The Influence of Non-Linear Dependencies on the Basis Risk of Industry Loss Warranties

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THE INFLUENCE OF NON-LINEAR DEPENDENCIES ON THE BASIS RISK OF INDUSTRY LOSS WARRANTIES

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ABSTRACT

Index-linked catastrophic loss instruments represent an alternative to traditional reinsurance to hedge against catastrophic losses. The use of these instruments comes with benefits, such as a reduction of moral hazard and higher transparency. However, at the same time, it introduces basis risk as a crucial key risk factor, since the index and the company's losses are usually not fully dependent. The aim of this paper is to examine the impact of basis risk on an insurer's solvency situation when an industry loss warranty contract is used for hedging. Since previous literature has consistently stressed the importance of a high degree of dependence between the company's losses and the industry index, we extend previous studies by allowing for non-linear dependencies between relevant processes (high-risk and low-risk assets, insurance company's loss and industry index). The analysis shows that both the type and degree of dependence play a considerable role with regard to basis risk and solvency capital requirements and that other factors, such as relevant contract parameters of index-linked catastrophic loss instruments, should not be neglected to obtain a comprehensive and holistic view of their effect upon risk reduction.

JEL-Classification: G13; G22; G28; G32
Keywords: Index-linked catastrophic loss instruments; solvency capital requirements; copulas; non-life insurer

1. INTRODUCTION

Index-linked catastrophic (cat) instruments, such as industry loss warranties, cat options and other derivatives, constitute an alternative to traditional reinsurance to hedge against losses caused by natural catastrophes. Even though these instruments come with benefits, such as a reduction of moral hazard or the raising of new capital in the reinsurance market, the usefulness of index-linked cat instruments is affected by the crucial factor of basis risk, which can be described as the potential loss if an insurer's position is hedged with an instrument, the payoff of which is not fully dependent upon

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the insurer's portfolio (see, e.g. Meyers, 1998; Harrington and Niehaus, 1999). Previous papers define basis risk in very different ways and analyze necessary conditions (e.g. correlation, diversification) for index-linked products to represent successful instruments for managing risks and an effective hedging of portfolio losses. The aim of this paper is to provide an overview of existing basis risk definitions and, based upon them, to analyze basis risk in more depth by comparing the hedging effectiveness of industry loss warranties with respect to the insurer’s solvency situation under varying types and degrees of dependence between the insurer’s losses and the industry index.

The literature includes steady research on index-linked cat instruments. After the implementation of futures based upon catastrophic losses in 1992, D’Arcy and France (1992) describe and critically discuss potential advantages and drawbacks of these derivatives and analyze their use for dealing with an insurer’s risks. Niehaus and Mann (1992) compare insurance futures contracts with traditional reinsurance, regarding each as a method to trade underwriting risk. Furthermore, Harrington, Mann and Niehaus (1995) take into account that insurance derivatives can reduce the need for equity capital. For example, instead of increasing ex ante capital, which usually increases costs, the insurer holds an insurance future to cover high losses. Due to basis risk, a tradeoff exists between lower costs and the possibility of nonperformance of the derivative contract. To draw conclusions about the severity of basis risk, several articles analyze the potential hedging effectiveness of index-linked instruments. Variance reduction of the insurer’s losses by means of linear hedging for different lines of insurance based on empirical data, for instance, is examined by Harrington and Niehaus (1999). To point out the differences in hedging effectiveness between zip-based and statewide indices, a simulation analysis is conducted by Major (1999). Cao and Thomas (1998) apply the same method for measuring hedging effectiveness as in Major (1999) and empirically estimate the impact of using a hedging instrument based upon the Guy Carpenter Catastrophic Index, instead of using a simulation analysis.

Cummins, Lalonde and Phillips (2004) analyze the effectiveness of catastrophic loss index options using a windstorm simulation model. Based on previous research, Zeng (2000, 2003, 2005) introduces a new definition of basis risk and compares the hedging effectiveness of index-linked instruments to traditional reinsurance. Doherty and Richter (2002) analyze the tradeoff between moral hazard and basis risk using utility theory, while Lee and Yu (2002) develop a model to price cat bonds taking into account basis risk and moral hazard. A simultaneous analysis of pricing and basis risk of industry loss warranties based on different measures of basis risk and several actuarial and financial pricing approaches is conducted in Gatzert, Schmeiser and Toplek (2007). After analyzing recent developments of the market for cat bonds and other index-linked instruments, Cummins (2008) concludes that basis risk is one of the main impediments to the success of index-linked instruments. Cummins and Weiss (2009) give an overview for index-linked instruments, describe particular attributes and information, and discuss, among others aspects, the relevance of basis risk for the respective instrument.

In this paper, we expand previous work in several ways. To measure basis risk adequately, we first review and condense different definitions of basis risk in the previous literature into two main definitions: a) the hedging effectiveness of index-linked instruments, in which we extend previous viewpoints by additionally calculating the former with regard to an increase in the insurer's free surplus, which comes along with a reduction of solvency capital requirements, using the value at risk; and b) the conditional probability that the index does not exceed the trigger level given the insurer's losses exceed a critical level, thus implying a zero payoff of the hedging instrument. Furthermore, early and recent studies consistently stress that a high correlation between the index and the insurer's loss experience is an obvious and necessary condition for a beneficial use of these instruments. Despite the significance of dependence, focus has been laid upon linear relationships between industry index and an insurance company's losses. Thus, we extend previous analyses by modeling the dependence structure between the company's losses and the index as well as between high-risk and low-risk investments using non-linear dependencies by applying the concept of hierarchical copulas, which, to the best of our knowledge, is done for the first time. By varying both the degree (Kendall's rank correlation) and type of dependence between the company's losses and the index (using Gauss, Clayton and Gumbel copulas), the effect of basis risk can be analyzed in more depth. With regard to the index-linked instrument for risk management, we consider an industry loss warranty (ILW) contract and compare it to a traditional reinsurance contract. In addition, we conduct numerical sensitivity analyses to examine the effect of changes in characterizing parameters of an ILW, such as the attachment point or price differences to traditional reinsurance. This allows the identification of other crucial parameters that contribute to an increase in the effectiveness of index-linked instruments. Overall, the analysis reveals the conditions under which index-linked instruments should be preferred compared to traditional reinsurance products.

Our findings show that consideration of basis risk in the presence of non-linear dependencies is essential for the success of industry loss warranties in improving an insurer's solvency situation and that both the type (Gauss, Gumbel, Clayton copula) and degree (varying values of Kendall's tau) of dependence play an important role herein. However, basis risk is not the sole factor that influences the hedging effectiveness of index-linked instruments. We find that, even when basis risk (measured with the conditional probability of non-payment) remains unchanged, the effectiveness of ILWs can be raised by adjusting contract parameters, such as the insurance company's loss attachment point. Furthermore, ILWs can be more effective than traditional reinsurance depending upon the premium loading and the type of dependence between the index and the company's loss.

The remainder of this paper is structured as follows. In Section 2, previous definitions of basis risk are discussed and their similarities and differences are analyzed. Section 3 contains the model framework of a non-life insurer including the dependence structure between assets and liabilities. Numerical results are discussed in Section 4, and Section 5 concludes.

2. Index-Linked Catastrophic Loss Instruments and Basis Risk

2.1 The use of indices to transfer insurance risks

Due to an increasing frequency and intensity of catastrophic events, such as hurricanes and earthquakes, the traditional insurance and reinsurance markets need efficient alternatives for risk transfer. With the first industry loss warranty contracts in the 1980s (see SwissRe, 2009) and the implementation of futures based upon catastrophic losses in 1992 by the Chicago Board of Trade (see D'Arcy and France, 1992), the first index-based instruments to manage catastrophe risk were introduced.

An ILW is a reinsurance contract, the payoff of which does not solely depend upon the protection buyer's loss but is also linked to an industry loss index. The buyer receives a payment only if both the insurance company loss[1] and the industry loss index exceed

[1] In the following, the protection buyer is synonymously named the "company", the "insurer" or the "insurance company".

a certain threshold.[2] An alternative to hedge against catastrophic losses is an insurance future, which is a forward trade with standardized features. The future's payoff is related to an index, which is correlated to the insurer's exposure, whereby the insurer's losses can be hedged by entering into a long position of the contract. Another possibility to transfer catastrophic risks to the capital markets arose in the mid 1990s through index-based insurance-linked securities. Usually, a special purpose vehicle enters into a reinsurance contract with the cedent and, at the same time, issues bonds to investors (see SwissRe, 2009). As long as no pre-defined loss event occurs, such as, for example, that the index exceeds a certain barrier, the investors receive coupon payments and the principal. After a predefined loss event, the coupon payments, the principal, or both are reduced. Irrespective of the instrument, the index itself can depend upon catastrophe losses of the insurance-industry or parametric values, which are usually physical characteristics of catastrophic events (see Cummins, Lalonde and Phillips, 2004).

By virtue of the correlation between an index and an insurance company's losses, the index-based instrument can be used by an insurer for hedging catastrophe risk and hence constitutes an alternative to traditional reinsurance or raising capital to maintain solvency (see Meyers, 1998). The use of such an instrument offers several advantages compared to traditional reinsurance. For certain index-linked instruments, such as industry loss warranties, transaction costs are lower than for traditional reinsurance due to a high transparency of the index, which simplifies the underwriting process. Additionally, legal costs and due diligence are substantially reduced (see Gatzert and Schmeiser, 2010). Furthermore, if index-linked catastrophe instruments are available for the capital markets, they can help to finance catastrophic losses. This way, by trading options or futures based upon catastrophic indices, the capital markets bear financial consequences of major catastrophes (see Cummins, Lalonde and Phillips, 2004).

An impediment for insurers and reinsurers is moral hazard, which occurs if the insured changes his or her behavior after closing the contract, thus influencing the probability of a loss (see Kangoh, 1992). An insurer may neglect its risk management or change the reporting behavior concerning own losses after buying reinsurance. By using an

[2] ILWs can also be structured as binary (derivative) contracts that do not depend on the insurance company's loss. In this case, the buyer receives a fixed payment if the index exceeds a predefined trigger level (see Zeng, 2000). However, ILWs typically involve an indemnity-based trigger in addition to the index trigger (see, e.g., SwissRe, 2006).

index-linked instrument, however, moral hazard can be eliminated, because the index depends upon a parametric value, such as wind speed or seismological activities or upon losses of many insurers, such that an individual insurer has no significant impact on the changes of the index (see Doherty and Richter, 2002).

Despite these advantages, the usefulness of index-linked products strongly depends upon basis risk. In the previous literature, basis risk is often described as the risk that a low correlation between an insurer's book of business (and thus the losses resulting out of it) and the index could lead to potential losses if the underlying index is used to hedge a position of the insurer (see, e.g. Meyers, 1998; Harrington and Niehaus, 1999; a detailed overview of basis risk definitions is provided in Section 2.2). This is of high relevance for the buying insurance company of an index-linked reinsurance product, if, for example, the company's losses exceed a critical level, but the industry index is not triggered, thus resulting in a zero payoff. Furthermore, basis risk is of relevance in the context of (future) solvency capital requirements. From an accounting and regulatory point of view, index-linked instruments, such as ILWs, are treated as reinsurance if an indemnity trigger is inherent in the contract (see, e.g. Cummins and Weiss, 2009; SwissRe, 2009). Under these circumstances, index-linked instruments can be used to reduce solvency capital requirements, because, instead of enhancing new risk capital, the risk is carried by the counterparty of the contract. Only if basis risk is reduced to a modest amount will the impact of index-linked instruments on solvency capital requirements be satisfied from a risk management's point of view. Thus, basis risk can be considered a crucial factor with regard to purchase decisions in the context of risk analysis and risk management.

2.2 Comparing definitions of basis risk

The impact of basis risk has been analyzed in several previous studies (see, e.g. Harrington and Niehaus 1999; Major 1999; Cummins, Lalonde and Phillips 2004; Zeng 2000, 2003; Gatzert, Schmeiser and Toplek 2007). Although all of these former studies deal with basis risk, they use different methods of quantification. In the context of risk management and for our analysis, it is indispensable to have a consistent perception of basis risk. Hence, the following section provides an overview on how basis risk is defined and quantified in selected articles to point out similarities and discrepancies between the different approaches.

During the first years after the introduction of index-linked instruments, the expression "basis risk" was not explicitly used in the literature. However, early studies (see, e.g. D'Arcy and France, 1992) already point out that a high correlation between the index and the insurer's loss experience is an obvious and necessary condition for a beneficial use of these instruments. The relevant literature is consistent with regard to this definition of basis risk. Most of the methods for quantifying basis risk presented in this section focus upon the impact of the index-linked instrument on the insurer's liability side. Accordingly, basis risk is often analyzed by measuring the potential hedging effectiveness, such as in lowering the volatility of an insurer's liabilities. Nevertheless, several differences can be found in these methods.

Harrington and Niehaus (1999)

Harrington and Niehaus (1999) examine basis risk inherent in catastrophe insurance derivative contracts. Therefore, a time series analysis is conducted based upon annual loss ratios for three business lines for individual insurance groups during the years from 1974 to 1994. The analysis of historical data aims at determining potential hedging effectiveness of catastrophe linked instruments and at providing additional information on the question of whether basis risk is a relevant impediment for this kind of hedging strategy. Harrington and Niehaus (1999) study the hedging effectiveness by comparing the variance of the insurer's loss position with and without a forward contract, whose payoff depends upon a state specific catastrophe loss ratio. The percentage variance reduction through the hedge is quantified by the coefficient of determination, which can be estimated by a time series regression between the loss ratio of the insurer and the catastrophe loss ratio. Hence, Harrington and Niehaus (1999) do not directly quantify basis risk, but detect the relevance of basis risk by analyzing the possible variance reduction by means of the coefficient of determination R^2.

Major (1999)

Another method of analyzing basis risk is conducted by Major (1999), who simulates losses for an insurer's book of business and a catastrophe index to examine the sample correlation between these parameters. Major (1999) describes basis risk as the random variation of the difference between the hedge contract payout and the actual loss experience of the subject portfolio. While Harrington and Niehaus (1999) consider the effects of periods with non catastrophic events in an additional simulation analysis, Major (1999) integrates this impact in his definition of basis risk by dividing basis risk into conditional and unconditional basis risk. Conditional basis risk considers the effectiveness of the hedge given that a catastrophic event happened, whereas unconditional basis risk relates to all events, including the non-event. Similar to Harrington and Niehaus (1999), Major (1999) also draws conclusions about the impact of basis risk by examining the attained volatility of the insurer's losses through a linear hedge relative to the expected loss. Major's (1999) results show that hedging with statewide indices suffers from substantial basis risk. The hedged volatility of the loss position, achieved by zip-based indices, is lower than for the hedge with statewide indices. This can be ascribed to the major correlation between the index and the insurer's own losses in the case of a hedge with a zip-based index.

Cummins, Lalonde and Phillips (2004)

Similar to Major (1999), Cummins, Lalonde and Phillips (2004) conduct an analysis based upon simulated hurricane losses and determine the hedging effectiveness for insurers writing windstorm insurance in Florida. In contrast to Harrington and Niehaus (1999) and Major (1999), they consider a non-linear hedging program. The hedged position consists of the unhedged insurer's losses and a position in call option spreads based on a loss index, including statewide and intra-state regional indices. The analysis of basis risk is conducted in multiple ways. First, the performance of the hedge is measured relative to a perfect hedge, which can be described as a hedging strategy based on a loss index that is perfectly correlated to the insurer's losses. Second, the hedging is subject to a cost constraint, and, third, three different criteria are used to measure the hedging performance. In addition to the variance of the insurer's position, the value at risk and the expected exceedance value, which reflects the expected amount of loss given the extent to which the company's losses exceed a specified percentile of the insurer's loss distribution, are considered as functions to be minimized under the cost constraint. The proportionate reduction in the unhedged value of the risk measure then represents the hedging effectiveness for a respective risk measure. In contrast to previous methods, Cummins, Lalonde and Phillips (2004) also include measures in their analysis that allow a direct comparison to the perfect hedge. The hedging efficiency, for example, is defined as the hedging effectiveness of the index hedge relative to the perfect hedge and thus allows drawing conclusions about the severity of basis risk. A low value for the hedging efficiency suggests an ineffective hedge with the index compared to the perfect hedge. Hence, the dependence between own losses and the index seems to be insufficient, which results in substantial basis risk.

Zeng (2000, 2003)

Based upon an analysis of basis risk of ILWs, Zeng (2000) introduces an alternative measure with the intent to provide an easier understanding of basis risk. Basis risk is quantified as the conditional probability β that the industry loss does not exceed the ILW trigger given that the actual loss by the policyholder exceeds a predefined critical level.[3] This quantification is specified on ILWs, the payoff of which depends upon an industry loss index to be triggered but which can be used for other index-linked catastrophic instruments as well. Applying the definition of β on call option spreads, for example, the trigger could be replaced by the lower strike price. The critical loss level can be a predefined amount of loss that is crucial to the survival probability of the protection buying insurer or, in the case of an ILW, it could also be an indemnity trigger.

As an alternative, Zeng (2003) also uses the hedging effectiveness to quantify basis risk. Instead of a perfect hedge, a traditional indemnity reinsurance contract is integrated as the benchmark for the index hedge. The hedging effectiveness[4] is measured similarly to Cummins, Lalonde and Phillips (2004) by calculating the counter value of the hedging efficiency. If the hedging effectiveness of the index-linked instrument is less than that of the benchmark, a positive amount of basis risk remains.

Comparison of measures of basis risk

In summary, the presented basis risk quantifications in Table 1 show that basis risk is usually captured by means of the hedging effectiveness or the conditional probability, which will be the relevant definitions of basis risk used in the following analysis. The formal representation will be presented after introducing the notation of model variables.

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[3] A formal definition of the conditional probability β is provided in Section 3.

[4] Zeng (2003) describes this quantification in the case of an ILW. As discussed before, the methods can be transferred to similar index-linked instruments. An extension of the basis risk measures can be found in Gatzert, Schmeiser and Toplek (2007), who examine basis risk and pricing of industry loss warranties by comparing different measures of basis risk and several actuarial and financial pricing approaches.

Table 1: Comparison of selected measures of basis risk

Authors	*Quantification method*
Harrington and Niehaus (1999)	Hedging effectiveness measured by R^2 (= % variance reduction).[5]
Major (1999)	Hedging performance by minimum variance hedge.
Cummins, Lalonde and Phillips (2004)	Hedging effectiveness compared to the perfect hedge, proportionate reduction of the risk, measured based on variance of liabilities, value at risk or expected exceedance value.
Zeng (2000, 2003)	Conditional probability and hedging effectiveness compared to a benchmark (traditional reinsurance).

Consistent with the verbal definition of basis risk, which strongly refers to the correlation between the insurer's loss and the index, it is reasonable to use hedging effectiveness/conditional probability, which should be high/low if the correlation is high. The question of the most suitable quantification depends upon the problem at hand. To determine basis risk as a possible impediment for index-linked instruments, an isolated analysis of the index-linked instrument's hedging effectiveness seems to be sufficient without comparing the hedging results to the performance of a benchmark. The quantification using the conditional probability allows for an additional interpretation of basis risk, which may be helpful for an easier understanding. If the use of an index-linked catastrophe instrument is considered as an alternative to traditional reinsurance, an integration of a benchmark as illustrated by Cummins, Lalonde and Phillips (2004) or Zeng (2003) seems to be necessary.

3. MODEL FRAMEWORK OF A NON-LIFE INSURER

This section describes the model framework for a non-life insurance company. In a one-period setting, at time 0, shareholders make an initial contribution of E_0 (equity capital), and policyholders pay a premium π^{S_1} for insuring possible losses S_1 at time 1.

[5] The coefficient of variation R^2 is defined as $R^2 = Cov\left(LR_{jt}, LR_{Ct}\right)^2 / Var\left(LR_{jt}\right) Var\left(LR_{Ct}\right)$, with LR_{jt} denoting the *j*-th insurer's loss ratio at time *t* and LR_{Ct} the cat loss ratio for the state at time *t* (see Harrington and Niehaus, 1999).

Modeling the asset side

The total initial capital A_0 consists of equity capital and premiums and is invested in the capital market, whereby a fraction γ is invested in low-risk assets (denoted by '*L*') and the remaining part $(1-\gamma)$ is invested in high-risk assets (denoted by '*H*'). The value of the respective investment in asset class $i = L, H$ at time 1 is given by

$$A_{1,i} = A_0 \cdot e^{r_i},$$

where $r_i = \mu_i + \sigma_i \cdot Z_i$ denotes the continuous one-period return of the investment with respective annual expected value μ_i , respective annual standard deviation σ_i; and Z_i being a normally distributed random variable. Thus, the value of the asset portfolio after one period (at time $t = 1$) is determined by

$$A_1 = \gamma \cdot A_{1,L} + (1-\gamma) \cdot A_{1,H} = A_0 \cdot \left(\gamma \cdot e^{r_L} + (1-\gamma) \cdot e^{r_H}\right).$$

Modeling the liability side

After one period, the policyholders receive their claims payments, resulting in a stochastic company loss S_1. For risk management, the management can choose the portion invested in low-risk and high-risk assets and, in addition, decide to purchase an index-linked catastrophic loss instrument (here: *ILW*) or a traditional reinsurance (here: aggregate excess of loss, denoted by 're') contract.

Let S_1 denote the company's loss distribution in $t = 1$, A^i the attachment of the company loss and L^i the layer limit for the respective risk management instrument *i* = *ILW*, *re*. The aggregate excess of loss reinsurance contract is thus described by

$$X_1^{re} = \min\left(\max\left(S_1 - A^{re}, 0\right), L^{re}\right).$$

The index-linked contract analyzed in this paper is an indemnity-based industry loss warranty contract, which also contains an aggregate excess of loss contract and further incorporates a second trigger that is based upon the industry loss distribution I_1 in $t = 1$ (see Zeng, 2000; Wharton Risk Center, 2007). The industry loss trigger of the ILW contract is denoted by Y and $1\{I_1 > Y\}$ represents the indicator function, which is equal to 1 if the industry loss I_1 in $t = 1$ is greater than the trigger Y and 0 otherwise. Hence, the payoff of this double-trigger contract in $t=1$ can be expressed as

$$X_1^{ILW} = \min\left(\max\left(S_1 - A^{ILW}, 0\right), L^{ILW}\right) \cdot 1\{I_1 > Y\}. \quad (1)$$

The most frequently used reference indices for insured catastrophic events are those provided by the Property Claim Services (PCS) in the United States.[6] Thus, the industry loss is usually determined by referencing a relevant PCS index. Burnecki, Kukla, and Weron (2000) show that, in general, a lognormal distribution provides a good fit to the analyzed PCS indices. Hence, the industry loss index and the company loss are both assumed to follow a lognormal distribution.

Premium calculation

The premiums for the different contracts are determined based on the actuarial expected value principle using the expected contract payoffs $E(S_1)$, $E\left(X_1^{re}\right)$ and $E\left(X_1^{ILW}\right)$ with a percentage δ^i (≥ 0) as a loading of itself, where i = S_1, re, ILW. Hence, the premiums are given by

$$\pi^{S_1} = E\left(S_1\right)\left(1+\delta^{S_1}\right),$$
$$\pi^{re} = E\left(X_1^{re}\right)\left(1+\delta^{re}\right),$$
$$\pi^{ILW} = E\left(X_1^{ILW}\right)\left(1+\delta^{ILW}\right).$$

This approach is not risk sensitive, since it considers only the expected value and not the risk inherent in the contract. However, it requires only the first moment of the contract's loss distribution and can thus be easily implemented as well as easily be adjusted to other valuation approaches (for an overview of different valuation approaches, see Gatzert, Schmeiser and Toplek, 2007).

Risk measurement

Risk is assessed based on the insurer's solvency situation represented by the insurer's free surplus FS_a^i and the solvency capital requirements SCR_α^i, where i stands for the respective risk management strategy, i.e., purchasing no risk management instrument,

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[6] Since 2009, PERILS has launched a European industry index for CRESTA data.

or acquiring an ILW or a reinsurance contract (i = *without*, *ILW*, *re*). Thus, SCR_α^i is the amount of capital needed at time zero to meet future obligations over a fixed time horizon for a required safety level α and is calculated based upon the distribution of the change in the economic risk-based capital over one year,

$$\Delta RBC^i = e^{-r_f} \cdot RBC_1^i - RBC_0^i,$$

where r_f represents the riskless interest rate and RBC_t^i denotes the risk-based capital at time $t = 0,1$, given by the difference between assets and liabilities. Hence, at time 1,

$$RBC_1^i = \left(\pi^{S_1} + E_0 - \pi^i\right) \cdot \left(\gamma \cdot e^{r_L} + (1-\gamma) \cdot e^{r_H}\right) + X_1^i - S_1,\ i = without, ILW, re, \tag{2}$$

where $\pi^{without} = X_1^{without} = 0$. In the current discussions of the Solvency II framework for insurance companies in the European Union, solvency capital will most likely be determined by using the value at risk concept with a confidence level of 99.5% (corresponding to a safety level of α = 0.5%). Hence, the SCR_α^i can be calculated from (see, e.g., Gatzert and Schmeiser, 2008)

$$SCR_\alpha^i = -VaR_\alpha\left(\Delta RBC^i\right),\ i = without,\ ILW,\ re,$$

where VaR_α is the value at risk for a confidence level α, given by the quantile of the distribution $F^{-1}(\alpha) = \inf\left\{x : F(x) \geq \alpha\right\}$. We assume that regulators expect the solvency capital requirements not to exceed the value of the available risk-based capital at time 0, $RBC_0^i \geq SCR_\alpha^i$, $i = without, ILW, re$, implying that the free surplus FS_a^i should be positive:

$$FS_a^i = RBC_0^i - SCR_\alpha^i = VaR_\alpha\left(e^{-r_f} \cdot RBC_1^i\right) \overset{!}{\geq} 0,\ i = without, ILW, re.$$

Thus, if the free surplus falls below zero, measures regarding, e.g., the insurer's risk management or underwriting strategy should be taken to avoid sanctions by regulatory authorities. Therefore, in the following, the free surplus FS_a^i is used to analyze the efficiency of risk management measures, whereby a decrease in SCR_α^i generally causes an increase in FS_a^i. The amount of FS_a^i depends upon the choice of the stochastic model of assets and liabilities and upon the input parameters of these models.

Dependence structure

When calculating FS_{α}^{i} in non-life insurance, the type of dependence plays a major role (see Eling and Toplek, 2009; Shim, Lee, MacMinn, 2009; Zhou, 2010). The dependence between investment classes (low-risk and high-risk), between the losses (company and industry), and between assets and liabilities are of high relevance for an analysis of basis risk and the insurer's solvency situation. To avoid restrictive assumptions concerning the dependence, the concept of copulas is applied when generating random numbers for risk factors. A fundamental benefit of copulas is that they are not restricted to linear dependencies and allow the involvement of such characteristics as upper- and lower-tail dependencies between risk factors (see Embrechts, Lindskog and McNeil, 2003, p. 4). To determine the impact of different dependence structures, we compare three copulas, the Gauss, Clayton and Gumbel copula. The Gauss copula is given by

$$C_P^{Gauss}\left(u_1,\ldots,u_n\right)=\Phi_P\left(\Phi^{-1}\left(u_1\right),\ldots,\Phi^{-1}\left(u_n\right)\right)$$

and represents the copula of a multivariate normal distribution that does not exhibit tail dependence (see McNeil, Frey and Embrechts 2005, p. 191). Φ_P is the joint distribution function of the *n*-variate standard normal distribution function with linear correlation matrix *P*, and Φ denotes the standard univariate normal distribution function (see McNeil, Frey and Embrechts 2005, p. 193).

The Clayton and Gumbel copulas are explicit copulas and belong to the family of Archimedean copulas. In contrast to the Gauss copula, Archimedean copulas have closed-form solutions and can be constructed by using generator functions $\phi^{Cl}(t)$ for the Clayton copula $\left(Cl\right)$ and $\phi^{Gu}(t)$ for the Gumbel copula $\left(Gu\right)$ (see McNeil, Frey and Embrechts 2005, p. 221). An *n*-dimensional copula $i=Cl,Gu$ is constructed by

$$C_{\theta}^{i}\left(u_1,\ldots,u_n\right)=\phi^{i^{-1}}\left(\phi^{i}\left(u_1\right)+\ldots+\phi^{i}\left(u_n\right)\right), \qquad (3)$$

using the respective generator and its inverse. Table 2 exhibits the generator functions and their inverse for the Clayton and Gumbel copulas, where the parameter θ determines the degree of dependence. For $\theta\to\infty$, both copulas imply perfect dependence. Independence is implied for $\theta\to 0$ in the case of the Clayton copula and for $\theta\to 1$ in the case of the Gumbel copula. Depending on which Archimedean copula is employed, upper- or lower-tail dependence persists. Tail dependence measures the dependence between extreme values and, thus, the strength of the tails in a bivariate distribution. Upper-tail dependence (λ_u) between two random numbers, X_1 and X_2 with distribution functions F_1 and F_2 can be defined as the conditional probability that X_2 exceeds its q-quantile, given that X_1 exceeds its q-quantile. Then, considering the limit as q goes to infinity, upper-tail dependence is given by

$$\lambda_u\left(X_1, X_2\right) = \lim_{q \to 1^-} P\left(X_2 > F_2^{\leftarrow}(q) \mid X_1 > F_1^{\leftarrow}(q)\right).$$

Table 2: Generator functions and its inverse for the Clayton and Gumbel copula

Copula	Generator $\phi^{i}(t)$	Inverse $\phi^{i^{-1}}(t)$	Parameter Range	Tail Dependence
C_θ^{Cl}	$\phi^{Cl}(t) = \frac{1}{\theta}\left(t^{-\theta} - 1\right)$	$\phi^{Cl^{-1}}(t) = \left(\theta \cdot t + 1\right)^{-\frac{1}{\theta}}$	$0 \le \theta < \infty$	lower
C_θ^{Gu}	$\phi^{Gu}(t) = \left(-\ln t\right)^{\theta}$	$\phi^{Gu^{-1}}(t) = \exp\left(-t^{\frac{1}{\theta}}\right)$	$1 \le \theta < \infty$	upper

Provided a limit $\lambda_u \in [0;1]$ exists, X_1 and X_2 show upper-tail dependence if $\lambda_u \in (0;1]$. The higher the value for λ_u, the stronger is the degree of upper-tail dependence. If $\lambda_u = 0$, X_1 and X_2 are asymptotically independent in the upper-tail. Analogously lower-tail dependence can be derived from

$$\lambda_l\left(X_1, X_2\right) = \lim_{q \to 0^+} P\left(X_2 \le F_2^{\leftarrow}(q) \mid X_1 \le F_1^{\leftarrow}(q)\right),$$

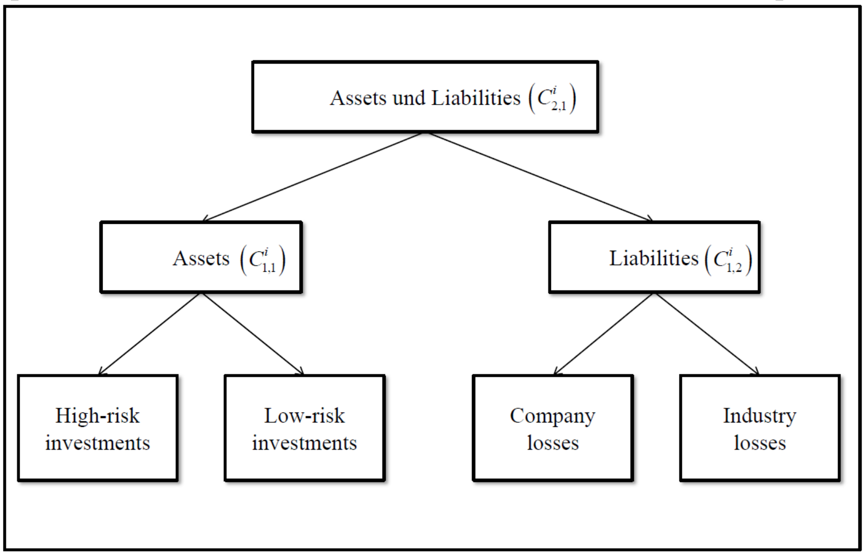
if $\lambda_l \in [0;1]$ exists (see McNeil, Frey and Embrechts 2005, p. 209). Lower-tail dependence is given in the case of the Clayton copula, upper-tail dependence in the case of the Gumbel copula.

A special case of dependence, which will be used in the numerical analysis, is perfect dependence between n random variables and constructed by the comonotonicity copula, which is defined by

$$M\left(u_1, \ldots, u_n\right) = \min\left\{u_1, \ldots, u_n\right\}.$$

The disadvantage of multivariate Archimedean copulas is that their use for higher dimensional simulations is very limited if they are generated according to Equation (3), because they allow only equal dependence structures between several risk factors. Hence, using this type of dependence structure for our analysis would assume the same type and degree of dependencies between high-risk and low-risk investments, between the company losses and the index, and between assets and liabilities. This limitation can be eliminated by constructing a hierarchical Archimedean copula, which allows different degrees of dependencies within and between the risk groups.

Figure 1: Dependence structure with a hierarchical Archimedean copula



We will thus construct hierarchical Archimedean copulas with two levels as described in Savu and Trede (2006). Two pairs of standard uniform random variables (u_1, u_2) and (u_3, u_4) are linked with different copulas $C_{1,1}^{i}$ (for the dependence structure between high- and low-risk investments) and $C_{1,2}^{i}$ (for the dependence structure between company losses and the index) by their generator functions $\phi_{1,1}^{i}(t)$ and $\phi_{1,2}^{i}(t)$. Afterwards, both copulas are joined at the upper level with a third generator $\phi_{2,1}^{i}(t)$ resulting in a hierarchical Archimedean copula $C_{2,1}^{i}$ (for the dependence structure between assets and liabilities) with the analytical form:

$$C_{2,1}^{i}(u_1,u_2,u_3,u_4)=\phi_{2,1}^{i\;-1}\left(\phi_{2,1}^{i}\circ\phi_{1,1}^{i\;-1}\left(\phi_{1,1}^{i}(u_1)+\phi_{1,1}^{i}(u_2)\right)+\phi_{2,1}^{i}\circ\phi_{1,2}^{i\;-1}\left(\phi_{1,2}^{i}(u_3)+\phi_{1,2}^{i}(u_4)\right)\right). \quad (4)$$

Values for high- and low-risk investments are generated by applying the inverse transform method on u_1 and u_2 and for company and the index losses on u_3 and u_4 respectively. The generated dependence structure is illustrated in Figure 1 (see Eling and

Toplek, 2009). Necessary conditions for hierarchical Archimedean copulas as postulated in Savu and Trede (2006) are fulfilled in the present situation. For example, the degree of dependence for upper levels of the hierarchical copula has to be lower than for lower levels. Applied to our framework, this implies that the degree of dependence between assets and liabilities always has to be the lowest.

To make the different copulas comparable, we use Kendall's rank correlation ("Kendall's tau") ρ_τ. For the Gauss copula, we use the following relation between Kendall's rank correlation ρ_τ and the off-diagonal elements ρ_{ij} of the correlation matrix (see McNeil, Frey and Embrechts 2005, p. 215):

$$\rho_\tau\left(X_1, X_2\right) = \frac{2}{\pi} \arcsin \rho_{ij} . \tag{5}$$

The Clayton and Gumbel copulas are also calibrated by the relationship between θ and ρ_τ, which is defined for the Clayton copula by

$$\rho_\tau = \frac{\theta}{\theta + 2} \tag{6}$$

and for the Gumbel copula

$$\rho_\tau = 1 - \frac{1}{\theta}, \tag{7}$$

respectively.

Definitions of basis risk

While Section 2 presented different definitions and quantifications of basis risk given in the literature, we will use two concrete types of quantifications in the following numerical analysis: the conditional probability and the counter value of the hedging efficiency CHE_m^b, where b represents a benchmark for the ILW hedge, which in our case is given by traditional reinsurance (denoted with superscript re) or a perfect hedge with an ILW (denoted with superscript pe, assuming that the company loss and industry loss are fully dependent), and m denotes a certain risk measure and is attached as a subscript to the relevant variables. Other methods to quantify the severity of basis risk such as the hedging effectiveness or the value of the risk measure after hedging can be derived from the hedging efficiency. Hence, there is no need to implement other quantifications separately. The conditional probability β is defined by

$$\beta = \mathrm{P}\left(I_1 < Y \mid S_1 > S_1^{\varepsilon}\right), \tag{8}$$

where the critical level of the company's losses S_1^{ε} is given by the $(1-\varepsilon)$-quantile $(0 \le \varepsilon \le 1)$ of the loss distribution S_1. The counter value of the hedging efficiency of an ILW contract CHE_m^b is defined based on the ratio of the hedging effectiveness of the ILW and the hedging effectiveness of a chosen benchmark *b*, using a risk measure *m*. In general, the hedging effectiveness HE_m^i measures the proportionate reduction in the risk measure *m*, which can be attained by acquiring an ILW, traditional reinsurance or a perfect hedge as a hedging instrument (*i* = *ILW*, *re*, *pe*) as compared to the case where no hedging instrument is purchased. The company's loss varies, depending upon which hedging instrument is used. In the case of hedging with an ILW, for instance, the company's loss is given by the difference between the losses resulting from the company's underwriting business and the payoff of the ILW. As an alternative, traditional reinsurance and a perfect hedging instrument are considered, such that

$$S_1^i = S_1 - X_1^i,\ i = ILW, pe, re,$$

where *i* stands for the instruments ILW, perfect hedge and reinsurance. In the numerical analysis, the hedging effectiveness of a hedging instrument is measured by means of the proportionate reduction in $m = Sd, VaR_{\alpha}$, where Sd stands for the standard deviation of the company's losses, given by

$$HE_m^i = 1 - \frac{m\left(S_1^i\right)}{m\left(S_1\right)}, i = ILW, pe, re.$$

Depending on the chosen benchmark contract and the risk measure, the hedging efficiency RHE_m^b of the ILW contract can then be calculated by the ratio

$$RHE_m^b = \frac{HE_m^{ILW}}{HE_m^b}.$$

With these definitions, CHE_m^b is quantified through

$$CHE_m^b = 1 - RHE_m^b . \tag{9}$$

In addition, basis risk is also interpreted as the impact of dependence on the insurer's solvency situation measured by FS_α^{ILW}. In contrast to the other definitions, this offers a comprehensive view on the insurer's risk situation that includes assets and liabilities. We thus consider the quantifications and interpretations of basis risk as summarized in Table 3.

Table 3: Quantifications of basis risk

Quantification	*Description*
$\beta = \mathrm{P}\left(I_1 < Y \mid S_1 > S_1^\varepsilon\right)$	The conditional probability, implying a zero payoff for the industry loss warranty
$CHE_{VaR}^{pe} = 1 - \dfrac{1 - \dfrac{VaR_\alpha(S_1^{ILW})}{VaR_\alpha(S_1)}}{1 - \dfrac{VaR_\alpha(S_1^{pe})}{VaR_\alpha(S_1)}}$	The counter value of the hedging efficiency using the value at risk as the relevant risk measure and the perfect hedge (perfect dependence between the company's losses and the index) as a benchmark
$CHE_{VaR}^{re} = 1 - \dfrac{1 - \dfrac{VaR_\alpha(S_1^{ILW})}{VaR_\alpha(S_1)}}{1 - \dfrac{VaR_\alpha(S_1^{re})}{VaR_\alpha(S_1)}}$	The counter value of the hedging efficiency using the value at risk as the relevant risk measure and a traditional reinsurance contract as a benchmark
$CHE_{Sd}^{pe} = 1 - \dfrac{1 - \dfrac{Sd(S_1^{ILW})}{Sd(S_1)}}{1 - \dfrac{Sd(S_1^{pe})}{Sd(S_1)}}$	The counter value of the hedging efficiency using the standard deviation as the relevant risk measure and the perfect hedge (perfect dependence between the company's losses and the index) as a benchmark
$CHE_{Sd}^{re} = 1 - \dfrac{1 - \dfrac{Sd(S_1^{ILW})}{Sd(S_1)}}{1 - \dfrac{Sd(S_1^{re})}{Sd(S_1)}}$	The counter value of the hedging efficiency using the standard deviation as the relevant risk measure and a traditional reinsurance contract as a benchmark
$FS_\alpha^{ILW} = VaR_\alpha\left(e^{-r_f} \cdot RBC_1^{ILW}\right)$	Free surplus if the insurer purchases an industry loss warranty contract

4. NUMERICAL ANALYSIS

This section studies the effectiveness of an ILW contract under non-linear dependence in the presence of basis risk by examining its impact on the risk-based capital of an insurer and several other definitions of basis risk. Its effectiveness is further compared to a traditional excess of loss reinsurance contract as introduced in the previous section. By means of sensitivity analyses, the following examples aim to examine whether, and if so, the extent to which, different types and degrees of dependence are relevant in the context of basis risk and to identify key drivers for basis risk and an insurer's solvency situation.

Input parameters for the reference contract

The input data for the reference contract are summarized in Table 4, where the expected value of the company loss $E(S_1)$ and the respective standard deviation $\sigma(S_1)$ are based on empirical data of a non-life insurer as presented in Eling, Gatzert and Schmeiser (2009). The expected value of the industry loss $E(I_1)$ and its standard deviation $\sigma(I_1)$ are adopted from Gatzert, Schmeiser and Toplek (2007), referring to Hilti, Saunders and Lloyd-Hughes (2004). Furthermore, expected value and standard deviation for the return of high-risk assets r_H are assumed to be 8% and 20%, and 5.5% and 6.5% for the return of low-risk assets r_L. These values are based on data from representative indices such as the S&P 500 and the DAX for high-risk assets, and US treasury bills and international government bond indices, e.g., Meryll Lynch Global Government Bond Index, for low-risk assets.[7] Since the basis risk measure β (see Equation (8)) is the conditional probability that the ILW does not pay off but the insurer faces large losses, we set the critical loss level to 95% and thus assume that it is critical for an insurer if losses exceed the 95%-quantile S_1^{ε} of S_1.[8] All other input parameters such as the safety level and riskless interest rate are chosen for illustration purposes and were – similarly to the critical loss level – subject to robustness tests to

7 Depending on the estimated time interval, expected values and standard deviations of returns for the S&P 500 and the DAX vary between 6% and 11%, as well as 13% and 25%, respectively. Analogously, expected values and standard deviations for the returns of U.S. treasury bills and the Meryll Lynch Global Government Bond Index range between 3% and 6%, as well as 3.5% und 8%, respectively.

8 The 95%-quantile is derived through Monte Carlo simulation for all copula cases and verified by directly calculating the 95%-quantile of the lognormal distribution of S_1 for the given input parameters.

ensure the stability of the general findings. Numerical results are obtained using Monte Carlo simulation with 500,000 sample paths for each process (see Glasserman, 2008), in which we use the same set of random numbers for each simulation run.[9] The copulas are generated by using the algorithms in McNeil, Frey and Embrechts (2005).

In the following analysis, we examine the influence of the degree of dependence using Kendall's rank correlation ρ_τ between company loss S_1 and industry loss I_1 as well as the type of dependence on an insurer's solvency situation and different definitions of basis risk. We further study the influence when varying the attachment of the company's loss for the ILW contract A^{ILW}, the premium loadings on ILW and reinsurance contract, and the volatility of the company's losses, keeping everything else constant.

To keep all cases comparable, we fix the dependence parameter ρ_τ for different types of dependence structures. In particular, we compare the cases for the Gauss and two hierarchical copulas (see Equation (4)) using the Gumbel or Clayton copula to generate the hierarchical structure.[10] To ensure comparability between the different copula cases, values for Kendall's rank correlation ρ_τ are converted into the respective dependence measures for each copula using Equations (5), (6) and (7).

Table 4: Input parameters for the reference contract

Available equity capital	E_0	\$40 million
Expected value and standard deviation of company loss (lognormally distributed)	$E(S_1), \sigma(S_1)$	\$117 million, \$66 million
Expected value and standard deviation of industry index (lognormally distributed)	$E(I_1), \sigma(I_1)$	\$1,450 million, \$3,550 million
Expected value and standard deviation for the return of high-risk assets (normally distributed)	μ_H, σ_H	8%, 20%
Expected value and standard deviation for the return of low-risk assets (normally distributed)	μ_L, σ_L	5.5%, 6.5%
Riskless interest rate	r_f	2%

9 To ensure the robustness of the results, all graphs have also been generated using different sets of sample paths and a different number of simulation runs.

10 Further analyses have shown that combine the Clayton and Gumbel copula in the hierarchical structure does not have a significant impact on our results and hence, the same copula is used to generate hierarchical copulas.

Investment in low-risk assets	γ	60%
Safety level for risk-based capital $\left(VaR_{\alpha}\right)$	α	5%
Kendall's tau for low-risk and high-risk assets	$\rho_{\tau}\,(A_{1,L},A_{1,H})$	0.2
Kendall's tau for company and index losses	$\rho_{\tau}\,(S_1, I_1)$	0.6
Kendall's tau for assets and liabilities	$\rho_{\tau}\,(A_1, L_1)$	0.1
Premium loading insurance contract	δ^{S_1}	30%
Premium loading reinsurance contract	δ^{re}	0%
Premium loading ILW	δ^{ILW}	0%
Layer limit for ILW and reinsurance contract	L^{ILW} , L^{re}	$200 million
Industry loss trigger	Y	$3,000 million
Attachment of the company's loss for ILW	A^{ILW}	$100 million
Attachment of the company's loss for reinsurance	A^{re}	$100 million
Critical level of company loss (95%-quantile of S_1)	S_1^{ε}	$242 million

Comparison of basis risk definitions for different types and degrees of dependence

In a first step, we compare different definitions of basis risk for different types and degrees of dependence. Thus, the left column in Figure 2 shows the conditional probability β and the counter value of hedging efficiency CHE_m^b, which is displayed for two functions of risk measures m (value at risk and standard deviation of the company's losses) and two benchmarks b (perfect hedge and traditional reinsurance) for Gauss, Clayton and Gumbel copula. In addition, one major question for insurers is whether required solvency capital can be reduced effectively such that the free surplus increases when purchasing an ILW. In this respect, the type and degree of dependence will play an important role for basis risk and thus FS_{α}^{ILW} .

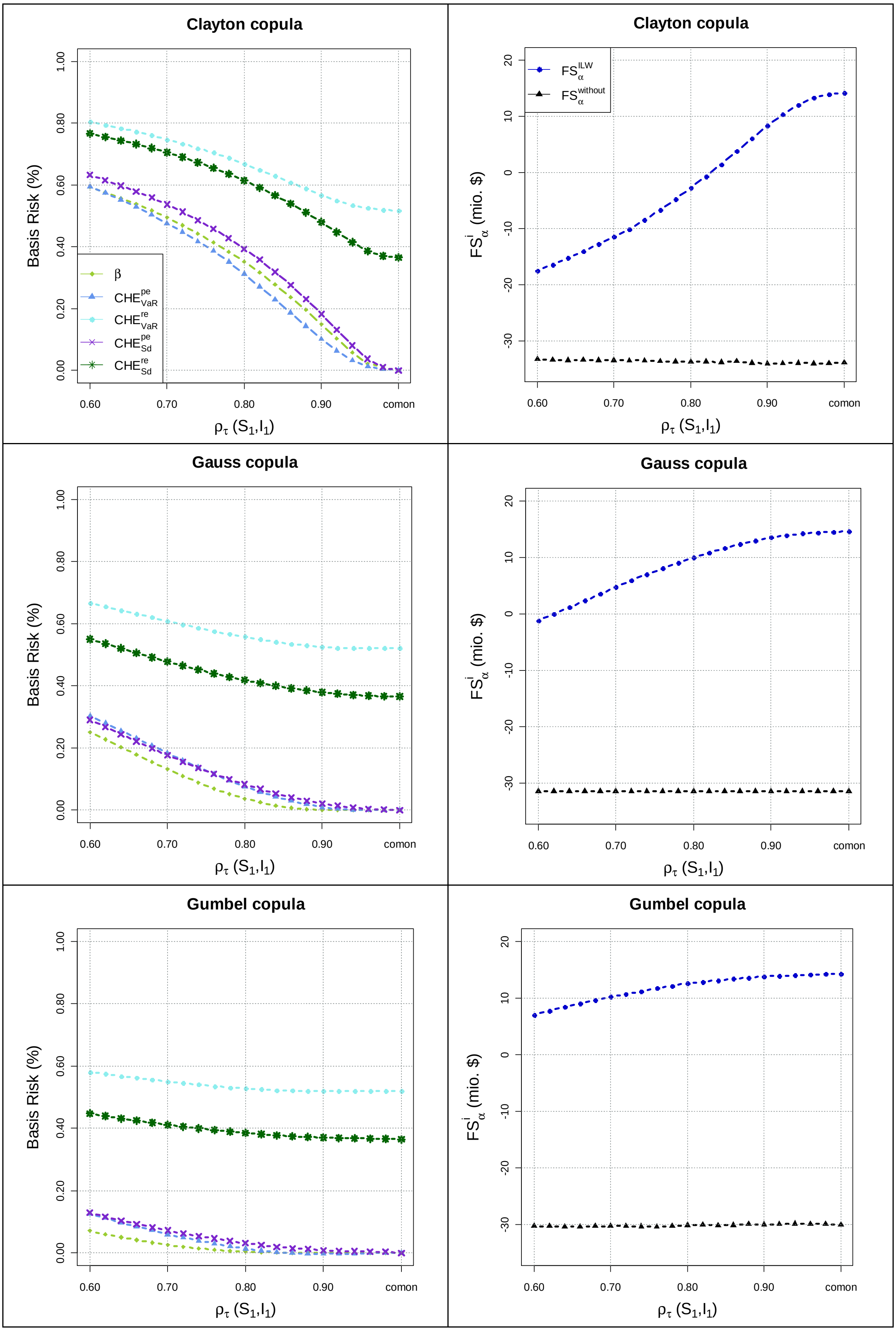
Therefore, to obtain a holistic picture of the impact of dependence on basis risk and as described in Table 3, we study the impact of dependence on FS_{α}^{ILW} in the sense of basis risk. Hence, for all three copulas, the right column in Figure 2 presents the free surplus without hedging[11] and with hedging using an ILW. From Figure 2, it can be seen that for all chosen basis risk measures, basis risk decreases for higher degree of de-

[11] Note that in general, $FS_{\alpha}^{without}$ is constant for each copula case. The slight variations in the case of the Clayton and the Gumbel copulas result from minor variations of gamma and alpha-stable random numbers, respectively, that are influenced by the degree of dependence $\rho_{\tau}\left(S_1, I_1\right)$.

pendence between the company and the industry losses (Kendall's tau) in a similar way, up to the case of perfect dependence between the company's losses and the index, which is generated through the comonotonicity (comon) copula (and represents the case of a perfect hedge). Furthermore, the insurer would be closed down by the regulators without taking any risk management measures, as a negative value of the free surplus $FS_{\alpha}^{without}$ implies that the solvency capital requirements exceed the available risk-based capital at time 0, such that $SCR_{\alpha}^{without} \geq RBC_{\alpha}^{without}$ (right column).

Figure 2 shows that the type of dependence structure is a key component regarding the effectiveness of an ILW for increasing FS_{α}^{ILW} and reducing basis risk. Even if the dependence parameter $\rho_{\tau}\left(S_1, I_1\right)$ is the same for different dependence structures (copulas), the *type* of (nonlinear) dependence has a great impact on basis risk and the insurer's solvency situation. In particular, the ILW yields the best results (highest FS_{α}^{ILW}, and lowest CHE_m^b and β) if the dependence structure between index and company loss is described by a Gumbel copula, the dependence structure of which is upper-tail dependent. This impact is reasonable, since, as described in Section 3, upper-tail dependence reflects the conditional probability that a random number X_2 exceeds the q-quantile, given that the other random number X_1 exceeds the q-quantile. Thus, in the present case, upper-tail dependence increases the payoff probability for an ILW due to an increasing probability for high values of the index, conditional on a high value of the company's loss. The lowest free surplus FS_{α}^{ILW} occurs in the case of the lower-tail dependent Clayton copula and can be explained analogously. In addition, with an increasing *degree* of dependence (Kendall's tau) FS_{α}^{ILW} rises considerably, in line with the decrease of other measures of basis risk (CHE_m^b and β). Thus, it is the combination of both type and degree of dependence that is relevant when assessing the attractiveness of ILWs with respect to, for example, improving an insurer's solvency situation in the presence of basis risk.

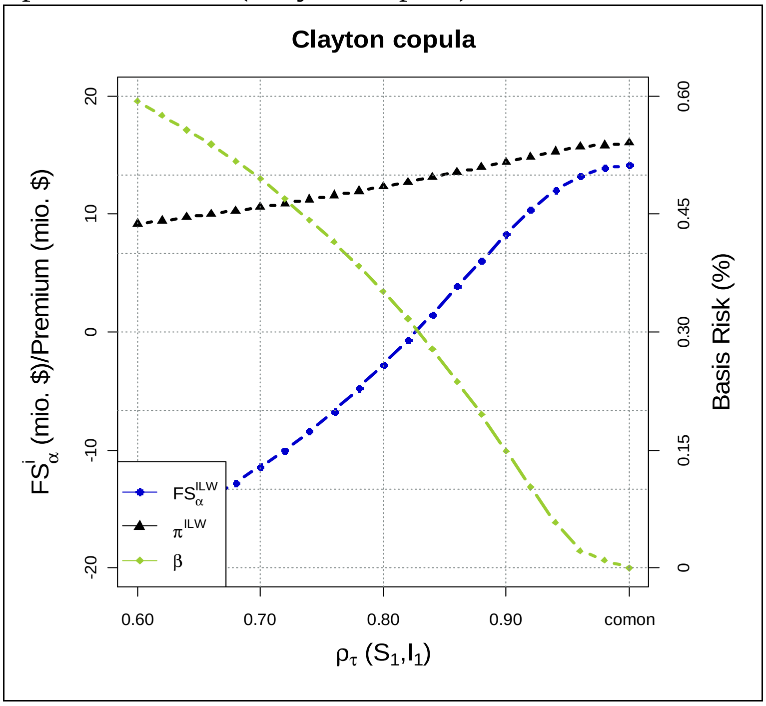
Figure 2: Basis risk measures and levels of free surplus for different types and degrees of dependence between company's losses S_1 and industry index I_1



The tradeoff between basis risk, premiums and free surplus

Next, we analyze the tradeoff between basis risk using the conditional probability β, the ILW premium, and the FS_{α}^{ILW} of an insurer using an ILW as a risk management tool as displayed in Figure 3 for varying values of Kendall's tau in case of a hierarchical Clayton copula.[12] Figure 3 illustrates that, for higher degrees of dependencies and thus lower levels of basis risk, the premium of the ILW increases, which is due to an increasing expected payoff of the ILW. Despite the increase in the premium, the free surplus increases along with decreasing values of basis risk.

Figure 3: The effect of the degree of dependence on basis risk (β), free surplus FS_{α}^{ILW}, and ILW premium π^{ILW} (Clayton copula)



Thus, basis risk constitutes a dominant factor with regard to the free surplus FS_{α}^{ILW}. As a consequence, paying higher prices for ILWs is justified if basis risk can be sufficiently reduced and if the increase of FS_{α}^{ILW} and thus the reduction of SCR_{α}^{ILW} is a main purpose of the insurer's risk management strategy. For instance, buying an ILW the payoff of which is based upon a regional index instead of a state wide index would increase its price, but at the same time would imply an enhancement of FS_{α}^{ILW} to a

[12] The results for other copulas or basis risk measures differed in the level of risk but were otherwise robust and in tendency similar.

significantly higher value, a trade-off that should be taken into account by insurers when deciding whether to buy index-linked cat instruments.

The impact of reinsurance premium loadings

If an insurer has the choice between an ILW and a traditional reinsurance contract, another crucial factor besides the basis risk inherent in ILW contracts that influences this decision is the degree of price difference between these alternatives, which has an impact on the insurer's solvency situation as well due to the inherent tradeoff exhibited in Figure 3. In practice, traditional reinsurance contracts are often more expensive[13] compared to ILW contracts due to an extensive underwriting and higher transaction costs.[14] In addition, the attachment point of the company loss in case of an ILW is typically set to a low level to ensure exceedance (see Cummins and Weiss, 2009).

To analyze the relationship between the surcharge of the traditional reinsurance and its advantageousness on lowering the required solvency capital and, hence, increasing the free surplus, we only vary the loading of the traditional reinsurance for different values of Kendall's tau between the company's losses and the index and keep the loading of the ILW at zero. Thus, the surcharge on the traditional reinsurance can be interpreted as a relative difference in premium loadings.

In general, the free surplus FS_{α}^{re} decreases substantially if the premium loading of the reinsurance contract as the relevant risk management tool is raised, as shown in Figure 4. The higher the degree of dependence $\left(\rho_{\tau}\left(S_1, I_1\right)\right)$ between the industry index and the company loss and thus the lower the basis risk of the ILW, the smaller is the reinsurance loading, which makes the ILW contract more attractive to the buyer than the reinsurance contract (e.g., a loading of around 125% for perfect dependence, i.e. $\rho_{\tau}\left(S_1, I_1\right)=1$). For low dependencies ($\rho_{\tau}\left(S_1, I_1\right) = 0.6$), the loading of the traditional reinsurance has to be around 150% in the example considered and thus more than twice as much as the ILW to be less favorable.[15]

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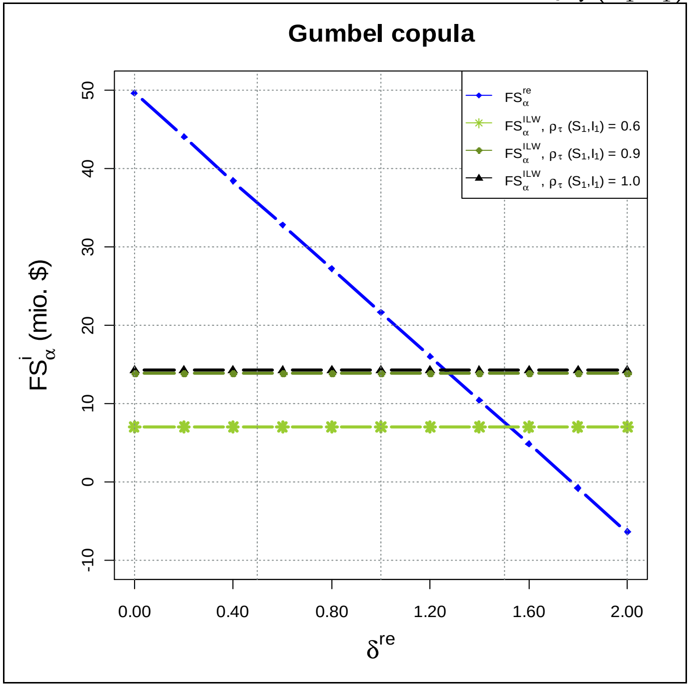
13 Froot (2001) empirically observes reinsurance premiums that amount to several times the actuarial price of the reinsured risk.

14 Doherty (2000), e.g., already pointed out that transaction costs of reinsurance account 20% of premiums or more.

15 A Kendall's tau of 0.9 or higher between the industry index and the company's losses can be attained using ZIP-based indices. E.g., assuming a linear relationship between the index and the in-

Figure 4: Comparison of free surplus FS_{α}^{i} for ILW and reinsurance for varying premium loadings for reinsurance coverage δ^{re} and different $\rho_{\tau}(S_1, I_1)$ (Gumbel copula)



Hence, the individual situation of an insurer, such as its dependence between the company's losses and the index or the availability and costs for a traditional reinsurance contract, should be analyzed in detail to make conclusions about the advantageousness of an ILW or traditional reinsurance, respectively. These findings again stress the point that basis risk and the type and degree of dependencies between an industry index and the company's losses play an important role for the use of ILWs in the context of an insurer's solvency situation. However, the surcharge on traditional reinsurance is not the sole parameter in addition to basis risk that plays an important role on the favorability of ILWs. In the previous examples, the attachment point is still the same for both reinsurance and ILW contracts. Varying the attachment point of the ILW has a strong influence on its payoff structure and thus must be investigated to evaluate the impact on basis risk and FS_{α}^{ILW} .

Varying the attachment point for ILW

Typically, the attachment point of the company's losses A^{ILW} is set to a lower level in the case of an ILW and often included to establish resemblance to reinsurance to have

surer's exposure, Major (1999) observes correlation coefficients close to 1 for ZIP-based indices compared to a correlation of 0.66 for a statewide index.

the ILW accepted as a risk transfer instrument. Figure 5 illustrates how the FS_{α}^{ILW} varies substantially when varying A^{ILW} for the company's losses and the dependence structure (degree and type of dependence). This finding is important, because the basis risk measure β (conditional probability) of the ILW, for instance, remains unchanged[16] for a given copula notwithstanding changes in the attachment point A^{ILW}.[17] Hence, a simultaneous consideration of FS_{α}^{ILW} and basis risk is important instead of focusing only upon one or the other.

Given a certain dependence structure (Clayton, Gauss or Gumbel copula) between the company's losses and the index and, thus, given a certain level of basis risk, there is an optimal attachment level that minimizes the required solvency capital and hence maximizes the free surplus of the insurer. This can be explained by examining the tradeoff between the price of an ILW and its payoff with regard to risk-based capital. Reconsidering the payoff structure of the ILW (see Equation (1)), its premium calculation and the equation for risk-based capital (see Equation (2)), it can be seen that, for decreasing attachment points, the expected payoff and consequently the ILW premium increases. Concerning the risk-based capital, two effects can be observed. An increasing premium for the ILW lowers the initial capital, which is invested in assets at time 0 and, thus, in principle reduces the available capital at time 1. In contrast, with a lower attachment point, the probability of payment of the ILW at time 1 is higher. These effects offset each other at the optimal attachment point, which varies for different levels of basis risk. For example, for the Gumbel copula and Kendall's tau of 0.6, β is about 7%. In the considered range for the discrete values of A^{ILW}, it would be optimal to enter into an ILW contract with an attachment point of around 140, as it enhances the FS_{α}^{ILW} to the highest value of around 11. This observation, too, is relevant, when evaluating the use of an ILW for risk management in a comprehensive way. In addition, it can be seen that for an increase in A^{ILW} as well as for an increase in $\rho_{\tau}\left(S_1, I_1\right)$, free surplus values for different copula cases converge. Generally, the probability of an ILW payoff decreases for high values of A^{ILW}, irrespective of the type of dependence. This reduces the impact of different types of dependence and, consequently, FS_{α}^{ILW} behaves similarly and converges for different types of dependence between $\rho_{\tau}\left(S_1, I_1\right) = 0.60$ and $\rho_{\tau}\left(S_1, I_1\right) = 1$.

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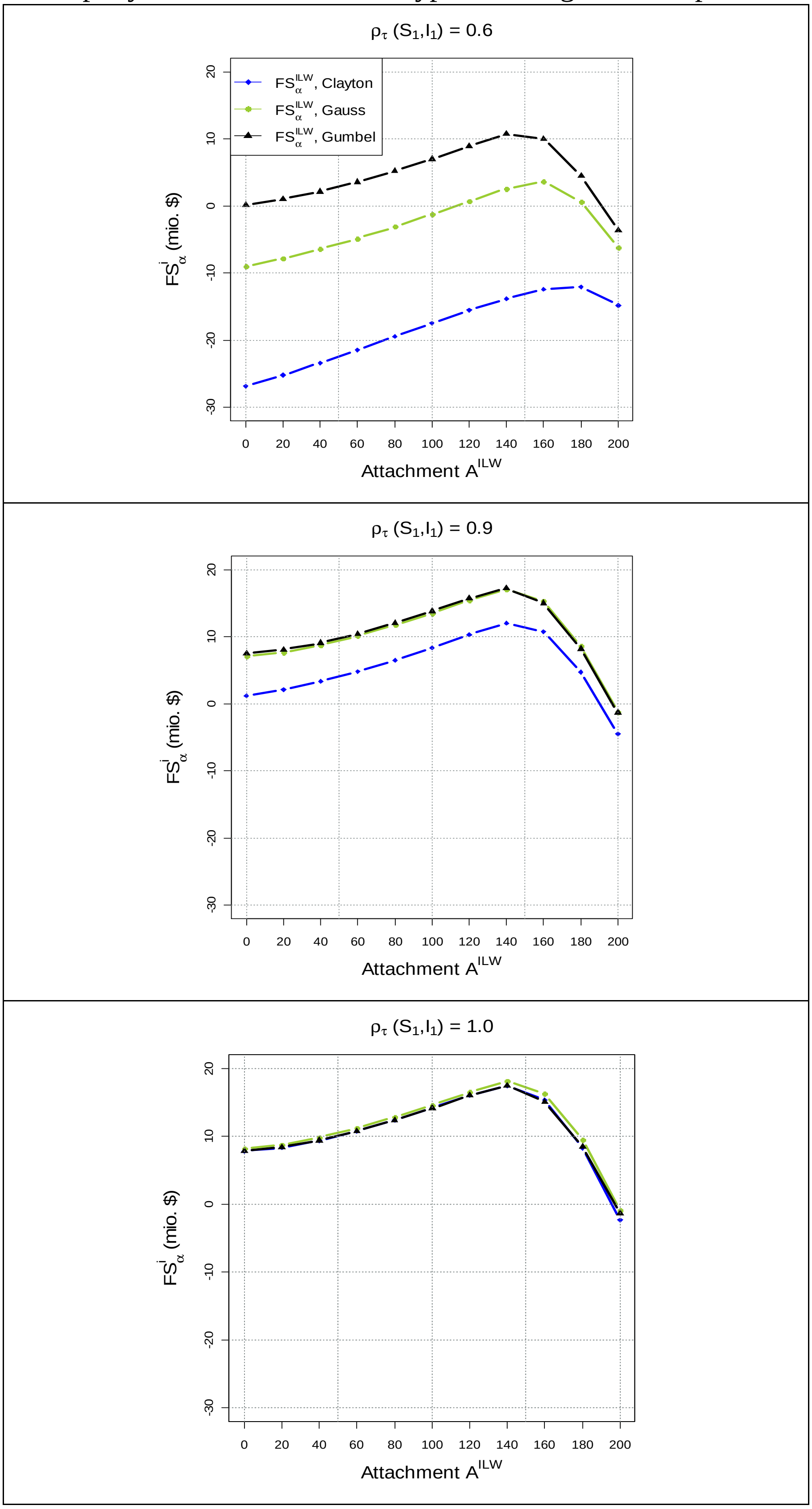
[16] The values for CHE_{m}^{b} vary only marginally as well. For example, CHE_{VaR}^{re} in the case of the Gauss copula varies within a range of 0.67 and 0.70.

[17] As illustrated earlier, although the basis risk values β for different copulas differ, for varying attachments A^{ILW}, each value is constant.

Figure 5: Comparison of free surplus FS_{α}^{ILW} for ILW with different attachment levels A^{ILW} for the company's losses for different types and degrees of dependence



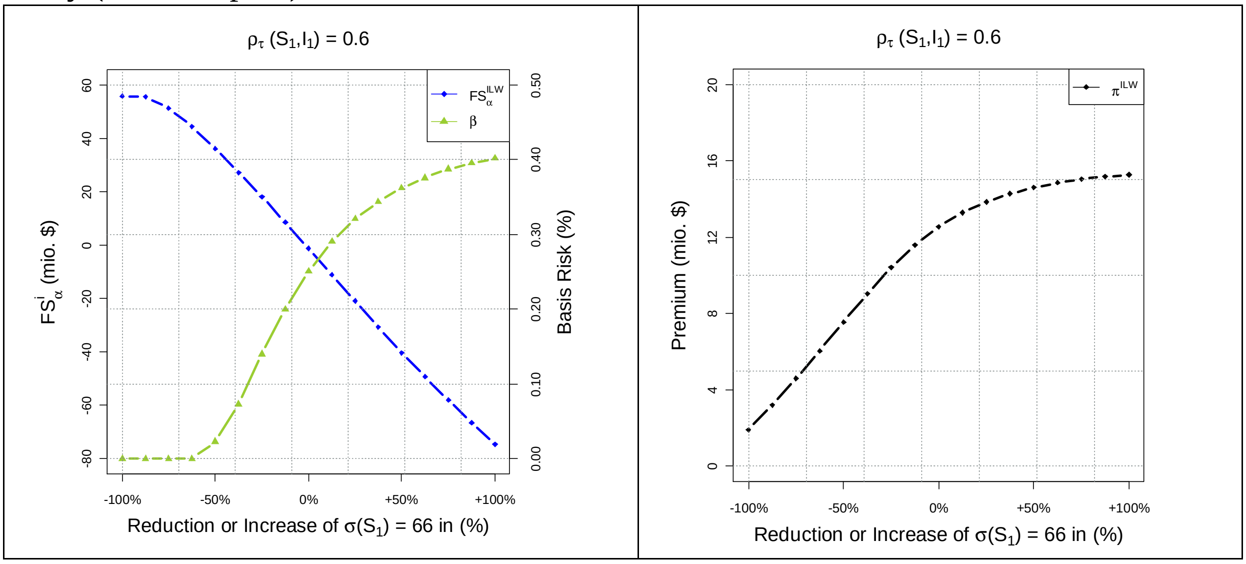
Varying the volatility of the company's loss

To conclude the numerical analysis, the effect of varying the volatility of the company's losses on the insurer's solvency situation is investigated. Figure 6 displays the values of FS_{α}^{ILW}, basis risk β and the premium π^{ILW} that an insurer would have to pay for an ILW in case of the Gauss copula and $\rho_{\tau}\left(S_1, I_1\right) = 0.60$. In the present setting, the lowest values for basis risk and π^{ILW} as well as the highest values for FS_{α}^{ILW} are given for low values of volatility.[18] In the special case of $\sigma\left(S_1\right) = 0$, β equals zero, as the critical loss level of the insurance company is never exceeded by the insurer's loss.

Due to a low volatility level, the probability for high company losses is also reduced, which leads to reduced payments by the ILW contract, hence to a reduced premium of the ILW and, at the same time, to a high level of FS_{α}^{ILW}, thus showing a tradeoff between an increasing probability for high company losses and rising ILW payoffs. In the considered range for volatility, FS_{α}^{ILW} decreases for increasing volatility values, whereas β and the ILW premium exhibit a concave progression. The behavior of the premium can be explained by the fact that for high volatility values of the insurer's loss, the shape of the loss distribution changes and becomes more skewed, which causes two effects. On the one hand, for increasing loss volatilities, the attachment A^{ILW} is less often triggered, but, on the other hand, if the attachment point is exceeded, the corresponding value of S_1 tends to be higher. However, in the present analysis, the ILW payment is limited by the layer limit L^{ILW} that dampens the second effect of higher payments if the insurer's loss reaches higher values.

[18] A similar pattern is observed in Gatzert, Schmeiser and Toplek (2007) in the context of basis risk measures that focus on the liability side and different pricing approaches.

Figure 6: Free surplus FS_{α}^{ILW} , premium payments (ILW), and basis risk β for an insurer with an ILW contract with respect to varying values of the company's loss volatility (Gauss copula)



Our results demonstrate that keeping the volatility of the company's losses at a low level in the present setting contributes to low levels of basis risk and thus to an increase in the effectiveness of an ILW in enhancing the free surplus FS_{α}^{ILW} . This result emphasizes the importance of diversification of underwriting risks, as the volatility of the insurance company's losses will generally decrease with an increasing degree of diversification of the insurance company's liabilities.

Implications

The model setup and our analysis are relevant to insurers and reinsurers in two ways. First, an insurer may fit its empirical data to the modeling framework by applying methods such as maximum likelihood estimation (see, e.g. Savu and Trede, 2006) for the type and degree of dependence between the insurer's and the industry losses.[19] By these means, an individual evaluation of whether the usage of an index-linked instrument is more favorable than alternatives for risk management, such as traditional reinsurance, can be conducted. Second, in the event of data limitations and against the background of model risk and possible mistakes in the estimation of input parameters, the model can be applied for stress testing purposes and an assessment of model risk. Results for the free surplus as well as the required solvency capital with and without

[19] See also, e.g., Shim, Lee and MacMinn (2009) for an application of non-linear dependencies in the context of U.S. property-liability insurance.

ILWs can vary tremendously for different types and degrees of dependence. Hence, comparing different scenarios with regard to dependence or varying contract parameters of the index-linked instrument, such as attachment point, layer limit or price, will help to provide deeper insight into an insurer's risk situation.

5. CONCLUSION

This paper examined the impact of basis risk on the usefulness of index-linked catastrophic loss instruments as risk management tools with regard to improve an insurer's solvency situation. We compared several definitions of basis risk, which were introduced in the previous literature, including hedging success (with focus on the liability side) and the conditional probability that one condition for the payment of an index-linked instrument is not fulfilled, given that the other necessary condition is satisfied. Moreover, we extended the view on basis risk measures that only concerns the liability side and also considered the effectiveness of ILWs in reducing required solvency capital and hence increasing the free surplus in the presence of basis risk, thus including assets and liabilities.

Furthermore, previous literature consistently points out that a high correlation between the index and the insurer's loss experience is an obvious and necessary criterion for an effective use of index-linked instruments. Therefore, we explicitly modeled and distinguished the type and degree of dependence between an insurance company's losses and the industry index as well as between high- and low-risk investments with copulas, thus allowing non-linear dependencies. To analyze the impact of different types and degrees of dependence structures on basis risk and the solvency situation, we applied a hierarchical Clayton and a hierarchical Gumbel copula, generating lower- and upper-tail dependencies, as well as the Gauss copula, exhibiting no tail dependence. Kendall's rank correlation was used to calibrate the dependencies for different copulas, thus making them comparable. By fixing the degree of dependence, the effect of different types of dependencies was isolated, thereby permitting its impact on improving the insurer's solvency situation solely to be measured.

To study the influence of basis risk, we set up a model, based upon which we measured basis risk and its impact upon required solvency capital and free surplus in a simulation analysis. To evaluate the success of an index-linked instrument, the hedging results using an ILW were examined in comparison to non-hedging or buying a traditional reinsurance contract. Sensitivity analyses were conducted concerning the type and degree of dependence between the company's losses and the index, the price difference between an ILW and a traditional reinsurance contract, the attachment of an ILW and the company's loss volatility.

Our numerical results revealed that, with increasing *degree* of dependence (Kendall's rank correlation) between the company's loss and the index, basis risk measures decrease and the insurer's solvency situation improves for all analyzed dependence structures despite the tradeoff between the increasing premiums of an ILW and a decreasing level of basis risk. Regarding the results of the solvency situation for different *types* of dependence structures, we found that the type of dependence (lower, upper or no tail dependence) plays an important role. Even if the degree (strength) of dependence between the company's losses and the index is identical, by fixing Kendall's tau for all three copulas, the values for the required solvency capital and the free surplus differ substantially.

For the insurer's decision between an ILW and traditional reinsurance, the price difference plays an important role in addition to the prevalent dependence between the insurer's own losses and the index. For lower values of the dependence between the company's losses and the industry index, the ILW is less advantageous compared to traditional reinsurance, except if the surcharge on the traditional reinsurance is relatively high. On the other hand, given higher dependencies between the company's losses and the industry index, even smaller loadings on the traditional reinsurance contract can make the reinsurance less favorable. In practice, traditional reinsurance contracts are often more expensive than ILWs due to an extensive underwriting process. This implicates that, if the portfolio of an insurer features high dependencies (e.g. due to the availability of a regional index) with an index, the ILW is likely to achieve better results than the traditional reinsurance in the context of a reduction of required solvency capital and an increase in the free surplus.

Regarding the chosen attachment point of an ILW, we found that, even if basis risk persists on a constant (conditional probability) or almost unchanged (counter value of hedging efficiency) level, the efficiency with regard to lowering required solvency capital and hence increasing the insurer's free surplus can be improved by varying the attachment point. The sensitivity analysis of the attachment point showed that there exists one, which maximizes the free surplus. The variation of company loss volatility demonstrated that keeping the volatility for the company's losses at a lower level contributes to achieving good results concerning basis risk and the insurer’s solvency situation.

Investigating an ILW contract provided insight into the determining factors influencing the application and success of such index-linked cat loss instruments for risk management in regard to an insurer’s solvency situation. The results pointed out that basis risk is a key risk factor in this context, but that it does not exclusively determine the success of index-linked instruments with regard to improve the solvency situation. In addition, we found that the combination of both *type* and *degree* of dependence is relevant when assessing the attractiveness of ILWs with respect to reducing risk. Hence, basis risk, contract parameters, and dependencies between risk factors should be taken into consideration simultaneously when insurers make decisions whether to include an index-linked cat loss instrument for risk management.

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