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On the Management of Life Insurance Company Risk by Strategic Choice of Product Mix, Investment Strategy and Surplus Appropriation Schemes

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ABSTRACT

The aim of this paper is to analyze the impact of management's strategic choice of asset and liability composition in life insurance on shortfall risk and the shareholders' fair risk charge. In contrast to previous work, we focus on the effectiveness of management decisions regarding the product mix and the riskiness of the asset side under different surplus appropriation schemes. We propose a model setting that comprises temporary life annuities and endowment insurance contracts. Our numerical results show that the effectiveness of management decisions in regard to risk reduction strongly depends on the surplus appropriation scheme offered to the customer and their impact on guaranteed benefit payments, which thus presents an important control variable for the insurer.

Keywords: Participating life insurance, surplus distribution, risk-neutral valuation, management mechanisms

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1. INTRODUCTION

Management decisions regarding asset and liability composition can considerably impact a life insurer's risk situation and also the fair risk-adjusted compensation for the company's shareholders. Decisions can relate to various factors, including a dynamic adjustment of the portion invested in high-risk assets, the portfolio composition on the liability side as well as the type of surplus appropriation scheme, which at the same time influences the extent of the long-term guarantees typically embedded in these contracts.

Life insurance contracts in many European countries contain a legally enforced participation mechanism through which policyholders participate in the company's surplus. This surplus participation represents an important factor in competition between insurers and is paid in addition to a guaranteed interest rate that is annually credited to the policyholder's account. In addition, it is not only the absolute amount of surplus distributed to the policyholders that has an effect on shortfall risk; the concrete way in which distributed surplus is credited to the policyholders also has a considerable influence on the value of the surplus participation part of the contracts (see Bohnert and Gatzert, 2012). These so-called surplus appropriation schemes also impact the risk profile of the insurance company due to their varying characteristics of turning surplus into guarantees. Policies may feature various appropriation schemes. In case of an endowment insurance contract, for instance, surplus is used to increase the death as well as the survival benefit, while interest-bearing accumulation increases the survival benefit only (and keeps the death benefit constant). In case of an annuity contract, surplus can be used to increase the annual annuity payments until maturity, or surplus can be directly paid out to the policyholders in the corresponding period (direct payment scheme).

Another important control variable besides the surplus appropriation scheme is the mixture of the product portfolio, e.g., the percentage of annuities and life insurance contracts that a company sells, which impacts liabilities and assets alike due to the different timing and amount of cash in- and outflows. In addition, a dynamic path-dependent asset strategy can be implemented regarding the riskiness of the asset portfolio to improve the insurer's solvency situation, as assets can be more easily adjusted over the contract term as compared to the liability side. The management of assets and liabilities for a life insurer with various product portfolios including a detailed modeling of surplus appropriation schemes can have an important impact on the company's shortfall risk as well as on the risk-adjusted compensation for shareholders. Therefore, the aim of this paper is to examine this issue in more depth, thereby ensuring a fair situation for shareholders.

In the literature, various papers examine participating life insurance contracts including surplus distribution mechanisms and interest rate guarantees, focusing on different aspects. The traditional actuarial surplus management focuses on balancing conservatism and fairness (also with respect to the equityholders) of surplus distribution schemes and has been discussed since as early as 1863 by Homans (1863) and by Cox and Storr-Best (1963). In the current literature, one aspect of special interest has been risk-neutral valuation, which has been researched by, amongst others, Briys and de Varenne (1997), Dong (2011), Grosen and Jørgensen (2000, 2002), Hansen and Miltersen (2002), Guillén, Jørgensen, and Nielsen (2006), Kling, Ruez, and Russ (2011), Tanskanen and Lukkarinen (2003), Siu (2005), Schmeiser and Wagner (2011), and Goecke (2013). In addition, several papers have focused on combining risk pricing and risk measurement, including Gatzert and Kling (2007) Kleinow and Willder (2007), and Gatzert (2008). Kling, Richter, and Russ (2007a, 2007b) analyze surplus distribution schemes and their effect on an insurer's risk situation while in Bohnert and Gatzert (2012) different surplus appropriation schemes in participating life insurance are analyzed for the first time from the policyholders' and the insurer's perspectives encompassing mortality and financial risk, thereby also studying the impact on default risk.

With respect to management discretion, Kleinow and Willder (2007) and Kleinow (2009) analyze the impact of management decisions on hedging and valuation of participating life insurance contracts, while Gatzert (2008) examines different asset management and surplus distribution strategies for participating life insurance contracts. A general asset-liability management framework for life insurance is provided by Gerstner et al. (2008) that allows the company to control the asset base, the bonus declaration mechanism and the shareholder participation. Furthermore, Huang and Lee (2010) deal with the optimal asset allocation for life insurance policies adopting a multi-asset return model that uses approximation techniques. The optimal portfolio composition for immunizing a life insurer's risk situation against changes in mortality has been studied in Gatzert and Wesker (2012) with a focus on endowment insurance contracts and annuities, but without including surplus distribution mechanisms or dynamic asset management strategies. Inspired by the products on the Danish market, Guillén et al. (2013a, 2013b) study the performance of Danish with-profit pension products and life cycle products, where they also account for management decisions such as asset management strategies.

Thus, while asset-liability management, portfolio composition and management rules in general have been researched previously, the effectiveness of management decisions regarding the asset and liability composition for different surplus appropriation schemes has not been examined so far, even though surplus appropriation schemes play a central role in traditional life insurance and can substantially impact shortfall risk and shareholder value due to their consequences for the long-term guarantees promised to policyholders. One major question is, therefore, how surplus appropriation schemes of different products impact the effectiveness of management discretion regarding path-dependent asset management strategies and product compositions on the liability side. Such an analysis will provide important insights in regard to the management of long-term guarantees induced by surplus appropriation schemes as well as complex interactions between assets and liabilities in life insurance and their effect on risk and a fair shareholder position.

Therefore, in this article, we extend previous literature by analyzing the effectiveness of management decisions regarding the asset and liability composition for a life insurance company selling endowment insurance contracts and annuities under different surplus appropriation schemes on the company's shortfall risk and on the fair compensation of shareholders. Toward this end, we provide a model setting including the two life insurance products with different surplus appropriation schemes. The smoothing surplus distribution scheme considered in the model is thereby similar to the mechanisms that have been used in, e.g., Denmark for a long time, implying that many important management decisions are now taken on the basis of this type of models. On the liability side, we consider the impact of the portfolio composition, thereby always ensuring a fair risk charge for shareholders. On the asset side, the effectiveness of management rules that modify the riskiness of the investment is studied, i.e., where funds are dynamically shifted from stocks to bonds to reduce volatility and vice versa using a constant proportion portfolio insurance (CPPI)-based investment strategy. These asset feedback mechanisms can have an impact on the overall amount of generated surplus and thus also on the policyholders' surplus participation and the induced increase in guaranteed benefits, which may imply complex interactions.

The remainder of the paper is organized as follows. Section 2 introduces the model framework of the insurance company, along with the management decisions and the surplus appropriation schemes. Numerical results are presented in Section 3 and Section 4 concludes.

2. MODEL FRAMEWORK

In what follows, we consider a life insurer that offers two products: temporary annuities and participating life insurance contracts (also referred to as endowment contracts) with different surplus appropriation schemes. We make use of the model framework introduced in Bohnert and Gatzert (2012) for surplus appropriation schemes in participating life insurance and expand their setting in various ways. In particular, we propose various company setups, where the product portfolio composition, surplus appropriation and asset strategies can be studied that are defined at inception of the contracts. The insurer's balance sheet at time t is laid out in Table 1.

Table 1: Balance sheet of a life insurance company at time t

Assets	Liabilities
A_t	E_t
	PR_t^R
	PR_t^S
	IA_t
	B_t
A_t	A_t

At the beginning of the first contract year (t = 0), equityholders make an initial contribution of $E_0 = l \cdot A_0$ and the collectivity of policyholders pay single premiums of $(1-l) \cdot A_0$.[1] The book values of the policy reserves for the annuities and the traditional endowment insurance contracts are given by PR_t^R and PR_t^S, respectively and IA_t denotes the book value of the endowment insurance contracts' interest-bearing accumulation system. The buffer account B_t is determined residually by subtracting equity, the policyholders' accounts and dividends paid to the equityholders from the market value of the assets (A_t), where equity (E_t) is assumed to be constant over time (see also Kling, Richter, and Russ (2007a, 2007b)).[2] Furthermore, a run-off scenario without new business is considered.

[1] The initial equity capital E_0 is set equal in case of annual premium payments for comparability reasons.

[2] Thus, B_t is a hybrid, since it is the difference between market and book values. This is a simplification of the actual procedures in an insurance company (see Grosen and Jørgensen, 2000).

2.1 The liability model

The insurance contracts

The company's range of products comprises temporary annuity and endowment insurance contracts with a contract term of T years.[3] We assume that a total number of N contracts is sold, which is distributed among life insurance and annuity holders, such that

$$N^R = \varphi \cdot N, \;\; N^S = (1-\varphi) \cdot N,$$

where φ is the percentage of annuity contracts. The annuities are sold against single premiums, whereas the endowment contracts are sold against single premiums as well as against annual premiums. We consider pools of contracts that are actuarially priced based on mortality tables. Thus, the single (net) premiums for the temporary annuity and the endowment contract for an individual policyholder are given by

$$P_{single}^R = R_1 \cdot a_{x:\overline{T}|}, \text{ and } P_{single}^S = S_1 \cdot A_{x:\overline{T}|},$$

respectively, where R_1 denotes the initially guaranteed annual annuity payment in case of survival (without any surplus) and S_1 denotes the initially guaranteed sum insured in the case of death or survival, both paid in arrears. The corresponding constant annual (net) premium for the endowment contract (paid in advance) is given by annuitizing the single premium, resulting in

$$P^S = S_1 \cdot \frac{A_{x:\overline{T}|}}{\ddot{a}_{x:\overline{T}|}}.$$

The actuarial present value of an endowment insurance with a sum insured of one and a contract term of T years ($A_{x:\overline{T}|}$) for an individual x-year old policyholder and the present value of

[3] In Germany, for instance, endowment insurance contracts and annuities together account for more than 60% of the premium volume in life insurance and thus represent a major product design in the life insurance sector (see GDV, 2013, Table 34). An endowment policy is a classical savings product and typically features a contractually defined duration after which a lump sum is paid out to the policyholder in case of survival. For comparability reasons, we further consider a temporary annuity with a contract term that is identical to the endowment policies rather than a lifelong annuity. The effects shown for these contracts types are also of relevance for other types of participating life insurance products with different surplus participation schemes.

an immediate temporary annuity for T years with an annual annuity payment of one in advance ($\ddot{a}_{x:\overline{T|}}$) and in arrear ($a_{x:\overline{T|}}$) is given by

$$A_{x:\overline{T|}} = \sum_{t=0}^{T-1} v^{t+1} \cdot {}_t p_x \cdot q_{x+t} + v^T \cdot {}_T p_x, \; \ddot{a}_{x:\overline{T|}} = \sum_{t=0}^{T-1} v^t \cdot {}_t p_x, \text{ and } a_{x:\overline{T|}} = \sum_{t=1}^{T} v^t \cdot {}_t p_x, \tag{1}$$

where $v = \left(1 + r^G\right)^{-1}$ describes the discount factor using an actuarial interest rate of r^G. The probability of an x-year old insured person surviving t years is given by ${}_t p_x$, while q_{x+t} states the probability of dying within one year for a policyholder aged $x+t$. The mortality probabilities that we use for pricing and reserving are based on the mortality tables by the German Actuarial Association that include a safety loading (first-order mortality basis). For annuities, mortality probabilities of the table "DAV 2004 R" are used, whereas the table "DAV 2008 T" is applied for the endowment insurance holders. To simulate the number of deaths in our later numerical analysis, we use the second-order mortality basis of the corresponding table, i.e. the tables' underlying best estimates.[4] An illustration of the evolution of cash flows resulting from the insurance products over time is given in Figure 1.

Figure 1: Development of cash flows from the insurance products over time ('–' denotes December 31st and '+' denotes January 1st for each year)

- 0 +		- 1 + …		- t + …		- T-1 +		- T +		time
x		$x+1$		$x+t$		$x+T-1$		$x+T$		cohort specific age
0	0	R_1	0	R_t	0	R_{T-1}	0	R_T	0	annuity payment
0	0	S_1	0	S_t	0	S_{T-1}	0	S_T	0	sum insured payment
0	P^R_{single}	0	0	0	0	0	0	0	0	single premium annuity
0	P^S_{single}	0	0	0	0	0	0	0	0	single premium endowment
0	P^S	0	P^S	0	P^S	0	P^S	0	0	annual premium endowment
0	0	0	D_1	0	D_t	0	D_{T-1}	0	D_T	dividend for equityholders

4 Note that one could alternatively also use mortality data from other countries as the model itself is generic. The "DAV" tables are the current mortality tables provided by the German Actuarial Association ("DAV"). The "DAV 2004 R" table is based on German mortality data for annuitants and includes safety loadings that account for model risk as well as the risk of a long-term change in the mortality trend. The "DAV 2008 T" is determined based on mortality data of those insured on endowment contracts, term life insurance and unit-linked policies. Here, safety loadings include the risk of random fluctuations, model risk and parameter risk, while the risk of a change in the mortality trend is neglected. Note that the considered tables without the safety loadings (best estimates) are not identical in case of the tables "DAV 2004 R" and "DAV 2008 T", since the corresponding cohorts that are used for constructing the tables differ. For further details, see www.aktuar.de.

Note that the premium payment(s) are constant, while the benefits from the contracts vary over time depending on the company's generated surplus and the selected surplus appropriation scheme.

Policy reserves

The policy reserves for the annuity and endowment insurance contracts are calculated on the same actuarial basis as the premiums. The policy reserves for the pool of annuity policies ($j = R$) and the pool of endowment insurance contracts ($j = S$) at the end of year t are given by

$$PR_{t^-}^j = \left(N^j - \sum_{i=1}^{t} d_i^j \right) \cdot {}_tV_x^j \, , \, j = R, S, \tag{2}$$

where ${}_tV_x^j$ represents the actuarial reserve for an individual annuity or endowment contract and d_i^j specifies the number of deaths (of year i) from the cohort of the initially sold contracts (N^j), which is determined based on the best estimates of the corresponding mortality table, i.e., the mortality tables without safety loadings.[5]

The prospective calculation for the actuarial reserve for an $x+t$-year old insured at time t for an individual annuity contract is thereby determined by

$${}_tV_x^R = R_{t+1} \cdot a_{x+t:\overline{T-t|}} \, , \tag{3}$$

and for an endowment contract, the individual actuarial reserve is calculated by

$${}_tV_x^S = S_{t+1} \cdot A_{x+t:\overline{T-t|}} - P_t^S \cdot \ddot{a}_{x+t:\overline{T-t|}} \, , \tag{4}$$

using the actuarial present values stated in Equation (1), where R_{t+1} is the guaranteed annuity that is paid at the end of year t (at time t+1) in case of survival, S_{t+1} denotes the guaranteed sum insured (that is paid out if death occurs during year t, i.e., from time t until time t+1, or if the insured survives until maturity) and P_t^S indicates the t-th premium payment. In case of a

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[5] For simplification purposes, we refrain from modeling systematic longevity, but we do take into account the difference between anticipated mortality including safety loadings (for pricing and reserving) and realized mortality. This also contributes to the natural generation of surplus and is typically conducted in life insurance.

single premium, $P_0^S = P_{single}^S$ and $P_t^S = 0$, $t = 1,\ldots,T$, whereas $P_t^S = P^S$ for a constant annual premium.

Buffer account

Surplus that has already been generated, but not yet been distributed to the policyholders and has thus not been transformed into guarantees yet, is saved in the buffer account. Funds in this account belong to the policyholders but are not guaranteed, as they can be used to compensate losses in years of low asset returns. At the end of year t, the buffer account is residually determined by

$$B_{t^-} = A_{t^-} - PR_{t^-}^R - PR_{t^-}^S - IA_{t^-} - E_t \,. \tag{5}$$

At the end of the last year, the value of the buffer account (minus dividend payments) determines the terminal bonus (TB_T) paid to policyholders, which cannot become negative and is given by[6]

$$TB_T = \max\left(B_{T^-} - D_T, 0\right) = \max\left(A_{T^-} - PR_{T^-}^R - PR_{T^-}^S - IA_{T^-} - E_T - D_T, 0\right).$$

2.2 The asset model

We assume that the investments on the asset side (I_t) evolve according to a geometric Brownian motion, which is given by

$$dI_t = \mu \cdot I_t \cdot dt + \sigma \cdot I_t \cdot dW_t^P ,$$

with an asset drift μ,[7] volatility σ and a P-Brownian motion W^P defined on the probability space $(\Omega, \mathcal{F}, P)$. The solution of this stochastic differential equation is given by (see Björk, 2009)

[6] We thereby take into account diversification effects between the pool of endowment insurance contracts and annuitants but, as in Bohnert and Gatzert (2012), we do not focus on substitution effects across generations. Analyses on generational substitution effects can be found in Døskeland and Nordahl (2008) and Faust, Schmeiser, and Zemp (2012), for instance.

[7] Under the risk-neutral pricing measure Q, the asset drift changes to the risk-free rate r_f, and the stochastic differential equation is thus given by $dI_t = r_f \cdot I_t \cdot dt + \sigma \cdot I_t \cdot dW_t^Q$ with a Q- Brownian motion W^Q.

$$I_t = I_{(t-1)} \cdot e^{\left(\mu-\sigma^2/2+\sigma\cdot\varepsilon_t\right)} = I_{(t-1)} \cdot e^{r_t},$$

with independent standard normally distributed random variables ε_t and a continuous one-period return r_t. We further assume that the total asset base is composed of stocks and bonds with a stock ratio a. To account for different stock ratios in the portfolio, we consider an adjusted return with corresponding drift and volatility for the aggregate asset portfolio, which satisfies

$$r_t = a \cdot r_S + (1-a) \cdot r_B,$$

with continuous one-year returns of stocks r_S and bonds r_B, corresponding volatilities and drifts σ_S and σ_B, expected values m_S and m_B ($m_i = \mu_i - \sigma_i^2/2$, i = B, S) and a correlation coefficient ρ. At the beginning of the first year, the initial asset value is composed of the equity capital and the (first) premium payments, i.e.,

$$A_{0^+} = P_{single}^R \cdot N^R + P_{single}^S \cdot N^S + E_0, \tag{6}$$

in the case of single premiums for both contract types and $A_{0^-} = 0$. In case of annual premiums for the endowment insurance, the corresponding single premium (P_{single}^S) has to be replaced by the first constant level premium P^S in Equation (6). During the contract term, annual annuity payments R_t have to be paid out to those annuitants still alive at that time and death benefits S_t are paid to the heirs of the policyholders who died during the contract year. Hence, the asset base at the end of year t is given by

$$A_{t^-} = A_{(t-1)^+} \cdot e^{r_t} - R_t \cdot \left(N^R - \sum_{i=1}^{t} d_i^R \right) - S_t \cdot d_t^S. \tag{7}$$

At the end of year t, which is assumed to be equal to the accounting date, two cases have to be distinguished for the further development of assets and liabilities. First, in case the insurer is solvent and assets are sufficient to cover the liabilities, i.e., $A_{t^-} \geq PR_{t^-}^R + PR_{t^-}^S + IA_{t^-}$, being equivalent to $B_{t^-} + E_t \geq 0$, a constant fraction β of the equityholders' initial contribution is paid out to as annual dividend payments D_t,[8] i.e.,

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8 If the insurer is solvent but does not have enough reserves to pay the dividends, i.e., if $B_{t^-} + E_t \geq 0$, but $B_{t^-} < D_t$, then $D_t = 0$. If the buffer account becomes negative, but equity capital is sufficiently high to cover the losses in this period, i.e., $B_{t^-} < 0$, but $B_{t^-} + E_t \geq 0$, the insurer still remains solvent. Here, equity capital

$$D_t = \beta \cdot E_0, \; if \; B_{t^-} \geq D_t .$$

This leads to $B_{t^+} = B_{t^-} - D_t$ and results in an asset base at the beginning of year t+1 of (see also Equations (6) and (7))

$$A_{t^+} = A_{t^-} - D_t + P^S \cdot \left(N^S - \sum_{i=1}^{t} d_i^S \right).$$

The last summand denotes the annual premium payments for the endowment contracts, which are set to zero in the case of single premiums. Second, in the case of an insolvency, liabilities cannot be covered by assets, i.e., $A_{t^-} < PR_{t^-}^R + PR_{t^-}^S + IA_{t^-}$ and equivalently $B_{t^-} + E_t < 0$, the company is liquidated by distribution of the remaining assets less bankruptcy/liquidation costs c $A_{(t-1)^+} \cdot e^{r_t} \cdot (1-c)$ to the policyholders who are still active.

2.3 Surplus distribution and appropriation

With respect to surplus appropriation, three schemes are considered. For the annuity, the direct payment scheme and the bonus system are used, whereas for the endowment insurance, the bonus system and the interest-bearing accumulation are applied based on the model in Bohnert and Gatzert (2012). While surplus is used to increase the initial annuity and guaranteed sum insured in case of the bonus system, surplus is saved on a separate account in case of the interest-bearing accumulation or directly paid out to the annuitants in case of the direct payment scheme.

Thus, in addition to the calculatory interest rate r^G (see Equation (1)), which has to be credited to the policy reserves annually and which thus constitutes a guaranteed interest rate, the policy interest rate r_t^P that includes surplus in addition to the guaranteed interest rate is determined using the reserve-based approach shown in Grosen and Jørgensen (2000),[9]

is reduced by the amount of the loss and the buffer account is set to zero. In the next year, the amount of equity capital is increased again to the original amount by using gains from the next period (see Equation (5)).

9 Traditional participating life insurance contracts with their surplus distribution and appropriation schemes make use of the process of collective saving (in contrast to individual saving as in case of unit-linked policies, for instance). An detailed analysis of return smoothing mechanisms is provided in Guillén, Jørgensen, and Nielsen (2006), while the merits of collective saving has been addressed in Goecke (2013), for instance.

$$r_t^P = \max\left\{ r^G, \alpha \cdot \left(\frac{B_{(t-1)^+}}{PR^R_{(t-1)^-} + PR^S_{(t-1)^-} + IA_{(t-1)^-}} - \gamma \right) \right\}, \tag{8}$$

with a target buffer ratio γ, i.e., the ratio of the free surplus or buffer divided by the liabilities belonging to the policyholders and a surplus distribution ratio α, which controls the extent of surplus that is distributed to the policyholders. The model proposed by Grosen and Jørgensen (2000) has certain aspects in common with an approach suggested in a report by the Danish Financial Supervisory Authority (1998). In particular, the idea to determine bonus (in excess of a guaranteed rate of return) as a fraction of an available buffer is a common characteristic of the two approaches and this is what has been long-term practice in Denmark, for instance.

The total amount of surplus for an individual contract in the t-th year is derived based on the individual reserves and defined by[10]

$$PR^j_{(t-1)^-} \cdot \left(r_t^P - r^G \right), \quad j = R, S \,.$$

Based on this surplus distribution approach, different appropriation schemes are applied, which have an impact on the overall dynamics of assets and liabilities.

Surplus appropriation: The bonus system for annuity and endowment insurance

The bonus system uses the annually distributed surplus amount as a single premium to increase the annuity R_1 and the initially guaranteed sum insured S_1 for the rest of the contract term. For an individual annuity contract, the additional annuity is calculated by

$$\Delta R_t = \frac{PR^R_{(t-1)^-} \cdot \left(r_t^P - r^G \right) \Big/ \left(N^R - \sum_{i=1}^t d_i^R \right)}{a_{x+t:\overline{T-t}|}}, \tag{9}$$

and for an endowment insurance, the additional sum insured is given by

[10] This is a typical approach to model surplus distribution when guaranteed interest rates are in place (see, e.g. Grosen and Jørgensen, 2000). A model for distributing surplus to policyholders and equityholders with specific characteristics of German regulation can be found in Maurer, Rogalla, and Siegelin (2013).

$$\Delta S_t = \frac{PR^S_{(t-1)^-} \cdot \left(r_t^P - r^G\right) \Big/ \left(N^S - \sum_{i=1}^{t} d_i^S\right)}{A_{x+t:\overline{T-t}|}}, \tag{10}$$

which results in an increased annuity and sum insured, respectively, of

$R_{t+1} = R_t + \Delta R_t$ and $S_{t+1} = S_t + \Delta S_t$.

In this setting, the surplus credit to the policyholders' reserves also participates in future surplus and is compounded at least with the guaranteed interest rate, thus inducing cliquet-style interest rate effects (see Equations (3)-(6) and (7)).

Surplus appropriation: Direct payment scheme for the annuity

The annuity's direct payment directly pays out surplus to the policyholders in addition to their originally guaranteed annuity in the subsequent year. In contrast to the bonus system, the annual surplus amount per annuitant only increases the next annuity payment, i.e.,

$$R_{t+1} = R_1 + PR^R_{(t-1)^-} \cdot \left(r_t^P - r^G\right) \Big/ \left(N^R - \sum_{i=1}^{t} d_i^R\right),$$

while the annuities after this additional payment are not affected by the surplus, but might again be increased by single surplus payments in the following years.

Surplus appropriation: Interest-bearing accumulation for the endowment contract

In case of the endowment insurance's interest-bearing accumulation, surplus is accumulated on a separate account IA_t, comparable to a bank account that is paid out to the policyholder at maturity in case of survival. In case of death during the contract term, funds are transferred to the buffer account and thus eventually to the collectivity of policyholders. The policyholders' heirs only receive the sum insured of S_1, which is constant throughout the contract term. The interest-bearing accumulation account at the end of year t (including new surplus) earns an interest rate $r^{IA} \geq 0$ and is given by

$$IA_{t^-} = IA_{(t-1)^-} \cdot \left(1 + r^{IA}\right) \cdot \left(1 - d_t^S \Big/ \left(N^S - \sum_{i=1}^{t-1} d_i^S\right)\right) + PR^S_{(t-1)^-} \cdot \left(r_t^P - r^G\right), \; IA_0 = 0.$$

2.4 Management decisions regarding assets and liabilities

The management of the insurer has several options for controlling the asset and liability side in order to positively influence the insurer's risk situation or shareholder value. The insurance company's risk profile can for instance be altered by means of the product portfolio composition by setting the fraction φ of annuities (and endowment contracts 1 - φ). The liability side can further be controlled by employing a specific type of surplus appropriation scheme for both products, which alter the implied guaranteed benefits. Regarding the asset side, a path-dependent adjustment rule of risk-relevant control variables can be implemented. In the following, we consider a dynamic CPPI-based feedback mechanism, where the stock portion at time t is given by

$$a_{t^+} = \min\left(\max\left(\frac{A_{t+} - PR^R_{(t-1)^-} - PR^S_{(t-1)^-} - IA_{(t-1)^-}}{A_{t+}} \cdot m, 0 \right), a_{\max} \right), \tag{11}$$

where m is a multiplier that controls the extent to which assets are shifted towards the risky investment and a_{max} represents the maximum stock portion allowed. The initial stock portion is denoted by a_0. The nominator thereby represents a buffer between the liabilities and the assets available to cover the liabilities. The lower the buffer becomes, the less is invested in risky assets and vice versa.

A more dynamic approach has recently been introduced by Guillén et al. (2013b) using an alternative feedback mechanism, where the stock portion at time t depends on the shortfall probability, which is given by the probability that the buffer ratio falls below a critical level. In particular, the stock portion is maximized while ensuring that the shortfall probability does not exceed a certain threshold in the subsequent period (e.g. according to a solvency level of 99.9%). This approach could be an interesting further development for future models. Note that other feedback mechanisms are possible as well and that the focus of this analysis is not on finding an optimal asset allocation strategy, but rather on studying the general impact and effectiveness of a feedback mechanism that depends on an insurer's risk situation with respect to different products including different ways of crediting surplus to policyholders. We thus focus on studying a set of relevant managerial control variables and their fundamental interplay in an insurance company and do not study how managerial discretion is affected when facing pressure from customer needs or when decisions dynamically depend on shortfall risk as is done in Guillén et al. (2013b), for instance, which we leave for future research.

2.5 Risk assessment and fair valuation from the shareholders' perspective

To determine the impact of management decisions regarding the riskiness of the asset investment, the portfolio composition and various surplus appropriation schemes, we calculate the life insurer's shortfall risk. A shortfall of the company occurs if the value of the assets A_{t^-} falls below the value of liabilities, $A_{t^-} < PR_{t^-}^S + IA_{t^-} + PR_{t^-}^R$ (or, equivalently, if $B_{t^-} + E_t < 0$). Hence, the shortfall probability under the real-world measure P is given by

$$SP = P\left(T_s \le T\right),$$

where the time of default is defined as $T_s = \inf\left\{t : A_{t^-} < PR_{t^-}^S + IA_{t^-} + PR_{t^-}^R\right\},\ t = 1,...,T$.

To ensure a fair situation from the shareholders' perspective, the constant dividend rate β is calibrated such that the value of the payments to the shareholders (dividends D_t and final payment E_T) is equal to their initial contribution E_0, which is calculated using risk-neutral valuation, i.e.,[11]

$$\begin{aligned} E_0 &= E^Q\left(e^{-r_f \cdot T} E_T + \sum_{t=1}^{T} e^{-r_f \cdot t} D_t\right) \\ &= E^Q\left(e^{-r_f \cdot T} \min\left\{E_0, E_0 + B_{T^-}\right\} \cdot 1\left\{T_S > T\right\} + \sum_{t=1}^{T} e^{-r_f \cdot t} \beta \cdot E_0 \cdot 1\left\{T_S > t\right\}\right). \end{aligned} \tag{12}$$

If the buffer account is nonnegative at maturity, i.e., there is no previous default, $E_T = E_0$. However, if the buffer account becomes negative at maturity but equity capital is sufficient to cover these losses (and, thus, the insurer still remains solvent), equity capital is reduced by the amount of the loss, i.e., $E_T = E_0 + B_{T^-}$ if $B_{T^-} < 0$.

3. Numerical Analysis

In what follows, numerical results are presented based on the previously introduced model with a focus on analyzing the insurer's risk exposure for fair dividend rates. After presenting the input parameters, we next study the general impact of surplus appropriation schemes for the two products, i.e., the annuity and the endowment insurance, on the fair dividend and the corresponding company's shortfall probability. Subsequently, we analyze to what extent deci-

[11] Since we use mortality tables that include safety loadings in order to price the life insurance products and thus deviations in mortality are priced in, Equation (12) mainly refers to financial risk.

sions with regard to management rules can reduce the shortfall risk. Numerical results are derived using Monte Carlo simulation based on 100,000 Latin hypercube samples (see Glasserman, 2010).

Input parameter

The underlying policies are annuities issued to $x_R = 60$ year old males and participating life insurance contracts issued to $x_S = 35$ year old males, both with a contract term of $T = 30$ years. The initial annual annuity is set to one and the actuarial present values of the benefits for the endowment insurance and the annuity (per insured) are equal in order to ensure comparability between the different cases considered. Thus the actuarial annual premium for one endowment contract is given by $P^S = 0.88$ and the corresponding single premium is $P^S_{single} = P^R_{single} = 18.83$, which is equal to the single annuity premium (due to the calibration of the initial guaranteed death benefit). According to this, the initial sum insured for the endowment insurance is $S_1 = 35.58$. The actual dates of death are simulated using the inverse transform method based on the mortality tables of the German actuarial association using the "DAV 2008 T" and the "DAV 2004 R" tables of second-order ("best estimates" without safety loadings) for a total number of $N = 100{,}000$ contracts sold. Assumptions about the evolution of the assets are based on the historical performance (1988 until 2009) of two representative German total return indices as given in Bohnert and Gatzert (2012).[12] The estimation for the stocks, which is based on monthly data for the German stock market index DAX, results in an expected one-year return $m_S = 8.00\%$ and a volatility $\sigma_S = 21.95\%$. The estimation for the bonds, which is based on monthly data for the German bond market index REXP, leads to an expected one-year return of bonds $m_B = 6.02\%$ and a volatility of bonds $\sigma_B = 3.30\%$. The estimated correlation coefficient of returns of the two indices is $\rho = -0.1648$.[13] To initiate the CPPI-based feedback mechanism, the initial stock portion is set to 1%, which is then immediately adjusted depending on the size of the free buffer (see Equation (11)). Furthermore, we assume the surplus distribution ratio to be $\alpha = 70\%$ and the target buffer ratio to be $\gamma = 10\%$. The initial equity capital E_0 is set to 1% of the total initial capital A_{0^+}.[14] The parameters are

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12 For calibrating parameters, German market data is used for illustration, but the analysis and the general results and interaction effects are also relevant to other insurance markets with life insurance contracts with surplus distribution mechanisms (e.g. Denmark).

13 The correlation coefficient is significant at a level of 0.01.

14 This assumption is based on the equity capital to balance sheet ratio of approximately 1% as in case of the German life insurer Allianz for the year 2010 (see www.allianz.de) and has also been subject to robustness checks.

chosen for illustration purposes only and were subject to sensitivity analyses.[15] Unless stated otherwise, we assume further relevant parameters to be those stated in Table 2.

Table 2: Parameters for the analysis

Expected one-period returns of stocks	m_S	8.00%
Volatility one-period returns of stocks	σ_S	21.95%
Expected one-period returns of bonds	m_B	6.02%
Volatility one-period returns of bonds	σ_B	3.30%
Correlation between stocks and bonds	ρ	-0.1648
Guaranteed interest rate	r^G	1.75%
Rate of interest for the interest-bearing accumulation account	r^{IA}	0%
Risk-free rate	r_f	3%
Number of contracts sold	N	100,000
Annual annuity payment in t = 0	R_1	1
Sum insured for the endowment in t = 0	S_1	35.58
Single premium for the annuity	P^R	18.83
Alternative single premium for the endowment	P^S_{single}	18.83
Level premium for the endowment	P^S	0.88
Equity in t = 0 (1% of total initial capital)	E_0	19,020
Contract term	T	30
Annuity policyholders' age in t = 0	x_R	60
Endowment policyholders' age in t = 0	x_S	35
Distribution ratio	α	70%
Target buffer ratio	γ	10%
Reduction coefficient for costs of insolvency	c	20%
Multiplier	m	1

The impact of surplus appropriation schemes on the effectiveness of management decisions in regard to shortfall risk

We first focus on the impact of the choice of the respective surplus appropriation scheme on the effectiveness of asset management strategies in regard to reducing shortfall risk for different portfolio compositions on the liability side. Results are displayed in Figure 2 for a (maximum) stock portion of 25% (left column) and 10% (right column). All numerical examples are based on fairly calibrated dividend rates β to ensure an adequate compensation for shareholders (see Figure A.1 in the Appendix).

[15] Note that when using a fixed stock portion, we compare two cases with 10% and 25% for illustration purposes, which are realistic assumptions for stock portions (shares held directly or in funds) for insurers operating in countries belonging to the OECD (see OECD, 2014). In Germany, stock portions are currently considerably lower and around 3 to 4% (see GDV, 2013). However, German insurers also currently aim to increase their average stock portions due to the low interest rate levels and insufficiently available other investment opportunities.

The first row (Figure 2a) contains results for portfolios of temporary annuities and endowment insurance contracts, both with the more risky bonus system, whereas the second row shows the case where the annuities are equipped with the direct payment scheme and the endowment contracts feature the interest-bearing accumulation, i.e. the second row represents the less risky scheme in each case.[16] On the *x*-axis, a portion $\varphi = 1$ represents a pool of contracts with 100% annuities and for $\varphi = 0$, one obtains a portfolio entirely composed of endowment contracts. In addition to the results for varying product portfolios, the shortfall risk is displayed without and with including the CPPI-based asset mechanism. Its effectiveness for reducing the firm's shortfall risk (in percent) is given on the right vertical axis.

Figure 2 illustrates that the *type of surplus appropriation scheme* plays an important role with respect to the insurer's risk situation,[17] especially for higher stock portions and thus a higher asset risk. Figure 2a (left graph) shows that the bonus system leads to a considerably higher shortfall risk as compared to Figure 2b with the endowment contracts' interest-bearing accumulation and the annuities' direct payment scheme in the case without a feedback mechanism and for higher portions of endowment contracts in the portfolio due to their cliquet-style interest rate guarantee effects. This is particularly evident for a maximum stock portion of 25% (compare left graphs in Figures 2a and 2b), where the bonus system increases the default risk by more than 20% as compared to the interest-bearing accumulation scheme. This is due to the higher expected returns associated with a higher stock portions that (on average) increase the surplus amount and thus the guaranteed death and survival benefits, while in the case of the interest-bearing accumulation, only the guaranteed survival benefit is increased. In the case of an annuity portfolio ($\varphi = 1$) and lower stock portions (e.g., 10%, see righthand graphs), the difference between the two surplus appropriation schemes is less distinct. This is in line with the fact that return guarantees that vary for different surplus appropriation schemes are generally more valuable when the investment process is more volatile. This observation is highly relevant in that it indicates that in the considered setting, surplus appropriation schemes of different types of products mainly impact shortfall risk heavily if the asset investment is too risky. Given the current trend of insurers of investing more in risky assets

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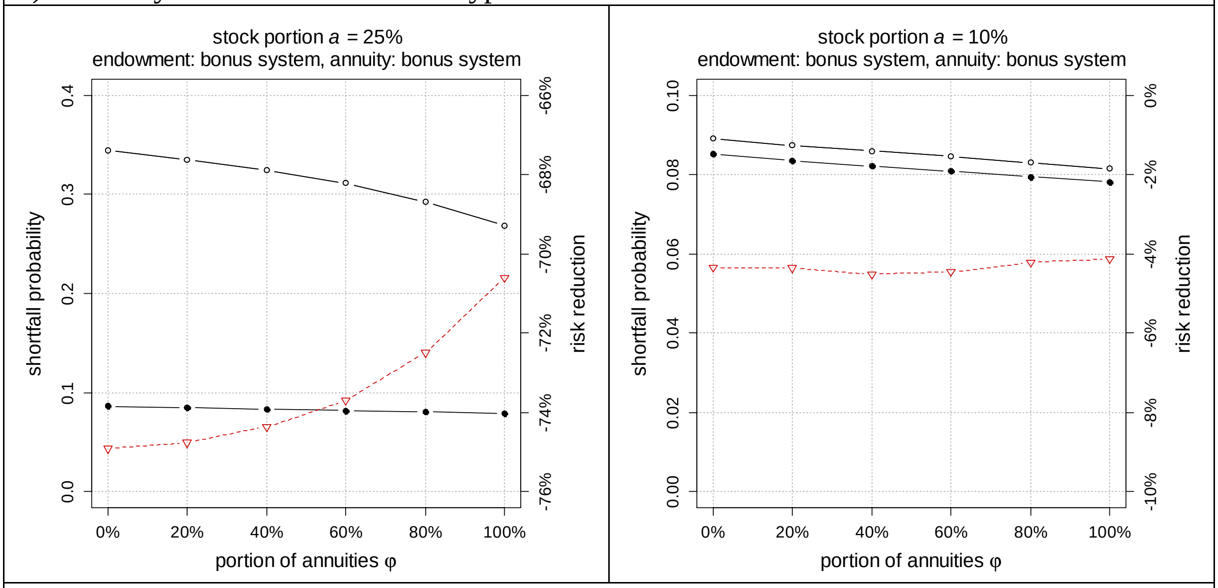
16 We directly compare the situation with the more and the less risky surplus appropriation scheme for both products, as we are interested in the impact of the surplus schemes on the interaction between both products. The impact of each scheme for each type of product individually is studied in Figure 3 in the following analysis.

17 This is consistent with the findings in Bohnert and Gatzert (2012), where, however, focus was not laid on annuities or their surplus appropriation schemes nor on the interaction between these products.

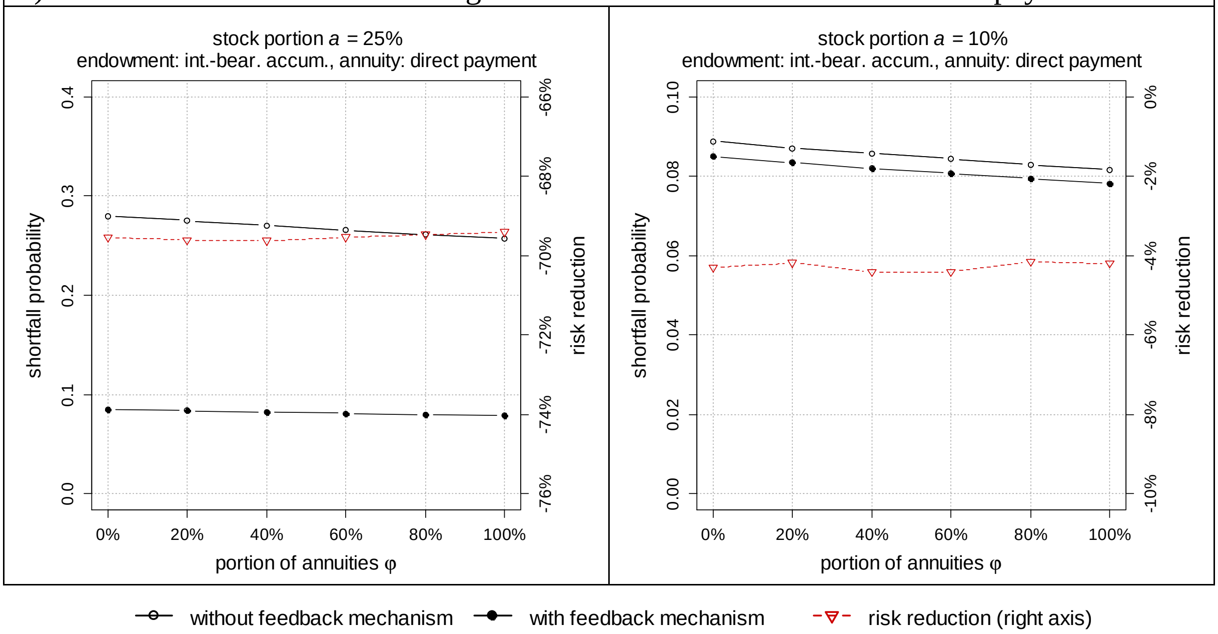
(stocks or credit risky securities) due to unattractive alternatives, the impact of the surplus appropriation schemes should be carefully monitored by insurers.

Figure 2: Shortfall probability for various portfolio compositions consisting of endowment contracts and temporary annuities for different surplus appropriation schemes without and with asset feedback mechanism[18]

a) Bonus system for both contract types



b) Endowment with interest-bearing accumulation and annuities with direct payment



Notes: In the case without the feedback mechanism, the stock portion is kept constant at a = 25% (left graphs) and a = 10% (right graphs) throughout the contract term. When applying the feedback mechanism, the specified stock portion denotes the maximum stock portion a_{max} *given in Equation (11).*

[18] Note the difference in scales when comparing the graphs in the left and right column.

Furthermore, Figure 2 shows that an insurance company's shortfall risk considerably depends on the *product portfolio composition*. In the case without asset management strategies, the company's shortfall risk decreases when increasing the portion of annuities in the portfolio. This stems from the development of the payouts to the policyholders over time of the two product types. Here, we consider endowment insurance contracts and temporary annuities that are both sold against single premiums with actuarial values being equal at contract inception. In case of the bonus system, surplus is used to increase the guaranteed sum insured S_t and the annual annuity payment R_t, respectively (see Equations (9) and (10) and upper right graphs in Figures A.2 and A.3 in the Appendix). Thus, in case of endowment contracts, surplus increases the guaranteed sum insured, which is paid out either in case of death or in case of survival at contract maturity, where especially the probability of survival for the next 30 years is relatively high for a 35-year old policyholder. In contrast, the annual annuity is only paid out to policyholders that are still alive. In case of death prior to contract maturity, the portion of the surplus that was used to increase the guaranteed annuity payment R_t is partly passed on to the collectivity of policyholders (which is not the case for the endowment contracts) and thus increases the collective buffer, which in turn reduces the shortfall risk. Thus, the increase in the long-term guarantees induced by the bonus system embedded in the considered annuities (by means of increasing the guaranteed annual annuity payment) in general carries less risk for the insurer than the bonus system embedded in an endowment contract. This effect can thereby vary depending on the considered age group of the policyholders and the premium payment scheme, which is addressed further below.

When including a *CPPI-based asset feedback mechanism* in this setting, this portfolio composition effect can still be observed, but is considerably dampened (see, e.g., upper left graph in Figure 2, case with and without the asset feedback mechanism). In particular, shortfall risk only slightly decreases for an increasing portion of annuities, but remains on a considerably lower level as compared to the case without asset management strategy for all portfolio compositions. In addition, the fair dividend rate decreases (see Figure A.1 in the Appendix) as the risk of default and thus non-payment is reduced. However, the effectiveness of risk reduction strongly depends on the portfolio composition, the respective surplus appropriation scheme (compare left graphs in Figure 2a and 2b), as well as the maximum stock portion allowed in the asset portfolio.

In particular, it can be seen that the considered CPPI mechanism is more effective for the pool of endowment contracts ($\varphi = 0$) as compared to the portfolio of annuities ($\varphi = 1$), which is especially pronounced for a higher maximum stock portion and the more risky bonus system (upper left graph in Figure 2). Furthermore, the different types of (long-term) guarantees imposed by the surplus appropriation schemes impact the effectiveness of the feedback mechanism in reducing risk, particularly for a higher stock portion (cf. left graphs in Figure 2). For instance, in the case that endowment contracts and annuities are both equipped with the bonus system, the CPPI mechanism reduces the shortfall risk of up to 75% in the case of a maximum stock portion of 25% and a portfolio of endowment contracts, while this amounts to around 69% in the case of a portfolio of annuities (Figure 2a, right axis in upper left graph). In cases where both products feature the less risky direct payment and interest-bearing accumulation scheme (Figure 2b, left graph), the asset strategy still implies a strong reduction in shortfall risk of around 69%, which, however, remains fairly stable for different portfolio compositions and is thus almost independent of the product type.

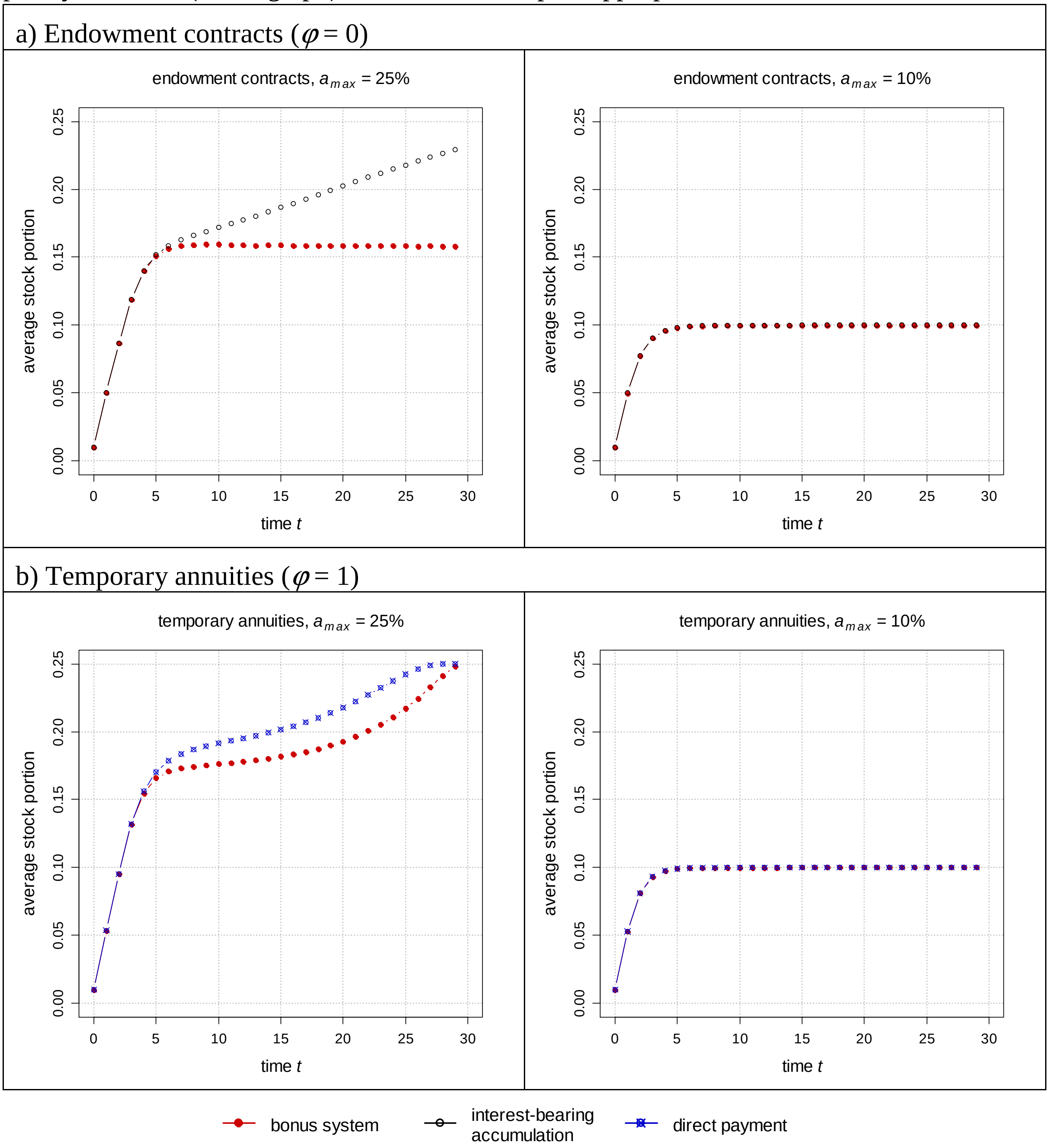
In addition, for a lower maximum stock portion of 10% (right graphs in Figure 2) the risk reduction only amounts to around 4% and does not vary much for different portfolio compositions. Thus, the feedback mechanism is much more effective in reducing shortfall risk in case of the bonus system and in case of a higher maximum stock portion and for a portfolio of endowment contracts.

The impact of management decisions on the development of account values and stock portions over time

In the following, we additionally study the development of account values and the stock portion over time to obtain further insight into the effectiveness of management decisions regarding the asset and liability side depending on the surplus appropriation scheme.

Figure 3 shows the average stock portion a_t over time when applying the asset feedback mechanism for the case of a maximum stock portion of 25% (left graphs) and 10% (right graphs) that correspond to Figure 2 with a portfolio of endowment contracts only ($\varphi = 0$) and for annuities only ($\varphi = 1$) (in the case without asset management strategy, the maximum stock portion is used for the whole contract term). Figures 4 and 5 additionally illustrate the average development of the assets, the bonus account and the policy reserves (and in case of the interest bearing-accumulation scheme also the interest-bearing accumulation account) over time for a portfolio of 100% endowment policies (see left graphs in Figure 4) and for 100% annuities (see left graphs in Figure 5). Furthermore, the development of the endowments' sum insured and annual annuity payments on average are displayed in the right graphs in Figures 4 and 5, respectively.

Figure 3: Average stock portion over time for endowment contracts (upper graphs) and temporary annuities (lower graph) with different surplus appropriation schemes



Notes: Average stock portion$_t = E\left(a_t \mid T_s > t\right)$, *where* T_s *denotes the time of default.*

Focusing on Figure 3, the graphs show that the average stock portion a_t over time (see Equation (11)) can be higher (if not capped by a maximum stock portion set up-front) for products with a less risky surplus appropriation scheme, i.e., a scheme that induces fewer (long-term) guarantees. As we assume for illustration purposes that the beginning of the contract coincides with the inception of the company, the bonus account starts with a value of zero (see Equation (5) for the calculation of the buffer account).[19] Thus, at the inception of the company that applies the dynamic CPPI-based feedback mechanism, we assume that the stock portion starts with a value of 1% that is in line with Equation (11) and the initial equity capital given in Table 2.[20] The buffer account especially plays an important role for investing in stocks under the feedback mechanism, since the stock portion cannot be increased without (enough) funds in the buffer account (see Equation (11)). This may also imply a firm survival effect in the results (at least partly contributing to the increasing stock portion in certain cases), as the reported results are averages from the surviving firms, which may imply a kind of evolutionary pressure in the sense of an increasing average firm equity over time. While this can be seen in Figure 4 in the case of endowments, for instance, where assets A_t increase relative to the policy reserves (and IA_t) with time (thus indicating an increasing share of stocks, which increases proportionally to the ratio of assets and policy reserves), Figure 5 displays the opposite result, implying that this firm survival effect does not provide an explanation in this case. Surplus is not paid out until the ratio of the buffer account divided by the policyholders' accounts reaches the target buffer ratio γ (see Equation (8)). Thus, the company's stock portion rapidly increases in the first contract years while surplus is accumulated and the buffer has to be built up over time (see Figures 3 to 5). After the target buffer ratio is reached and surplus is slowly starting to be paid out (see Equation (8) for the surplus distribution mechanism), the development of the stock portion differs considerably depending on the surplus appropriation mechanism, which can best be seen in the case of a maximum stock portion of 25%. Here, a higher stock portion is possible for surplus appropriation schemes with fewer (long-term) guarantees (given a fair situation from the shareholders' perspective).

In the case of the endowment contracts' bonus system, the stock portion does not increase to the maximum stock portion of 25%, but is instead capped to a stock portion of about 16% due the high guarantees induced by the bonus system. These guarantees transform the corresponding surplus entirely into policy reserves, which participate in future surplus and which are thus subject to the guaranteed interest rate (cliquet-style guarantee). The buffer therefore

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[19] In the case of a life insurance company with ongoing business, a part of the buffer account of one generation is passed on to the subsequent generation, i.e., a cross-subsidization takes place from insured members of early generations to insured members of later generations (see Døskeland and Nordahl, 2008). Hence, surplus can be paid out to policyholders from the beginning of the contract period, and in turn, policyholders are not entitled to receive the entire remaining buffer at the end of the contract term as a terminal bonus; instead, a fraction thereof has to be passed on to the next generation of policyholders.

[20] According to Equation (11), funds that do not belong to the policyholders' accounts on the liabilities side are invested in stocks, i.e., equity and the additional buffers are invested in stocks.

builds up more slowly (see also upper graphs in Figure 4), thus limiting the possibilities in stock investments according to Equation (11). In contrast to this, the endowment contracts' interest-bearing accumulation account does not involve cliquet-style guarantees, which leads to a faster increase of the buffer account over time. Thus, the contracts' corresponding stock portion can be higher (within the range set up-front) in the case of the interest-bearing accumulation due to fewer long-term guarantees (compare the buffer account in the left graphs in Figure 4). In case of a lower maximum stock portion of 10%, the stock portion differs only marginally.

In the case of annuities, the buffer account is first built up and then reduced over time (see left graphs in Figure 5). When considering the annuities' bonus system, it can be seen that the stock portion first increases to around 16% (see lower left graph in Figure 3), similar to the case of endowment contracts. After this initial increase, the further increase is slower as compared to the direct payment scheme, since the bonus system also induces cliquet-style guarantees as in the case of the endowment contracts. However, in contrast to the latter, the average stock portion still increases during the later contract years, which is due the considerably higher mortality probabilities towards the end of the contract term. As described above, in case of a policyholders' death, the corresponding policy reserves are passed on to the collectivity of policyholders, which increases the buffer account and thus the possibility for investment in stocks. The annuities' direct payment scheme does not increase the guarantees over time and thus the stock portion can be increased over time given fair contracts.

As in Figure 2, in Figures 4 and 5 we consider endowment insurance contracts and temporary annuities, respectively, that are sold against single premiums with actuarial values being equal at the inception of the contracts. The development of the corresponding average account values illustrates the various guarantees implied in the surplus appropriation schemes.[21] When comparing the two considered surplus schemes for the endowment contract in Figure 4, it can be seen that the policy reserves increase to a larger extent for the more risky bonus system (Figure 4a) as compared to the interest-bearing accumulation (Figure 4b). The policy reserves are subject to the guaranteed interest rate and surplus is paid on funds in this account, thus implying a strong impact of the type of surplus scheme on long-term guarantees (in contrast to the interest-bearing accumulation scheme).

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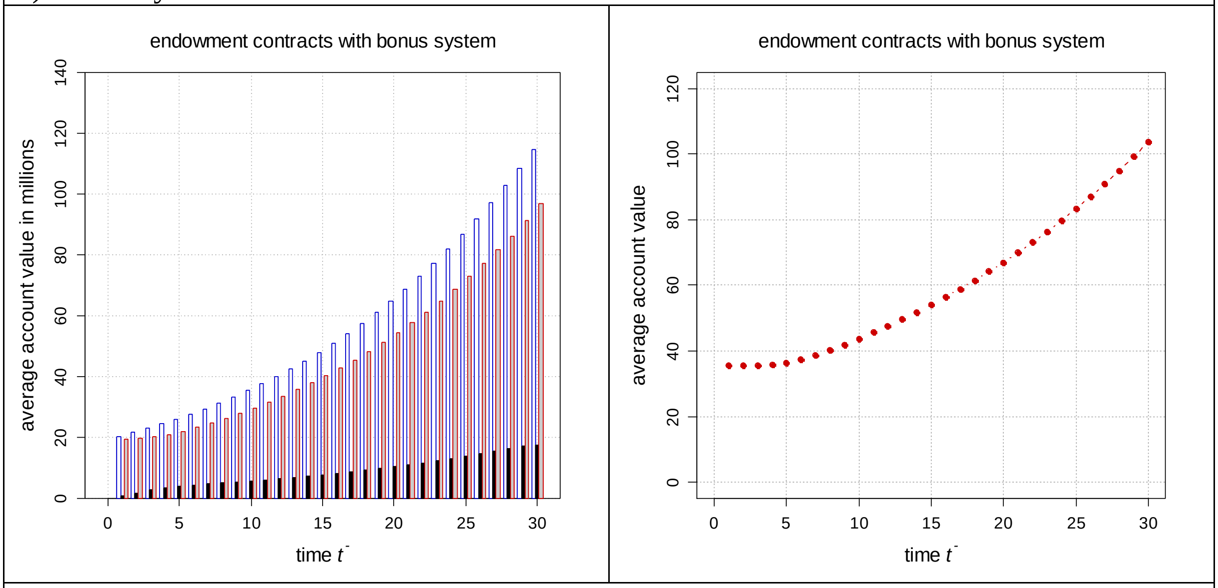
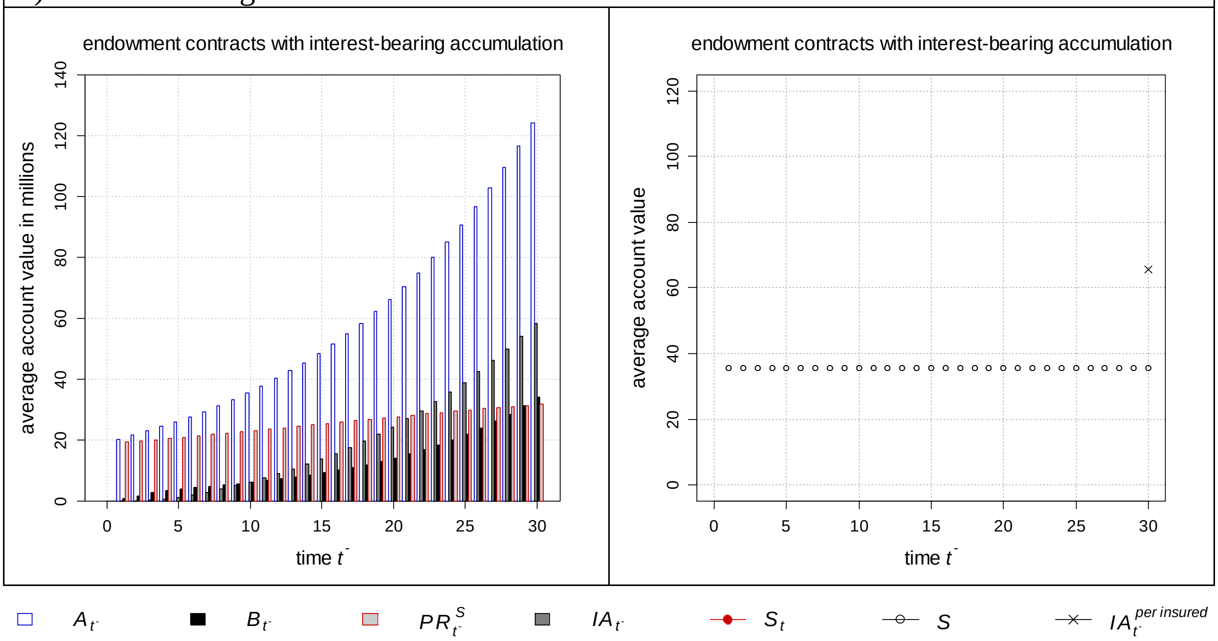
[21] In case of the bonus system, surplus is subject to compound interest (cliquet-style guarantee) and thus the corresponding payments are exponentially shaped over time.

Figure 4: Average account values over time for the *endowment contracts* for a maximum stock portion of a_{max} = 25% in the case with feedback mechanism

a) Bonus system

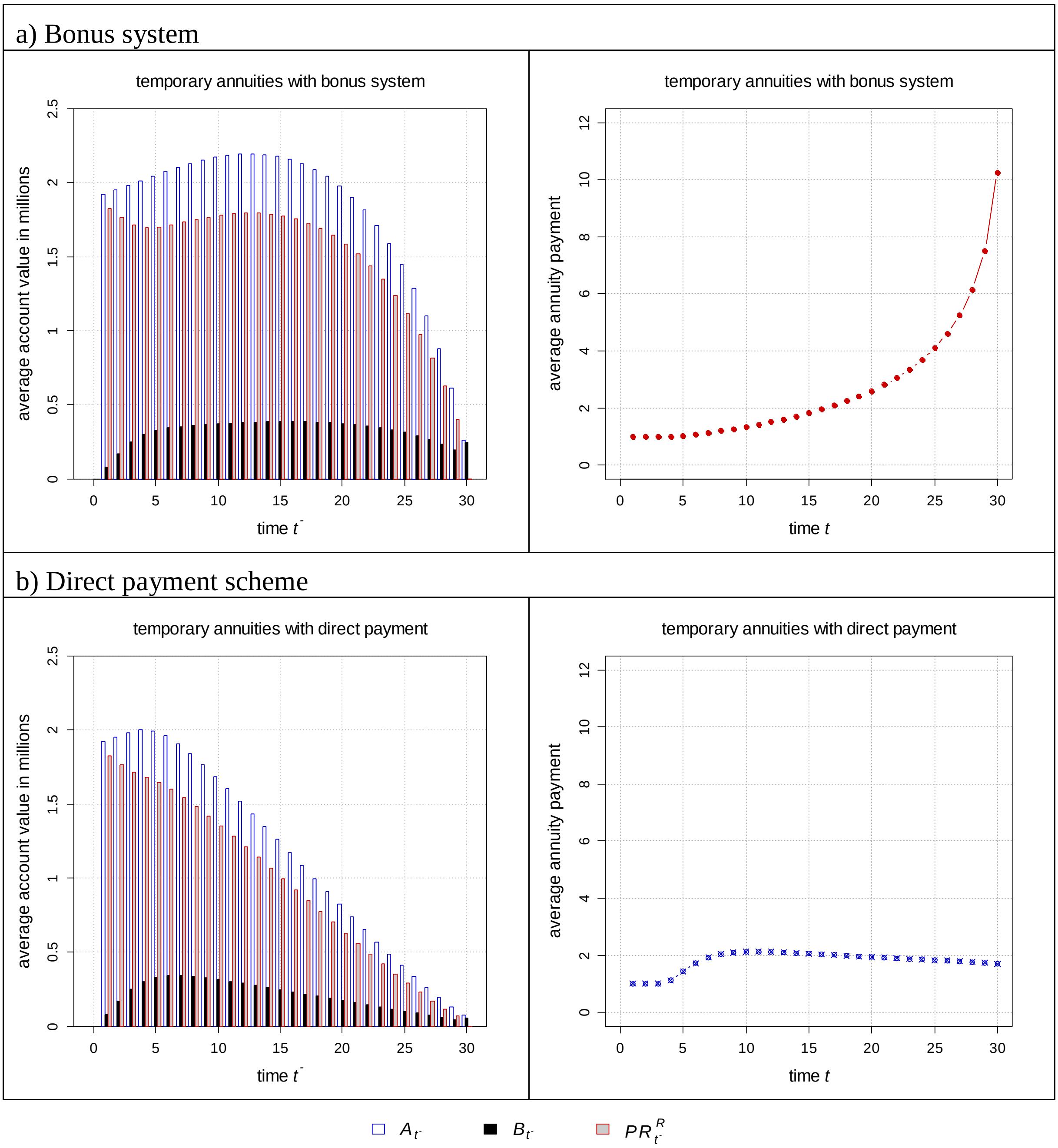


b) Interest-bearing accumulation



Notes: Average account value$_t = E\left(account\ value_t \mid T_s > t\right)$, where T_s denotes the time of default and account value$_t \in \left\{A_{t^-}, B_{t^-}, PR_{t^-}^S, IA_{t^-}, S_t\right\}$; the interest-bearing accumulation account per insured that is alive at the end of the contract term equals to $IA_{t^-}^{per\ insured} = E\left(IA_{t^-} \Big/ \left(N^S - \sum_{i=1}^{T} d_i^S\right) \mid T_s > t\right)$.

Figure 5: Average account values over time for the *temporary annuities* for a stock portion of a_{max} = 25% in the case with feedback mechanism



Notes: Average account value$_t = E\left(\textit{account value}_t \mid T_s > t\right)$, *where* T_s *denotes the time of default and account value*$_t \in \left\{A_{t^-}, B_{t^-}, PR^R_{t^-}, R_t\right\}$.

In Figure 5, the stronger guarantee implied by the annuities' bonus system in comparison to the direct payment scheme is also illustrated by the average development of the corresponding accounts. Here, the policy reserves of the annuities with bonus system as well as assets and the buffer account first increase and then decrease to a far smaller extent over time as compared to the corresponding accounts of the direct payment scheme due to the cliquet-style guarantees inherent in the bonus system. When comparing the average annual annuity payments over time, one can see that surplus is partly shifted to later contract years in case of the bonus system, while for the direct payment scheme, surplus is directly paid out to the policyholders, which can serve different policyholders' needs (see right graphs in Figure 5).

Further findings revealed that the impact of management discretion focusing on the asset side of the balance sheet is considerably greater than the impact of management rules that solely affect the liability side, e.g., by means of adjusting the surplus participation rate during the contract term depending on the insurer's solvency situation. Moreover, the results reveal that the management rules' ability to reduce shortfall risk heavily depends on the chosen parameter setting. Here, the multiplier m plays an important role. It controls the sensitivity of the management rules' reactions on the asset side based on the company's economic environment, i.e., it specifies the extent to which free surplus is invested in stocks. Analogously to a regular CPPI controlled investment strategy, the multiplier indicates the risk attitude, i.e., the lower the multiplier is, the more risk-averse is the investment strategy and vice versa. Thus, increasing the multiplier implies an increase in shortfall risk (and in the fair dividend payments). Additional analyses also revealed that an increase in the surplus participation rate α or a reduction in the target buffer ratio γ can considerably increase the gap between the risk levels for different surplus appropriation schemes and portfolio compositions. Further analyses demonstrated a considerable impact of the type of premium payment scheme (annual versus single premiums) in case of the endowment contracts. When considering a portfolio of only endowment contracts with the bonus system ($\varphi = 0$), for instance, it can be seen that annual premiums lead to a considerably lower shortfall risk as compared to single premiums. This is due to the fact that the contracts' policy reserves are built up more slowly than in the case of a single up-front premium and thus less surplus is generated for each single contract, which could be turned into long-term guarantees.

Implications regarding the customers' perspective

As emphasized by the previous analyses, the mechanisms for the distribution and appropriation of surplus to the policyholders imply (long-term) guarantees that can have a considerable impact on the effectiveness of management decisions for reducing shortfall risk. In this regard, one main question to be studied in future work concerns the impact of these surplus appropriation schemes and the implied guarantees on the performance or value of contracts from the customers' perspective. Such an analysis could be based on the approach as presented in Guillén et al. (2013a), for instance, which allows a ranking of products based on a set of criteria. The authors study several Danish pension (life cycle) products with surplus distribution and interest rate guarantees based on the contracts' performance and by using various risk measures as well as the fair value of guarantees. Their results show that all seven considered pension products containing various guarantees are outperformed by trivial benchmark investment strategies that have the same estimated long-term risk but higher long-term mean / median returns. In Guillén et al. (2013b), a similar approach is used to study the impact of minimum interest guarantees in Danish with-profit pension policies on the return of the products, showing that the price of the guarantee implies a considerable loss in returns. Furthermore, Bohnert and Gatzert (2012) consider an endowment contract and examine the impact of different surplus appropriation schemes on the contract's net present value from the policyholder's perspective without focusing on management decisions, showing that the policyholders' net present value considerable differs for different surplus appropriation schemes.

As emphasized by the results in Gatzert et al. (2011), who find that the average willingness-to-pay for guarantees in unit-linked policies is generally below the theoretical price and as also pointed out by Guillén et al. (2013b), a higher transparency is needed in regard to risk-return profiles of the products to allow policyholders to make adequate purchase decisions. In particular, the consequences of embedding different types of guarantees (interest rate guarantees and/or guaranteed surplus distribution and appropriation schemes) in the contracts that can considerably reduce expected returns should be transparently communicated. Such an analysis should take into account management decisions, which impact the value of guarantees and the shortfall risk. Based on this information, policyholders can then decide whether they are willing to pay for a guarantee or a specific surplus appropriation scheme or whether they prefer a product with lower levels of guarantees. Future research should thus extend the present analysis with surplus distribution and appropriation schemes and consider more dynamic management decisions as well as their impact on the performance of contracts from the policyholders' perspective. The consideration of the recent developments in the literature as laid out above could improve the practice of dealing with these types of life insurance and pension contracts when included in future work.

4. SUMMARY

In this paper, we study how surplus appropriation schemes of different products influence the effectiveness of management's strategic decisions regarding asset and liability composition. This is of high relevance as surplus appropriation schemes can considerably impact long-term guarantees embedded in life insurance contracts, depending on how surplus is turned into guaranteed benefit payments and depending on the type of product (endowment versus annuities). Toward this end, we present a model setting for a life insurance company selling endowment insurance contracts and annuities equipped with different surplus appropriation schemes. A fair situation for the shareholders is ensured by calibrating the dividend rate using risk-neutral valuation. Regarding the management's decisions, on the asset side a rule is employed that modifies the riskiness of the investment, i.e., funds are shifted from stocks to a bond investment to reduce volatility and vice versa. Such asset investment decisions have an impact on the overall amount of generated surplus and thus also affect the policyholders' share in the surplus. In addition, the company can control the liabilities by means of the product mix by varying the portion of endowment contracts and annuities, which imply different exposures to risk and thus allow the exploitation of possible diversification benefits.

Our results show that management's strategic choices regarding assets and liabilities by means of investment strategy and product mix can substantially lower an insurer's shortfall risk, but that surplus appropriation schemes can considerably impact the effectiveness of these management strategies due to the different types of (long-term) guaranteed benefit payments induced by the respective surplus scheme. However, the extent of this effect strongly depends on the type of product in which the surplus scheme is embedded. For instance, given fairly calibrated dividend payments, the considered CPPI-based asset management strategy is more effective in reducing shortfall risk for the endowment contracts as compared to the annuities, which is especially pronounced for a higher maximum stock portion and the more risky bonus system (as compared to the interest-bearing accumulation scheme). In the case of annuities, the asset strategy is more effective when applying the bonus system scheme instead of the direct payment scheme. Thus, the product mix and the type of surplus appropriation scheme play a major role and represent important control variables for insurers and regulators.

Our findings also show that management's actions not only have a considerable impact on an insurer's risk level, but also on the fair risk-adequate position of shareholders, an issue that is particularly relevant for regulatory authorities. In addition, especially the type of surplus appropriation scheme considerably impacts the insurer's risk situation, even though the amount of surplus is derived in the same way for all surplus schemes using a reserve-based smoothing surplus distribution approach. We observe that the consequences of the surplus appropriation schemes on the company risk strongly depend on the type of product, since the bonus system for instance implies a higher shortfall risk when embedded in an endowment contract as compared to an annuity insurance product. Finally, we also find that in the considered setting, the riskiness of the asset base has a considerable effect regarding the extent of the impact of the type of surplus appropriation schemes for different types of products. In particular, the effects can be minor as long as the asset process is not too risky, while they are extensive when increasing the riskiness of the assets. This is of high relevance for insurers who currently think about investing a higher share in risky assets (stocks or credit risky securities, for instance), in which case the impact of the surplus appropriation schemes and the different types of products should be carefully monitored.

In summary, surplus appropriation schemes not only impact an insurer's shortfall risk depending on the respective product (endowment versus annuities), but can especially be of relevance for the effectiveness of asset management decisions due to the different ways in which surplus is transformed into (long-term) guarantees, a fact that should in any case be taken into account in practice when designing new life insurance products and in management's strategic choices of product mix, surplus appropriation schemes and asset investment strategy. These aspects can also have a considerable influence on the customer's perspective, which along with the recent developments in the literature in regard to performance analyses should be taken into account in future developments and when studying life and pension products with different types of guarantees and surplus schemes.

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APPENDIX

Figure A.1: Fair dividend for various portfolio compositions consisting of endowment contracts and temporary annuities for different surplus appropriation schemes without and with asset feedback mechanism

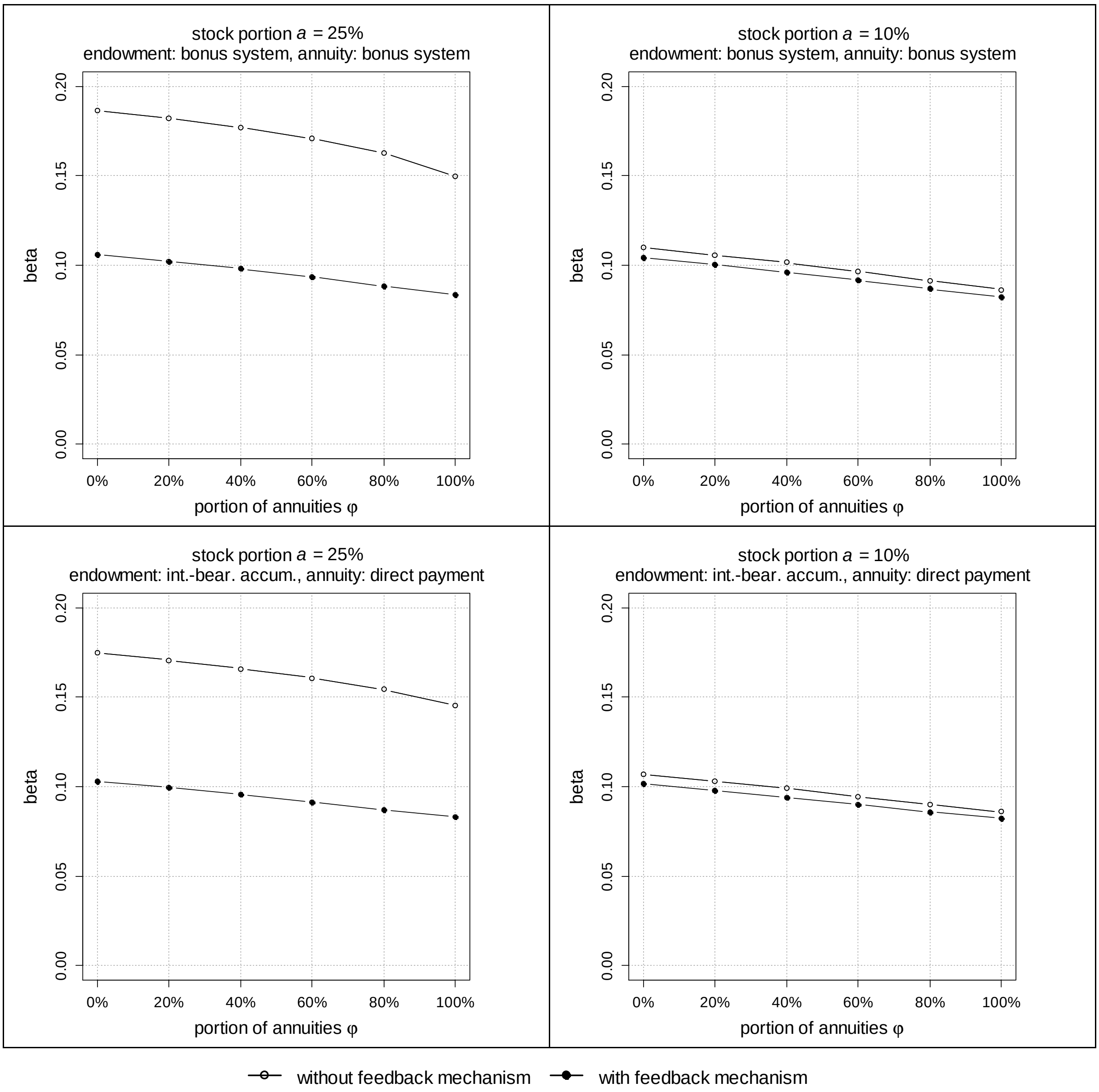
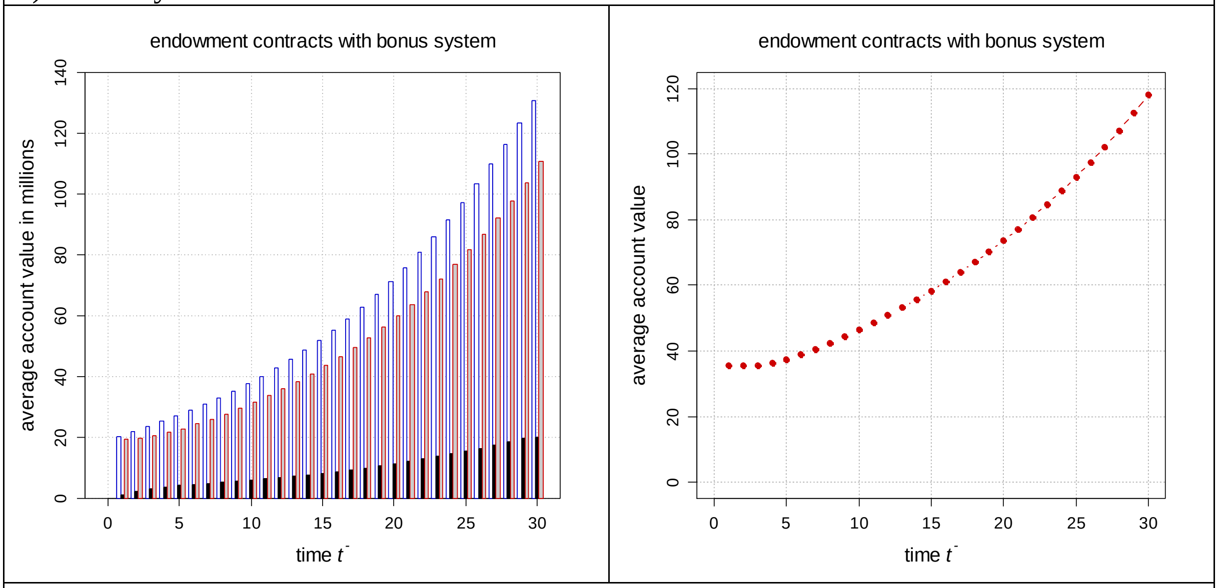
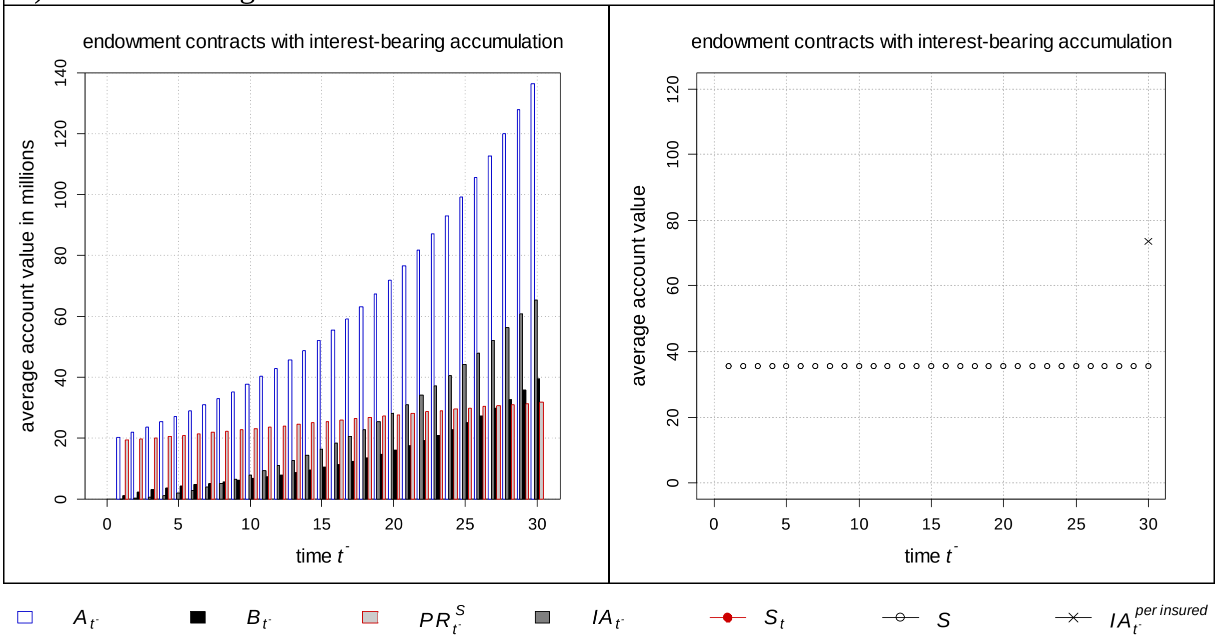


Figure A.2: Average account values over time for the endowment contracts for a maximum stock portion of a_{max} = 25% in the case without feedback mechanism

a) Bonus system

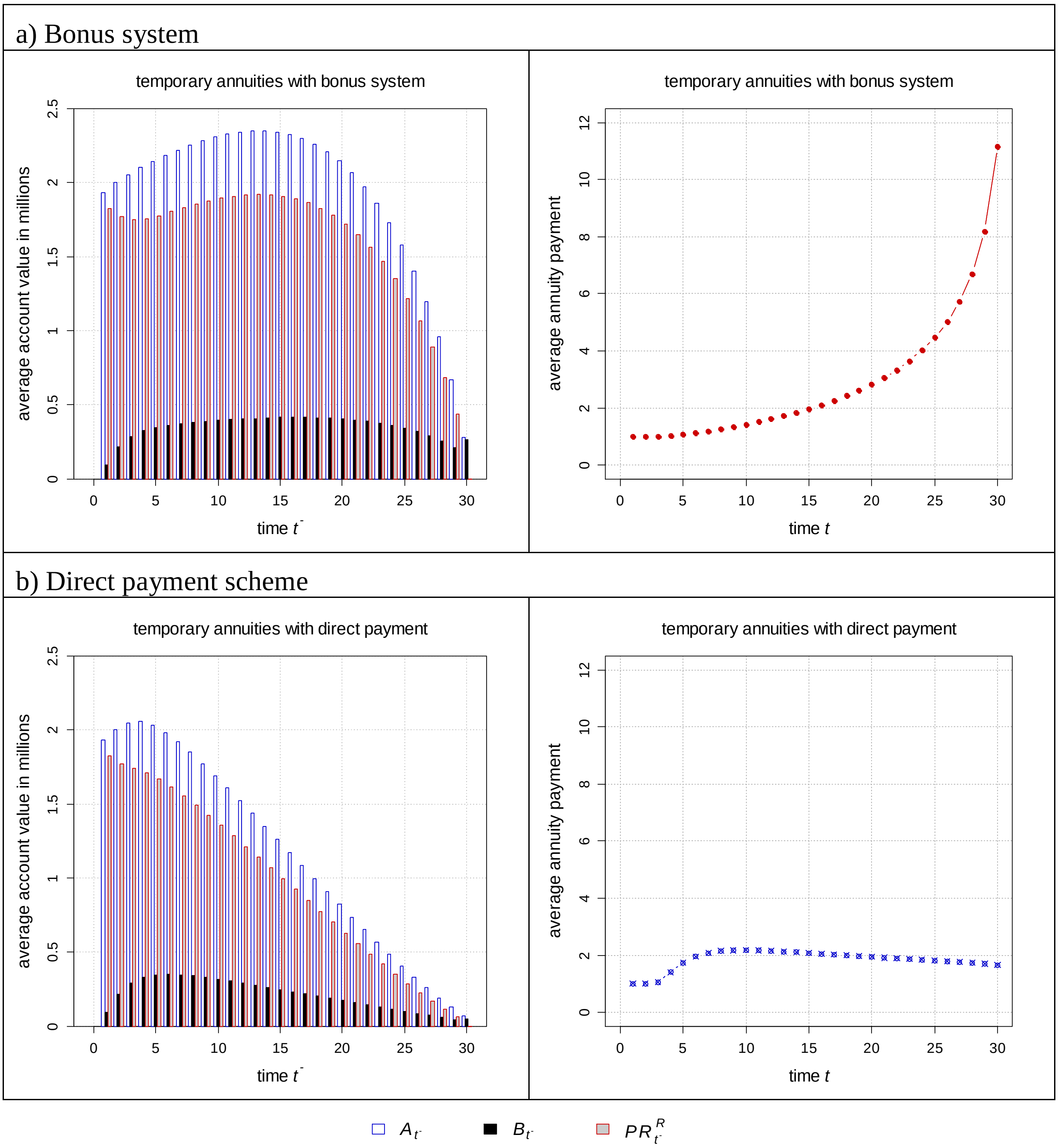


b) Interest-bearing accumulation



Notes: Average account value$_t = E\left(\text{account value}_t \mid T_s > t\right)$, where T_s denotes the time of default and account value$_t \in \left\{A_{t^-}, B_{t^-}, PR_{t^-}^S, IA_{t^-}, S_t\right\}$; the interest-bearing accumulation account per insured that is alive at the end of the contract term equals to $IA_{t^-}^{per\ insured} = E\left(IA_{t^-} \Big/ \left(N^S - \sum_{i=1}^{T} d_i^S\right) \mid T_s > t\right)$.

Figure A.3: Average account values over time for the temporary annuities for a stock portion of a_{max} = 25% in the case without feedback mechanism



Notes: Average account value$_t = E\left(account\ value_t \mid T_s > t\right)$, *where* T_s *denotes the time of default and account value*$_t \in \left\{A_{t^-}, B_{t^-}, PR_{t^-}^{R}, R_t\right\}$.