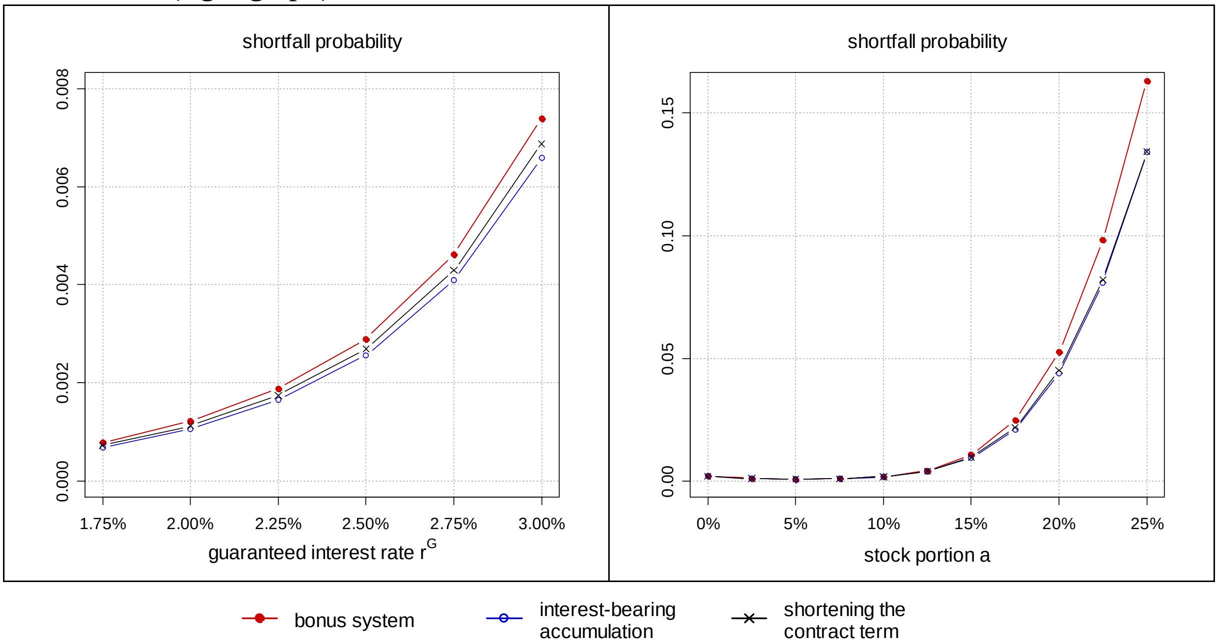
Figure 7: Shortfall probability as a function of the contract term T for a stock portion of a = 10% and for a = 25%



Generally, the shortfall probability decreases with an extension of the contract term of ten years. Figure 7 further reveals interaction effects between time to maturity and stock portion. Here, while the risk level is still decreasing for all three surplus schemes for a stock portion of a = 10% (left graph in Figure 7), for a stock portion of a = 25% (right graph in Figure 7), the level of shortfall risk for the bonus system remains stable, which can to a lesser extent also be observed for the system with shortening the contract term, while the shortfall probability of the interest-bearing accumulation still exhibits a clearly decreasing level. This shows that the gap in the shortfall probability between the bonus system and the interest-bearing accumulation as well as shortening the contract term increases considerably for riskier assets and an increased contract term (see also Figures 4, 5, and 6). This result is particularly relevant against the background of long-term contracts. Thus, shortfall risk cannot be as effectively reduced for a longer term in the case of higher stock portions when using the bonus system and also when shortening the contract term, i.e., in the case of the two other companies with emphasis on guaranteed death and survival benefits (possibly paid out earlier).

Another key risk driver in this context is typically the guaranteed interest rate. Results are displayed in Figure 8 (left graph) and show that a higher contractual guarantee in the form of a higher interest rate guarantee leads to higher shortfall risk for all three systems, stressing again that the bonus system is associated with the highest shortall risk, the system of shortening the contract term constitutes the second highest shortfall risk, and the interest-bearing accumulation has the lowest shortfall risk. It can be further noticed that these differences in shortfall risk increase with an increase in the guaranteed interest rate.

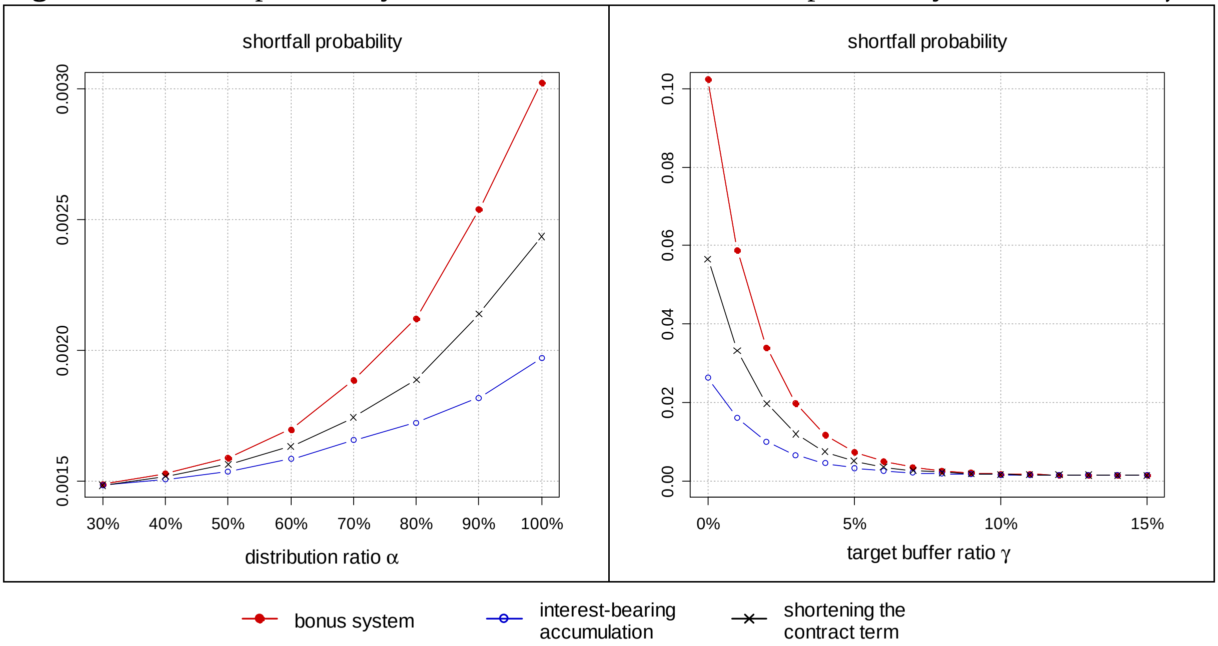
Figure 8: Shortfall probability as a function of the guaranteed interest rate r^G (r^{IA} = 0%, r^{RD} = 0%, left graph) and shortfall probability for different asset allocations for $r^G = r^{IA} = r^{RD}$ = 2.25% (right graph)



The right graph in Figure 8 shows the shortfall probability as a function of asset allocation, where the interest rates for the interest-bearing accumulation account and for shortening the contract term are increased and set equal to the guaranteed interest rate, i.e. $r^G = r^{IA} = r^{RD}$ = 2.25%. In this case, the shortfall risk of the interest-bearing accumulation is almost equal to the system of shortening the contract term. Nonetheless, these two schemes are significantly dominated by the bonus system with regard to shortfall risk.

Furthermore, it is relevant to assess the impact of the parameters for the surplus distribution mechanism on the shortfall risk with regard to the different surplus appropriation schemes. Figure 9 presents the shortfall probability for varying values of the surplus distribution ratio α and target buffer ratio γ. In the left graph, γ equals 10%, and in the right graph, α is set to 70%, while the stock ratio is kept constant at a = 10% for both. As can be seen in Equation (6), these parameters control the surplus distribution to the policyholders. In general, the insurer's shortfall risk varies considerably with a *ceteris paribus* increase in α and a *ceteris* *paribus* decrease in γ, respectively. A decrease of γ, e.g., from 10% to 5%, more than quadruples the shortfall probability for Company 1 (bonus system).

Figure 9: Shortfall probability as a function of α and shortfall probability as a function of γ



These effects, i.e. the more surplus that is paid to the insureds and thus transformed into guarantees, the higher the shortfall probability rises, do not occur equally for all three companies but depend on the extent of the guarantee and thus on the type of surplus appropriation scheme. The results demonstrate that particularly Company 1 with the bonus system is most sensitive to changes in those two parameters, followed by Company 3, which applies the system of shortening the contract term, and finally Company 2 with the interest-bearing accumulation.

The impact of surplus appropriation schemes on the net present value

We next study the impact of the three surplus appropriation schemes from an insured's point of view by examining the policyholder's (fair) net present value. Figure 10 displays results for different stock portions, shocks to mortality, guaranteed interest rates, and distribution ratios for a cost of insolvency of c = 20%. Results for, e.g., the target buffer ratio are similar and are thus omitted.

Figure 10: Policyholder net present value for varying assets allocations, shocks to mortality, guaranteed interest rate, and distribution ratio

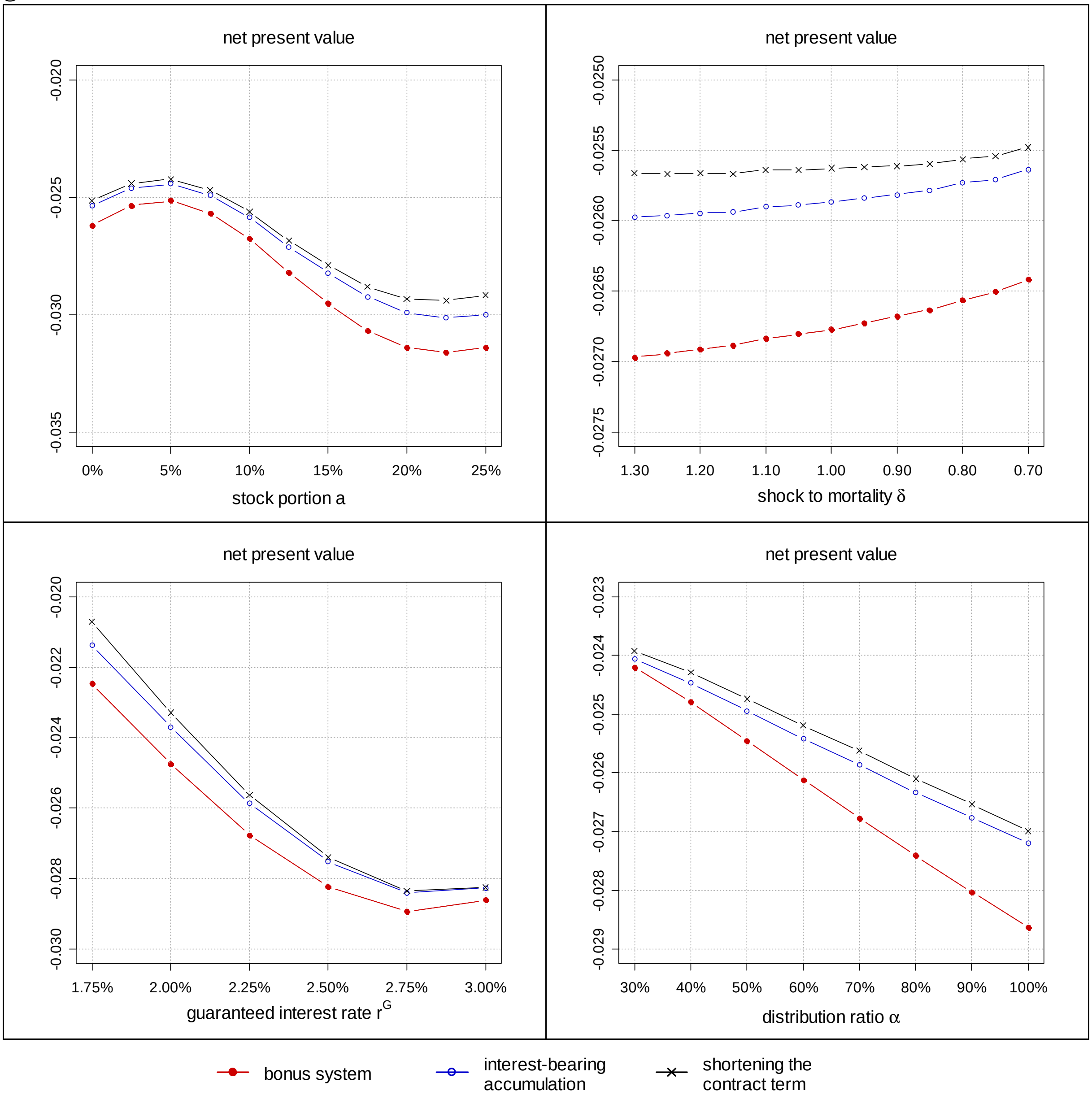


Figure 10 shows that, in the present setting, the *NPV* is slightly negative in all four cases, implying that the expected discounted sum of premium payments exceeds the present value of benefit payments, which is true for all three surplus appropriation schemes. This effect can be explained by the inclusion of an insurer insolvency, which reduces the benefit payments but is not considered in actuarial pricing. Furthermore, and in line with this, the *NPV* (except for certain cases) generally decreases for a higher stock portion, a higher guaranteed interest rate, or a higher surplus distribution ratio, i.e., with an increasing shortfall probability (see Figures

3, 8, and 9).[17] Moreover, the bonus scheme with the highest shortfall risk implies the lowest policyholder *NPV*. However, even though the company with a surplus appropriation mechanism used to shorten the contract term only has the second highest shortfall risk, it yields the highest *NPV* (instead of the interest-bearing accumulation with the lowest risk).

These general observations are different in the case of increasing mortality rates (upper right graph in Figure 10), where we observe an almost constant and slightly increasing *NPV*, as higher mortality probabilities lead to earlier death benefit payments before maturity, thus implying a tradeoff with respect to early default.[18] Here, the findings reveal a greater increase for the bonus system compared to the two other schemes, which is in line with the higher death benefit payments in case of the bonus system (see left graph in Figure 4).

In general, even when fixing the shortfall risk for the three companies, e.g. to a shortfall probability of 0.05 by adjusting the stock portion, the *NPV* can differ for the three surplus appropriation schemes. In particular, the *NPV* for Company 1 (bonus system) would be equal to -0.031 (a = 19.8%), the *NPV* of Company 2 (interest-bearing accumulation) is -0.030 and the *NPV* for Company 3 (shortening the contract term) would result to -0.029. While these differences appear minor at first glance, they have to be interpreted against the background of scaling, as the initial guaranteed sum insured is $S_1 = 1$. Furthermore, these differences in the *NPV* can increase depending on, e.g., the given level of shortfall risk and the shock to mortality, and result from the different assumptions concerning the actuarial pricing and the calculation of the net present value. While actuarial pricing is conducted under the real-world measure P and without including default, the net present value is calculated under the risk-neutral measure Q, thereby additionally taking into account default.[19]

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[17] In the case of no insolvency costs, i.e., all remaining assets are paid to the beneficiaries in the event of a shortfall, the *NPV* increases for riskier asset allocations, which is due to compensating relatively small losses in the event of default by high returns in periods of solvency. Furthermore, the order of the three schemes changes, with the interest-bearing accumulation yielding the lowest *NPV*. However, as this assumption of no insolvency costs does not seem realistic in practice, this case is not further considered.

[18] Note that, for higher costs of insolvency, the *NPVs* are decreasing as well.

[19] See Gatzert and Kling (2007) for similar arguments in the context of comparing the risk of fairly priced participating life insurance contracts (under the risk-neutral measure).

Comparison with previous literature

In general, our results can be considered to be in line with previous literature and to further extend earlier findings. However, the comparison of our results to other work has to be conducted against the background of differences in models and assumptions. In particular, the endowment insurance with death benefit (and without surrender) in Gerstner et al. (2008), denoted as product $p^{(2)}$, is comparable to our bonus system, where at least a minimum interest rate has to be paid to the policyholder accounts. In addition, the surplus distribution approach is the same as in the present paper, based on Grosen and Jørgensen (2000), but without modeling the concrete surplus appropriation scheme. Moreover, their model also accounts mortality risk; however, policyholders die according to the mortality table, while we forecast mortality and thus generate a surplus component that results from cautious mortality assumptions in actuarial pricing. Finally, Gerstner et al. (2008) consider a heterogeneous model portfolio, consisting of contracts that differ with respect to the initial age of policyholders, gender, and monthly premium, and take into account a dynamic asset allocation.

With respect to Kling, Richter, and Russ (2007b), their first and second surplus distribution system are comparable to our setting, as their first system incorporates a cliquet-style interest rate guarantee, where the guaranteed rate also has to be paid on distributed surplus. However, the smoothing scheme is different and features specific management rules. The second mechanism represents an interest-bearing accumulation, where surplus cannot be reduced once it has been credited to the policyholder's account and thus ensures an interest rate guarantee of 0% (without cliquet-style effects). In the third surplus model, surplus is credited to a terminal bonus account, which is not guaranteed, since the insurer can reduce the account in order to keep the insurance company in force. In contrast to our setting, mortality effects are not included and pricing is not conducted, as the company is assumed to be in a "steady state", implying that the amount of cash outflows equals cash inflows.

Even though the models are thus not fully comparable to ours due to different model approaches and varying assumptions, central previous results can still be confirmed and extended. For instance and most importantly, similar to Gerstner et al. (2008) and Kling, Richter, and Russ (2007b), the interest rate guarantee represents a key risk driver and implies a serious increase in shortfall risk. In addition to previous findings, our results demonstrate that this holds true even if surplus is appropriated in different ways.

Furthermore and also in accordance with our results, an increase in the stock portion leads to a rapid increase in the shortfall probability for different surplus systems in both papers. However, in case of Kling, Richter, and Russ (2007b), the first two surplus systems that are similar to our bonus and interest-bearing accumulation systems (implying a guarantee of past surplus) react similarly with respect to an increase in the stock portion, and differences mainly arise with respect to the third system, which allows the insurer to reduce past surplus, thus being superior to the other two mechanisms. However, in our setting, once surplus is credited to the policyholders' accounts, it cannot be reduced at any time in the future for all three surplus appropriation schemes. Despite this fact, we still observe a considerable increasing gap in shortfall risk between all three systems as the stock ratio grows. Thus, taking into account mortality risk and the concrete surplus appropriation scheme as is done here reveals a stronger reaction and an increasing gap for the three surplus appropriation schemes with respect to asset risks.

Moreover, observations for longer contract terms differ compared to Kling, Richter, and Russ (2007b), as their results show an increase in shortfall risk when raising the time horizon. In contrast, our findings reveal a rather constant or even decreasing shortfall risk, depending on the stock portion and the respective surplus appropriation scheme. This difference can be ascribed to the fact that for the initially guaranteed death and survival benefit, we explicitly determine the actuarially fair premium, which decreases for higher contract terms, thus deferring the surplus distribution mechanism as reserves build up more slowly. In addition and as described in the previous subsections, the discrepancy between the mortality table used in pricing and the projected and actually realized mortality rates increases, thus raising the insurer's buffer. Hence, a longer contract term implies a lower risk, which is not the case when using steady-state assumptions.

Finally, in line with results in Gerstner et al. (2008) and Kling, Richter, and Russ (2007b), the shortfall risk decreases with an increasing initial reserve and equity for all considered companies. Providing further detailed analyses, Gerstner et al. (2008) additionally show that due to the high initial buffer, default risk is almost zero during the first contract years. This can also be seen in Figure 6, which illustrates the shift of the number of shortfalls over time towards the end of the contract period for higher initial capital resources.

4. SUMMARY

In this paper, we examine the impact of different surplus appropriation schemes for participating life insurance contracts with respect to the insurer's risk exposure and the policyholder's net present value. Three systems for a participating life insurance contract are considered. First, the bonus system, which increases the guaranteed death and survival benefit; second, the interest-bearing accumulation, which accumulates surplus on a separate account, and thus keeps the death benefit constant while increasing the survival benefit; and, third, shortening the contract term by transforming surplus in an earlier payment of the survival benefit along with a reduced number of premium payments, while keeping the amount of the death and survival benefit constant, a system which has not been examined to date. In the analysis, we analyze the impact of asset and mortality risk as well as time to maturity and surplus distribution ratios on shortfall probability based on an actuarial reserving model. Mortality risk is modeled using a variation of the Lee-Carter (1992) model and assets are assumed to follow a geometric Brownian motion.

Our results show that, even if the amount of surplus is derived in the same way using a reserve-based smoothing surplus distribution approach, the concrete surplus appropriation scheme has a considerable impact on the risk situation of an insurer and the policyholder's net present value. In all cases, the bonus system implies the highest risk for the insurer, followed by the system of shortening the contract term, while the company that provides the interest-bearing accumulation scheme faces the lowest shortfall risk throughout our analyses. While increased mortality leads to generally higher shortfall probabilities, the differences in shortfall risk between the companies representing different surplus appropriation schemes roughly persists for different shocks to mortality and all are affected in a similar, severe way.

In contrast, for different asset allocations, the companies' shortfall risk exhibits an increasing gap between the three appropriation schemes for higher stock portions. Particularly in the case of the bonus system, a riskier investment strategy results in a considerably higher increase in the shortfall probability compared to the interest-bearing accumulation or shortening the contract term. Consequently, a company applying the bonus system is advised to compose its asset base in a more conservative way than a company with one of the other two appropriation schemes to achieve the same shortfall risk.

In line with these results, an increase of distributed surplus leads to generally higher shortfall probabilities. Here again, there are substantial differences in default risk between the types of surplus appropriation schemes. When fixing the level of shortfall risk, the interest-bearing accumulation allows a considerably higher policyholder surplus participation as compared to the system of shortening the contract term or, in particular, the bonus system. With respect to the policyholder's net present value, the three systems also differ, as the system of shortening the contract term induces the highest net present value from an insured's point of view, followed by the interest-bearing accumulation, while the bonus system implies the lowest net present value for the policyholders.

In summary, the combination of actuarial pricing and reserving approaches with financial approaches to measure shortfall risk and to evaluate the contract in a fair way from the policyholder's perspective allows a deeper insight into fundamental effects of surplus appropriation schemes. Thus, even if the smoothing surplus distribution scheme is the same, the way of using surplus with respect to guaranteed death or survival benefits or shortening the contract term substantially impacts an insurer's risk situation. In particular, these findings demonstrate that it is mainly the guaranteed death benefit or an earlier survival benefit that can increase the risk level and that a risk reduction for longer contract terms may not be as effective in the case of the very common bonus system as compared to shortening the contract term or interest-bearing accumulation.

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