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Risk- and Value-Based Management for Non-Life Insurers under Solvency Constraints

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Working Paper

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Version: January 2017

RISK- AND VALUE-BASED MANAGEMENT FOR NON-LIFE INSURERS UNDER SOLVENCY CONSTRAINTS

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This version: January 24, 2017

ABSTRACT

The aim of this paper is to study optimal risk- and value-based management decisions regarding a non-life insurer's investment strategy by maximizing shareholder value based on preference functions, while simultaneously controlling for the ruin probability. We thereby extend previous work by explicitly accounting for the policyholders' willingness to pay depending on their risk sensitivity based on the insurer's reported solvency status, which will be of great relevance under Solvency II. We further investigate the impact of the risk-free interest rate, dependencies between assets and liabilities as well as proportional reinsurance. One main finding is that the consideration of default-risk-driven premiums is vital for optimal management decisions, since, e.g., the target ruin probability implying a higher shareholder value differs for various risk sensitivities of the policyholders. Furthermore, in the present setting, reinsurance increases shareholder value only for non-risk sensitive policyholders.

Keywords: Risk- and value-based decision-making; non-life insurance; Solvency II; shareholder value optimization; default-risk-driven premium; market discipline

JEL Classification: G22; G28; G31

1. INTRODUCTION

An insurance company's equity capital can be exposed to large fluctuations, which may potentially result in severe solvency problems. These fluctuations can arise from both the investment side (due to an increasing volatility in the financial markets) as well as the underwriting side against the background of a rising frequency and severity of natural catastrophes in the last decades (see Swiss Re, 2016). In this context, risk- and value-based management is essential for the long-term success of insurance companies, in that investment as well as underwriting decisions should take into account risk and return in order to ensure an efficient and profitable use of capital and to control for default risk.

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In order to protect the policyholders and the stability of the financial system, Solvency II as the new European regulatory framework for insurers[1] came into force on January 1st, 2016. Pillar 1 introduces risk-based capital requirements by demanding sufficient equity capital to fulfill insurance contracts also under adverse events, Pillar 2 specifies qualitative requirements such as a governance system and the Own Risk and Solvency Assessment (ORSA), and Pillar 3 comprises reporting requirements. The insurer thereby has to provide information not only to the supervisor but also to the public by means of the Solvency and Financial Condition Report (SFCR), which is intended to provide transparency and enforce market discipline. Pillar 1's solvency capital requirements are thereby based on the Value at Risk with a 99.5% confidence level, implying a one-year ruin probability that does not exceed 0.50%. It can be derived either by means of a standard model provided by the regulatory authorities or based on a company-specific internal model that adequately reflects the firm's risks (see, e.g., Eling et al., 2009). An internal model should also be used in Pillar 2 for the firm's ORSA and should thus represent an integral part of an insurer's risk- and value-based management, i.e., to be applied for corporate risk management and asset allocation decisions, for instance.[2]

Against this background, the aim of this paper is to study optimal risk- and value-based management decisions regarding the investment strategy for a non-life insurer, which contribute to increasing shareholder value.[3] In particular, by adjusting the asset allocation to satisfy solvency capital requirements, the approach should be less costly than raising equity capital or adjusting the liability side (see Eling et al., 2009). Toward this end, we considerably extend the analyses and model frameworks in previous work (e.g., Eling et al., 2009; Zimmer et al., 2014; Braun et al., 2015) by using a more general model based on the literature on non-life insurance and ruin theory, thereby focusing on shareholder value based on preferences while simultaneously controlling for the insurer's ruin probability. In particular, we consider the impact of several new key features on risk- and value-based management regarding the investment strategy, including the policyholders' willingness to pay depending on the insurer's reported solvency status, which despite its great impact has not been studied to date in this

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[1] Furthermore, since many countries have a Solvency II equivalent (e.g., the Swiss Solvency Test or the China Risk Oriented Solvency System) or at least seek to establish one (Sub-Saharan African countries opt for regulation systems resembling a simplified form of Solvency II (EY, 2016)), these results are not limited to the impact of Solvency II but are generally of relevance.

[2] The quantitative approaches in Pillar 1 and 2 rely on a market consistent balance sheet of the insurer, which reflects an insurer's assets and liabilities and thus the shareholders' equity capital at a single point in time. For instance, Allianz Group as one of the most important insurance groups worldwide (and identified as a global systemically important insurer by regulators) exhibits a total value of assets of approximately 850 bn Euros and 66 bn Euros equity capital, implying an unweighted capital ratio of 7.8% (see Allianz Group annual report 2015, also for remarks regarding their internal model).

[3] Note that for simplicity, we use the expression "shareholder value", but assume preferences for the evaluation (instead of relative valuation).

context, the dependencies between assets and liabilities, the impact of reinsurance contracts as well as the risk-free interest rate.

In previous work, Eling et al. (2009) derive minimum requirements for a non-life insurer's capital investment strategy that satisfy solvency restrictions based on different risk measures. Using "solvency lines", i.e. isoquants of risk and return combinations of the asset allocation for a fixed safety level of the insurer, they determine admissible risk and return asset combinations given a certain liability structure, and then compare these to allocation opportunities actually available at the capital market based on portfolio theory. Similarly, but in a life insurance context, Braun et al. (2015) study optimal asset allocations taking into account restrictions from solvency capital requirements, thereby comparing the Solvency II standard formula with an internal model. In the context of a fixed investment decision and a default-risk-driven customer demand, Schlütter (2014) further studies an insurer who chooses insurance prices and an allowed solvency level when optimizing shareholder value given risk-based capital requirements or price regulation. In addition, experimental and empirical research (Wakker et al., 1997; Zimmer et al., 2009; Lorson et al., 2012; Zimmer et al., 2014) shows that an insurer's default risk can have a strong influence on customer demand, where lower safety levels can lead to a considerable reduction of achievable premiums. In this context, Zimmer et al. (2014) develop a risk management model assuming that the insurer's default risk is fully known to consumers, and based on this derive the solvency level that maximizes shareholder value, which is the case for a ruin probability of zero. These results emphasize that an insurer's safety level should be taken into account in risk- and value-based management as the reaction of customers to default risk (by way of the premium level) can considerably impact shareholder value. This will be even more relevant when insurers have to publicly report their solvency status under Solvency II. In the context of deriving minimum requirements for the investment strategy, Fischer and Schlütter (2015) further criticize that the standard model leads to an incentive to avoid diversification between assets and liabilities, as dependencies are not adequately taken into account in the standard model, which is in line with the results in Braun et al. (2015).

The ruin probability (as one basis of Solvency II) is also a classical topic of applied mathematics in non-life insurance as introduced by Lundberg (1903) and Cramér (1930), with the Cramér-Lundberg model being the classical model of risk theory in non-life insurance mathematics (Mikosch, 2009). Since then, various extensions have been developed, e.g., regarding the process of the total claims amount (e.g., the Sparre-Andersen (1957) model generalizes the total claims process to a renewal model, Albrecher and Teugels (2006) model the dependence between claim size and the inter-claim time) as well as the investment side of the insurer. Taking into account the possibility of investing in (risky) assets that influence the probability of ruin goes back to Segerdahl (1942) and Paulsen (1993) (see Paulsen, 2008). Some further recent works in this regard include Paulsen (2008), Klüppelberg and Kostadinova (2008), Heyde and Wang (2009), Hult and Lindskog (2011), Bankovsky et al. (2011), Hao and Tang (2012), and Ramsden and Papaioannou (2017). There are also a number of papers focusing on a discrete-time risk model where the insurer's surplus is controlled by i.i.d. discrete insurance and financial risk processes that are independent from each other (e.g., Nyrhinen, 1999; Tang and Tsitsiashvili, 2003, 2004; Yang and Zhang, 2006; Li and Tang, 2015).

In recent years, the applied math literature also focused on optimal investment and/or reinsurance strategies in the sense of minimizing the ruin probability or optimizing other objective functions. For example, Schmidli (2001, 2002) and Promislow and Young (2005) minimize the ruin probability in continuous time, and Schäl (2004), Diasparra and Romera (2009, 2010), Romera and Runggaldier (2012), and Lin et al. (2015) in discrete time. Another popular and relevant optimization criterion is the maximization of expected utility of the insurer's terminal wealth, e.g. in continuous time in general in Liang et al. (2011, 2012), Liang and Bayraktar (2014), and Huang et al. (2016), and in particular for mean-variance preferences in Bäuerle (2005), Bai and Zhang (2008), and Bi and Guo (2012). For discrete time, we further refer to Schäl (2004).

Another strand of the literature relates to the (frequent) assumption of independence between insurance and financial risk processes (most often assumed), which simplifies analytical solutions but is difficult for practical applications. Since discrete-time risk models create an efficient possibility to investigate the interplay of both risks, several papers drop this assumption, using, e.g., an insurance risk process and/or a financial risk process being a sequence of dependent random variables while keeping the independence between the two processes (see, e.g., Chen and Yuen, 2009; Collamore, 2009; Shen et al., 2009; Weng et al., 2009; Zhang et al., 2009; Yi et al., 2011). Alternatively, Chen (2011), Yang et al. (2012), Yang and Hashorva (2013), and Yang and Konstantinides (2015) allow for dependences between the insurance and financial risk processes assuming each process is i.i.d.

The purpose of this paper is to contribute to the existing literature in various relevant ways. First, in contrast to Braun et al. (2015), we study a non-life insurer instead of a life insurer and do not use the standard model. Moreover, we extend the analysis in Eling et al. (2009) and Braun et al. (2015) by focusing on the firm's shareholder value under risk- and value-based management decisions and by considering the policyholders' willingness to pay when deriving admissible asset allocations under solvency constraints. The insurer's surplus process is modeled by a discrete time representation of the Sparre-Andersen (1957) model in the presence of risky investments and reinsurance similar to Huang et al. (2016), Jin et al. (2016), and Yang and Zhang (2006) in order to derive the discrete one-year ruin probability as used in Solvency II. We thus combine approaches in the economic literature (e.g., Braun et al., 2015, Eling et al., 2009) and the applied math literature on ruin theory by generalizing the economic model for the total claims payments using a probabilistic framework with stochastic frequency and severity and by applying this model to a shareholder value optimization problem. Moreover, the dependence structure between the insurance and financial risk processes is modeled similar to Chen (2011) and Yang and Konstantinides (2015) by means of copulas and we consider the possibility of purchasing proportional reinsurance. We further extend the economic and applied math literature by explicitly taking into account the policyholders' willingness to pay in our model framework when deriving admissible asset allocations under solvency constraints that are determined by the ruin probability. We thereby generalize the approach in Lorson et al. (2012) and Zimmer et al. (2014) and model the achievable premium in the presence of market discipline as a function of the insurer's safety level and the policyholders' risk assessment. In contrast to the mentioned applied math literature, we study the optimal investment problem regarding the shareholders' expected utility while simultaneously controlling for the ruin probability given several influencing factors. In this setting, we follow Eling et al. (2009) and link the admissible risk-return combinations of the insurer's asset portfolio (i.e. those that are allowed under solvency constraints) to allocation opportunities actually attainable at the capital market using Tobin (1958)'s capital market line.

In a numerical analysis, we first investigate the impact of a default-risk-driven premium income on the maximum shareholder value for an insurer facing solvency constraints and, second, the impact of changing the reported target solvency status on the shareholder value in the presence of market discipline. One main finding is that the consideration of policyholders' willingness to pay based on the ruin probability is of great relevance when deriving optimal risk-return asset allocations under solvency constraints (e.g., the target ruin probability implying a higher shareholder value differs for various risk sensitivities of the policyholders) and that reinsurance can considerably impact the results, depending on the level of the policyholders' risk sensitivity.

The remainder of this paper is structured as follows. Section 2 presents the model framework for a non-life insurer, while Section 3 focuses on the derivation of attainable and admissible risk-return combinations as well as shareholder value maximization. An application of the developed approach based on several underlying simplified assumptions with sensitivity analyses for various policyholders' risk sensitivities, asset-liability dependence, proportional reinsurance and risk-free interest rate is provided in Section 4. The last section summarizes our main findings.

2. MODEL FRAMEWORK

2.1 Model foundations

We consider the problem of ruin in a risk model of a non-life insurer. Given a stochastic basis $\left(\Omega,\mathcal{F},\left(\mathcal{F}_t\right)_{t\geq 0},\mathbb{P}\right)$, we use the Sparre-Andersen (1957) model (see Andersen, 1957) for the insurer's surplus process in continuous time t

$$U_t = u + pt - S_t, \quad t \geq 0,$$

where $u > 0$ is the initial equity capital of the insurer, $p > 0$ is the constant premium intensity, and the total claims payments are modeled by a compound renewal process. Due to its tractability, this model is widely used in risk theory, see e.g., Li and Garrido (2004), Wu et al. (2007), Li (2012) and Jin et al. (2016).[4] The total claims amount

$$S_t = \sum_{i=1}^{N_t} X_i, \quad t \geq 0,$$

is thereby given by a sequence of i.i.d. a.s. positive random variables $X_i, i \in \mathbb{N}$ denoting the claim size sequence, where N_t is the number of claims up to time t with $N_t, t \geq 0$ being a renewal process generated by an i.i.d. sequence of a.s. positive inter-arrival times $D_i, i \in \mathbb{N}$, i.e.,

$$N_t = \inf\left\{ j \geq 1 : \sum_{i=1}^{j} D_i \geq t \right\}, \quad t \geq 0 .$$

Moreover, the two random variable sequences $X_i, i \in \mathbb{N}$ and $D_i, i \in \mathbb{N}$ are independent and have finite variance.

Since in Solvency II, the discrete time one-year ruin probability is used (see, e.g., Luca and Schmeiser, 2017), we now switch to a discrete-time representation $U_n, n \in \mathbb{N}$ of the risk process with unit intervals (e.g., years) $n = 1,2,3,\ldots$, such that

$$U_n = u + pn - S_n, \quad n \in \mathbb{N}.$$

Additionally, we introduce a financial process, which is defined as a sequence of normally distributed i.i.d. random variables $r_n, n \in \mathbb{N}$ generating the return on investment from year n-

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4 Regarding risk models used in non-life insurance, we further refer to Mikosch (2009), Asmussen and Albrecher (2010), and Beard et al. (2013) for an introduction to risk theory.

1 to year n. The distribution assumption is made in order to be consistent with the assumptions of Tobin (1958)'s capital market line and can be justified by the Euler discretization of the return process as used in Hult and Lindskog (2011) and Kabanov and Pergamenshchikov (2016). We further assume that the insurer may additionally purchase a proportional reinsurance contract (see, e.g., Diasparra and Romera, 2010; Liang et al., 2012; Huang et al., 2016). For each n, the insurer reinsures a fraction of its claims with retention level $q \in [0,1]$, i.e. the insurer pays $q \cdot 100\%$ of each claim occurring at time n. The corresponding reinsurance premium rate is given by $\pi(q)$. Hence, the discrete time surplus process evolves according to

$$U_n = u + \sum_{i=0}^{n-1} \left(U_i + p - \pi(q)\right) r_n + \left(p - \pi(q)\right) n - q S_n, \quad n \in \mathbb{N}. \tag{1}$$

Recursively, we thus obtain the insurer's surplus accumulated until the end of year $(n-1, n]$ as

$$U_n = (1 + r_n)\left(U_{n-1} + p - \pi(q)\right) - q\left[S_n - S_{n-1}\right], \quad U_0 = u, n \in \mathbb{N}.$$

We thus follow Yang and Zhang (2006) and assume that the premium income (considering reinsurance) is received at the beginning of the (n-1)-th year and that both surplus and premiums are then invested in a risky asset portfolio with discrete return $r_n \sim N\left(\mu_r, \sigma_r^2\right)$. The claims of the policyholders (considering reinsurance) have to be paid at the end of the (n-1)-th year resulting in a surplus U_n. Alternative approaches of discrete time ruin theory do not distinguish between the end of year n-1 and the beginning of year n, assuming that both premiums and claims cash flows only occur at the end of the year (e.g., Paulsen, 2008), i.e. $U_n = (1 + r_n) U_{n-1} + p - \pi(q) - q\left[S_n - S_{n-1}\right]$, or at the beginning of the year with $U_n = (1 + r_n)\left(U_{n-1} + p - \pi(q) - q\left[S_n - S_{n-1}\right]\right)$ (e.g., Tang and Tsitsiashvili, 2003).

Solving the recurrence equation, we obtain (see the Appendix for a detailed derivation)

$$U_n = B_n\left(U_{n-1} + p - \pi(q)\right) - q\left[S_n - S_{n-1}\right] = \left(u + p - \pi(q)\right)\prod_{i=1}^{n} B_i + \sum_{i=1}^{n-1} A_i \prod_{j=i+1}^{n} B_j - q\left[S_n - S_{n-1}\right],$$

with a financial process $B_n = 1 + r_n$ and an insurance process $A_n = \left(p - \pi(q)\right) - q\left[S_n - S_{n-1}\right]$ with convention $\prod_{j=n+1}^{n} = 1$ (see, Tang and Tsitsiashvili, 2003, Equation (2.3) for a similar representation of alternative surplus calculation approaches).

Typically, the insurance process and the financial process are assumed to be independent from each other (see, e.g., Tang and Tsitsiashvili, 2003, 2004; Diasparra and Romera, 2009, 2010; Romera and Runggaldier, 2012). To drop this assumption, we use the concept of copulas. Similar to Chen (2011) and Yang and Konstantinides (2015), we assume that the insurance process and the financial process (more precisely $S_n - S_{n-1}$ and r_n) form a sequence of i.i.d. random variables, and that the dependence structure is given by a copula C, which is a popular tool for modeling dependencies (see, e.g., in general Joe, 2001; Nelsen, 2006), also in discrete ruin theory (e.g., Chen, 2011; Yang and Konstantinides, 2015).

In particular, according to Sklar's Theorem (see Sklar, 1959), for any multivariate distribution function F on $\mathbb{R}^d$ with univariate margins F_i, a unique function $C:[0,1]^d \to [0,1]$ exists, such that $F(x) = C\left(F_1(x_1),...,F_d(x_d)\right) \forall x \in \mathbb{R}^d$. In order to model the dependence structure between risk processes, one can use elliptical copulas generalizing the Gaussian copula and the t-copula, which only capture elliptical symmetry or Archimedean and hierarchical Archimedean copulas to obtain asymmetric dependencies, for instance (see McNeil et al., 2005).

2.2 One-period model (discrete one-year ruin probability)

The time of ruin is given by $\tau = \inf\{n : U_n < 0\}$ with $\tau = \infty$, if U stays non-negative. The probability of ruin in finite and infinite time is then given by $\mathbb{P}(\tau \le n \mid U_0 = u)$ and $\mathbb{P}(\tau < \infty \mid U_0 = u)$, respectively. Besides the planning horizon, one can also distinguish the frequency of observing an insurer's solvency status, where both have a non-negligible effect on the result (for a discussion on these aspects, see, e.g., Bühlmann, 1996).

Solvency II requires insurers to derive solvency capital requirements such that the one-year ruin probability does not exceed 0.50%. Most insurers issue financial reports once a year, thus exhibiting an annual frequency (see Luca and Schmeiser, 2017), leading to

$$U_1 = (1+r_1)\left(U_0 + p - \pi(q)\right) - qS_1, \quad U_0 = u, \tag{2}$$

and

$$RP = \mathbb{P}(\tau = 1 \mid U_0 = u) = \mathbb{P}(U_1 < 0 \mid U_0 = u) = \mathbb{P}\left((1+r_1)\left(u + p - \pi(q)\right) - qS_1 < 0\right).$$

2.3 Premium income and policyholders' risk sensitivity

In ruin theory, the premium income of the surplus process is mostly modeled by a constant rate $p > 0$ (Mikosch, 2009). In what follows, we use the expected value principle to determine the premium rate

$$p = (1+\delta) \cdot \mathbb{E}[S_1]$$

for some positive premium loading δ, which results in a premium income that on average exceeds the total claims payments and can thus absorb fluctuations of the claims amount. However, if the insurer imposes an overly large premium loading, it becomes less competitive compared to other premiums offered at the market. Thus, we assume a fixed premium loading that is exogenously given in the sense of a market standard in the insurance industry.

Moreover, according to experimental and empirical research (Wakker et al., 1997; Zimmer et al., 2009; Lorson et al., 2012; Zimmer et al., 2014), an insurer's default risk can strongly impact customer demand, with lower safety levels leading to a considerable reduction of the achievable premiums. Whereas expected utility theory suggests that a small increase in the ruin probability should only reduce the policyholders' willingness to pay in a marginal way, Wakker et al. (1997) observe that the actual willingness to pay decreases sharply in the context of "probabilistic insurance", i.e. insurance contracts with a small non-zero ruin probability. They explain this phenomenon based on Kahneman and Tversky's (1979) prospect theory, according to which individuals tend to overweigh small (extreme) probability events and vice versa. The low probabilities of an insurer's default are thus assigned a weight higher than the objective probability, implying a considerable reduction of premiums below the actuarially fair premium, which also directly impacts shareholder value.

Hence, we adapt our model framework in order to take into account the policyholders' willingness to pay as a reaction to the reported solvency status depending on their risk sensitivity, which is especially relevant under Solvency II's Pillar 3. In the presence of market discipline, customers could be influenced by the reported solvency status when comparing it to other insurers. A potentially induced change of customer demand could thereby incentivice insurers to achieve a higher solvency status than the required regulatory minimum. To take this aspect into account, we use and extend the approach in Lorson et al. (2012), who calculate the premium reduction compared to the premium offered by a default-free insurer as a function of the reported one-year ruin probability *RP*. In particular, to model the premium reduction function *PR* (which is to be distinguished from the default-free premium p) depending on the ruin probability, we follow Lorson et al. (2012) and choose a log-linear model given by

$$PR(RP) = a \cdot \ln(RP) + b, \quad RP \in (0,1]. \tag{3}$$

For an increasing safety level, i.e. a decreasing ruin probability *RP*, the premium reduction function decreases until the default-free state is reached and the policyholders exhibit full willingness to pay; the highest premium reduction results for a ruin probability converging to 1. However, since the policyholders' risk assessment is not known, i.e. how well-informed the policyholders are and if they can assess the numerical ruin probability correctly, the actual premium reduction might differ. To take into account the policyholders' risk sensitivity, we thus extend the model in Lorson et al. (2012) and not only link the premiums paid in $t = 0$ to the insurer's reported ruin probability $RP = \alpha$, but also to the policyholders' risk sensitivity represented by a scaling parameter ξ similar to Gatzert and Kellner (2014). The premium payments in $t = 0$, $P(RP,\xi)$, are then given by

$$P(RP,\xi) = p \cdot \max\left(1-\xi \cdot PR(RP),0\right), \quad \xi \geq 0,\ RP = \alpha \in (0,1],\ p = (1+\delta)\cdot E[S_1], \tag{4}$$

where $RP = \alpha$ represents the reported one-year ruin probability and PR the premium reduction function in Equation (3). Note that the functional form of the latter is chosen for illustration purposes and can also be adjusted (e.g. depending on the type of contract). For $\xi = 1$, we obtain the risk sensitivity modeled in Lorson et al. (2012).

We further assume that the reinsurance premium is also calculated according to the expected value principle, resulting in

$$\pi(q_1) = (1+\delta_{re})\cdot(1-q)\cdot \mathbb{E}[S_1].$$

3. Optimal Attainable and Admissible Risk-Return Combinations

To derive optimal risk- and value-based management decisions, we next consider minimum solvency requirements in a risk-return (asset) context (here: expected return and standard deviation of assets that are compatible with the solvency requirements) and link these "admissible" risk-return combinations of the insurer's asset portfolio to allocation opportunities actually "attainable" at the capital market as is done in Eling et al. (2009). In contrast to previous work, however, we generalize this model by explicitly taking into account the policyholders' willingness to pay depending on the insurer's solvency level, dependencies between assets and liabilities as well as the effect of reinsurance contracts, which impact the liability side of the insurer's balance sheet.

3.1 Capital market line: "Attainable" risk-return combinations

To identify the risk-return profiles (σ_r, μ_r) that are actually attainable at the market, we follow the classical approach of Tobin (1958) and derive the capital market line ($\mu_r = CML(\sigma_r)$) representing the set of efficient risk–return combinations (σ_r, μ_r) given risk-free lending and borrowing. The solution of the optimization problem can also be determined by using the analytical expression in Merton (1972). Note that a derivation of efficient risk-return combinations that takes into account the insurer's liabilities can be found in Brito (1977) and Mayers and Smith (1981), for instance.

3.2 Solvency lines: "Admissible" risk-return combinations

Having identified the risk-return combinations that are actually attainable at the market, we next link them with the minimum requirements for the insurer's investment performance based on a target ruin probability. To derive risk-return combinations (σ_r, μ_r) for the insurer's investment strategy that are compatible with solvency requirements, we fix the insurer's desired ruin probability *RP* at time $t = 1$ to a prescribed maximum value α denoted "target ruin probability" (note that the risk measure can as well be changed as is done in Eling et al. (2009), but closed-form solutions may not be derivable). The insurer's fixed ruin probability simultaneously impacts the achievable premiums as illustrated in Section 2 (Equation (4)). At time $t = 0$ the insurer sets the maximum value α as a target level, which is then communicated and revealed to the policyholders, who in turn adapt their willingness to pay based on this information. Based on the resulting amount of premium income (see Equation (4)), the insurer makes the actual investment decision (i.e., chooses a risk-return asset combination), which must be compatible with the announced target level α to preserve the policyholders' trust and confidence. Thus, the real ruin probability *RP* must not exceed the reported target ruin probability α, i.e.

$$RP = \mathbb{P}\Big(\big(1+r_1\big)\big(u+P(\alpha,\xi)-\pi(q)\big)-qS_1<0\Big)\overset{!}{\leq}\alpha. \tag{5}$$

For a given σ_r, we can solve for μ_r and obtain the so-called "solvency lines" *SolvL*. The notion follows Eling et al. (2009) who derived closed-form solutions for a one-year period model using a normal power approximation for the difference between assets and liabilities. Thus, the solvency lines are (σ_r, μ_r) -combinations that satisfy Equation (5), which implies that for a given risk (here measured with the standard deviation σ_r of returns of the asset portfolio) the expected return μ_r needs to be at least as high to ensure that the ruin probability does not exceed the given target level α.

3.3 Maximizing shareholder value given attainable and admissible risk-return combinations

Among the typical firm objectives is the creation of shareholder value through risk- and value-based decision making regarding assets and liabilities. Toward this end, our model can be used for deriving the shareholders' maximum expected utility while maintaining a minimum (typically regulatory required) solvency level in order to protect the policyholders.

Let Ψ denote the shareholders' preference function to determine their expected utility dependent on the (σ_r, μ_r)-combination of the insurer's asset allocation. In its decisions regarding the investment portfolio, the insurer can take into account the shareholders' preferences with-in the limits of attainable and admissible investment opportunities, leading to the following optimization problem

$$SHV^{\max,\Psi} = \max_{(\sigma_r,\mu_r)} \Psi, \tag{6}$$

subject to the constraints

$$c = \begin{pmatrix} \sigma_r \geq 0 \\ \mu_r \leq CML(\sigma_r) \\ \mu_r \geq SolvL(\sigma_r) \end{pmatrix}.$$

The constraints ensure that only risk-return combinations are taken into consideration that are attainable at the market and that are admissible according to solvency restrictions. This leads to the feasible set

$$\Upsilon = \left\{ \left(\sigma, \mu(\sigma)\right) : \sigma \in \left[0, \sigma_{IP}\right], \mu(\sigma) \in \left[SolvL(\sigma), CML(\sigma)\right], SolvL(\sigma) \leq CML(\sigma) \right\}, \tag{7}$$

with σ_{IP} denoting the σ-coordinate of the intersection point *IP* between the solvency line and the capital market line.

The optimal (σ_r, μ_r)-combination that solves the optimization problem (6) depends on the actual preference function Ψ. Since decisions based on any utility function can be well approximated by assuming mean-variance preferences (see Kroll et al., 1984 in general, and Bäuerle, 2005; Bai and Zhang, 2008; Bi and Guo, 2012 in ruin theory), we assume that Ψ is based on the expected value and variance of the shareholders' wealth at the end of the period (see, e.g., Gatzert et al., 2012; Braun et al., 2015).[5] One should thereby take into account that shareholders generally have limited liability and thus at most lose their initial equity capital in case of insolvency.[6] Due to limited liability, the shareholders' wealth at t = 1 is given by $\max(0,U_1)$ and, hence, the preference function Ψ^1 is given by

5 Note that full compatibility of mean-variance analysis and expected utility theory is given in case of normally distributed returns or a quadratic utility function. Even if these conditions do not hold, decisions of any utility function can be well approximated by those based on mean-variance preferences as shown in Kroll et al. (1984).

6 In general, the premium level should thus correspond to the value of indemnity payments less the default put option arising from the shareholders' limited liability, which we have not taken into account in pricing as we assume (in the sense of a behavioral-type approach) that policyholders are willing to pay a premium that depends on their risk sensitivity and that may thus exceed the expected payoff. These assumptions can also be changed, but closed-form solutions for the optimization problem as well as the solvency lines are no longer possible, such that one has to revert to numerical simulation approaches.

$$\Psi_k^1 = \mathbb{E}\left(\max\left(U_1, 0\right)\right) - \frac{k}{2} \cdot \mathbb{V}\left(\max\left(U_1, 0\right)\right), \tag{8}$$

where k represents the risk aversion coefficient, with $k < 0$ implying a risk-seeking, $k = 0$ a risk-neutral, and $k > 0$ a risk-averse risk attitude.[7]

To simplify the optimization problem, one could also assume a preference function Ψ^2 that does not account for limited liability, i.e.

$$\Psi_k^2 = \mathbb{E}\left(U_1\right) - \frac{k}{2} \cdot \mathbb{V}\left(U_1\right).$$

3.4 Special case: The model in a Gaussian environment

We now consider a special case that bridges the gap to the simpler but more tractable model frameworks in economics that do not differentiate between the number and size of claims, as is done in, e.g., Eling et al (2009) and Braun et al. (2015). In particular, we derive closed-form solutions for the insurer's capital investment strategy that satisfy the solvency rules as well as for the associated maximum shareholder value under various conditions. This allows studying the impact of various economic factors on an insurer's optimal risk- and value-based management decisions and the derivation of key drivers.

The one-period model of the non-life insurer's surplus in t =1 is given by Equation (2),

$$U_1 = \left(1 + r_1\right)\left(U_0 + P\left(\alpha, \xi\right) - \pi\left(q\right)\right) - qS_1, \quad U_0 = u .$$

To simplify the framework and to model the total claims payments by one random variable, we make use of the central limit theorem to approximate the total claims amount using the normal distribution for the Sparre-Andersen model with finite variance of the inter-arrival times and claims sizes (see, e.g., Embrechts et al., 2000; Mikosch, 2009), i.e.,

$$\mathbb{P}\left(\left(S\left(t\right) - \mathbb{E}\left[S\left(t\right)\right]\right) / \sqrt{\mathbb{V}\left[S\left(t\right)\right]} \leq y\right) \to \Phi\left(y\right), \quad y \in \mathbb{R},$$

where Φ stands for the distribution function of the standard normal distribution, and thus

---

[7] Note that we hereby assume that the derivation of the CML (and thus the available asset combinations) is conducted in a first step by asset management in line with portfolio theory, assuming that risk-averse investors dominate the capital market. This is independent of individual preferences of the considered insurer's shareholders, such that the cases of risk-neutral and risk-seeking shareholders of a specific insurer are also considered.

$$S(t) \overset{approx.}{\sim} N\left(\mathbb{E}\left[S(t)\right], \mathbb{V}\left[S(t)\right]\right).$$

The approximation by a normal distribution based on the central limit theorem is quite good in the center of the distribution, but not as good in case of tail probabilities. Nevertheless, the Gaussian dynamic allows obtaining first insight due to its simple calculation. More reliable results can alternatively be obtained by other tools such as, e.g., large deviation probabilities and saddle point approximations (see Mikosch, 2009).

The bivariate distribution function of the asset return r_1 and the total claims amount S_1 is fully captured by their marginal distribution functions that are both normally distributed with $r_1 \sim N\left(\mu_r, \sigma_r^2\right)$ and $S_1 \overset{app}{\sim} N\left(\mu_S, \sigma_S^2\right)$ and their copula. To derive closed-form solutions for the solvency lines, we use the Gaussian copula with correlation matrix Θ. The *2*-dimensional Gaussian copula is thereby given by

$$C_{\Theta}^{\text{Gauss}}\left(u_1, u_2\right) = \Phi_{\Theta}\left(\Phi^{-1}\left(u_1\right), \Phi^{-1}\left(u_2\right)\right),$$

where $\Phi_{\Theta}\left(\cdot\right)$ stands for the distribution function of the bivariate standard normal distribution and Θ for the correlation matrix with linear correlation ρ (see McNeil et al., 2005).

Assuming a correlation ρ between the return on assets r_1 and the total claims payments S_1, the resulting surplus (i.e. equity capital) at time t = 1 is also normally distributed with

$$U_1 = \left(1+r_1\right)\left(u + P\left(\alpha, \xi\right) - \pi\left(q\right)\right) - qS_1 \sim N\left(\mathbb{E}\left[U_1\right], \mathbb{V}\left[U_1\right]\right),$$

where the expected value is given by

$$\mathbb{E}\left[U_1\right] = \left(1+\mu_r\right)\left(u + P\left(\alpha, s\right) - \pi\left(q\right)\right) - q\mu_S \tag{9}$$

and the variance is

$$\begin{aligned} \mathbb{V}\left[U_1\right] &= \left(u + P\left(\alpha, \xi\right) - \pi\left(q\right)\right)^2 \cdot \sigma_r^2 + q^2 \cdot \sigma_S^2 - 2 \cdot \left(u + P\left(\alpha, \xi\right) - \pi\left(q\right)\right) \cdot q \cdot Cov\left(r_1, S_1\right) \\ &= \left(u + P\left(\alpha, s\right) - \pi\left(q\right)\right)^2 \cdot \sigma_r^2 + q^2 \cdot \sigma_S^2 - 2 \cdot \left(u + P\left(\alpha, s\right) - \pi\left(q\right)\right) \cdot q \cdot \sigma_r \cdot \sigma_s \cdot \rho. \end{aligned} \tag{10}$$

Overall, these assumptions allow us to obtain simple expressions for the solvency lines and shareholder value functions. The real ruin probability *RP* not exceeding the reported target ruin probability α can thereby be expressed as

$$RP = \mathbb{P}(U_1 < 0) = \Phi\left(-\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right) \overset{!}{\leq} \alpha.$$

With N_α denoting the α-quantile of the standard normal distribution, we obtain

$$N_\alpha = -\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}} = -\frac{(1+\mu_r)(u+P(\alpha,\xi)-\pi(q))-q\mu_S}{\sqrt{\mathbb{V}[U_1]}}.$$

For a given σ_r (reflected in the variance of equity capital $\mathbb{V}[U_1]$), we can solve for μ_r and obtain the solvency lines *SolvL* as closed-form solutions

$$\mu_r = SolvL(\sigma_r) = \frac{q\mu_S - N_\alpha \cdot \sqrt{\mathbb{V}[U_1]}}{u+P(\alpha,\xi)-\pi(q)} - 1. \tag{11}$$

The shareholders' preference functions Ψ^1 and Ψ^2 that are subject to the optimization problem deriving the shareholders' maximum expected utility while maintaining a minimum solvency level also become more tractable. Taking into account limited liability, we use the preference function Ψ^1 (see Equation (8)). In the present special case, the term max(0,U_1) can be interpreted as a normally distributed random variable censored at 0 from below. If φ stands for the corresponding density function of the standard normal distribution, the following holds for $Y \sim N(\mu,\sigma^2)$ (see, e.g., Greene, 2012)

$$\mathbb{E}[\max(a,Y)] = \Phi(\alpha)\cdot a + (1-\Phi(\alpha))(\mu+\sigma\cdot\lambda) \text{ and}$$

$$\mathbb{V}[\max(a,Y)] = \sigma^2 \cdot (1-\Phi(\alpha))\cdot\left(1-\Delta+(\alpha-\lambda)^2\cdot\Phi(\alpha)\right),$$

with $\alpha = \dfrac{a-\mu}{\sigma}$, $\lambda = \dfrac{\varphi(\alpha)}{1-\Phi(\alpha)}$ and $\Delta = \lambda^2 - \lambda\cdot\alpha$.

For the terms in Equation (8) we can thus derive the closed-form expressions

$$\mathbb{E}[\max(0,U_1)] = \mathbb{E}[U_1]\cdot\Phi(x) + \sqrt{\mathbb{V}[U_1]}\cdot\varphi(x) \text{ and}$$

$$\mathbb{V}[\max(0,U_1)] = \mathbb{V}[U_1]\cdot\Phi(x)\cdot\left[1-\left(\left(\frac{\varphi(x)}{\Phi(x)}\right)^2+\frac{\varphi(x)}{\Phi(x)}\cdot x\right)+\left(x+\frac{\varphi(x)}{\Phi(x)}\right)^2\cdot\Phi(-x)\right],$$

with $x = \dfrac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}$.

The expected value $\mathbb{E}\left[\max\left(0,U_1\right)\right]$ is monotonically increasing in $\mathbb{E}\left[U_1\right]$ (and thus in μ_r) as well as in $\mathbb{V}\left[U_1\right]$ (and thus in σ_r depending on the correlation, see also Equations (9) and (10)) as can be seen from the derivatives (see the Appendix for the detailed derivation)

$$\begin{pmatrix} \dfrac{\partial\mathbb{E}\left[\max\left(0,U_1\right)\right]}{\partial\mathbb{E}\left[U_1\right]} \\ \dfrac{\partial\mathbb{E}\left[\max\left(0,U_1\right)\right]}{\partial\mathbb{V}\left[U_1\right]} \end{pmatrix} = \begin{pmatrix} \Phi\left(\dfrac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) \\ \dfrac{1}{2\sqrt{\mathbb{V}\left[U_1\right]}}\cdot\varphi\left(\dfrac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) \end{pmatrix},$$

and thus

$$\begin{pmatrix} \dfrac{\partial\mathbb{E}\left[\max\left(0,U_1\right)\right]}{\partial\mu_r} \\ \dfrac{\partial\mathbb{E}\left[\max\left(0,U_1\right)\right]}{\partial\sigma_r} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial\mathbb{E}\left[\max\left(0,U_1\right)\right]}{\partial E\left[U_1\right]}\cdot\dfrac{\partial\mathbb{E}\left[U_1\right]}{\partial\mu_r} \\ \dfrac{\partial\mathbb{E}\left[\max\left(0,U_1\right)\right]}{\partial\mathbb{V}\left[U_1\right]}\cdot\dfrac{\partial\mathbb{V}\left[U_1\right]}{\partial\sigma_r} \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} \Phi\left(\dfrac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right)\cdot\left(u+P\left(\alpha,\xi\right)-\pi\left(q\right)\right) \\ \dfrac{1}{2\sqrt{\mathbb{V}\left[U_1\right]}}\cdot\varphi\left(\dfrac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right)\cdot\left(2\cdot\left(u+P\left(\alpha,\xi\right)-\pi\left(q\right)\right)^2\cdot\sigma_r-2\cdot\left(u+P\left(\alpha,\xi\right)-\pi\left(q\right)\right)\cdot q\cdot\sigma_S\cdot\rho\right) \end{pmatrix},$$

while the derivatives of the variance $\mathbb{V}\left[\max\left(0,U_1\right)\right]$ cannot be expressed with a suitable closed-form expression. Therefore, in case of risk-neutrality (k = 0) and non-positive correlation ρ, Ψ^1 ($=\mathbb{E}\left[\max\left(0,U_1\right)\right]$) is monotonically increasing in μ_r and σ_r. In addition, since the capital market line $CML(\cdot)$ as the upper bound of attainable risk-return combinations is strictly monotonically increasing under the common assumption of a positive Sharpe ratio, i.e. a positive risk premium per unit of standard deviation, it holds that

$$\max_{(\sigma,\mu)\in\Upsilon}\mu = CML\left(\sigma_{IP}\right).$$

Under these assumptions, the maximization problem in (6) is thus solved by the intersection point IP between the capital market line and the solvency line

$$\left(\max_{(\sigma,\mu)\in\Upsilon}\sigma,\ \max_{(\sigma,\mu)\in\Upsilon}\mu\right) = \left(\sigma_{IP}, CML\left(\sigma_{IP}\right)\right)\in\Upsilon,$$

which is an element of the feasible set Υ. Hence, in this case, maximizing the shareholder value while maintaining a minimum solvency status in order to protect the policyholders requires the insurer to invest in the risk-return combination given by the intersection point between the capital market line and the solvency line.

However, in case of a risk-averse or risk-seeking attitude, general statements about the impact of μ_r and σ_r on the preference function Ψ^1 cannot be derived due to a lack of knowledge about the gradient of the variance $\mathbb{V}\left[\max\left(0,U_1\right)\right]$, which is why we later use numerical analyses to obtain more insight regarding the optimal risk-return combination that maximizes this preference function (see Section 4).

Alternatively, one could assume preference function Ψ^2 that does not account for limited liability, i.e.,

$$\Psi_k^2 = \mathbb{E}\left(U_1\right) - \frac{k}{2}\cdot \mathbb{V}\left(U_1\right). \tag{13}$$

When considering the gradients of the preference function Ψ^2

$$\begin{pmatrix} \dfrac{\partial \Psi_k^2}{\partial \mu_r} \\ \dfrac{\partial \Psi_k^2}{\partial \sigma_r} \end{pmatrix} = \begin{pmatrix} u + p - \pi\left(q\right) \\ -k\cdot\left(u + P\left(\alpha,\xi\right) - \pi\left(q\right)\right)^2 \cdot \sigma_r + k\cdot\left(u + P\left(\alpha,\xi\right) - \pi\left(q\right)\right)\cdot q\cdot \sigma_S \cdot \rho \end{pmatrix}, \tag{14}$$

one can see that the function is monotonically increasing in μ_r, but that the effect of σ_r again depends on the correlation between assets and claims ρ as well as the risk aversion parameter k. In particular, Ψ^2 is monotonically increasing in σ_r if and only if

$$-k\cdot\left(u + P\left(\alpha,\xi\right) - \pi\left(q\right)\right)^2 \cdot \sigma_r + k\cdot\left(u + P\left(\alpha,\xi\right) - \pi\left(q\right)\right)\cdot q\cdot \sigma_S \cdot \rho \geq 0\,.$$

We first concentrate on the case where $k > 0$ (risk-aversion), which implies that this equation holds if the correlation satisfies

$$\begin{aligned} & k\cdot\left(u + P\left(\alpha,\xi\right) - \pi\left(q\right)\right)\cdot q\cdot \sigma_S \cdot \rho \geq k\cdot\left(u + P\left(\alpha,\xi\right) - \pi\left(q\right)\right)^2 \cdot \sigma_r, \\ & \Leftrightarrow \rho \geq \left(u + P\left(\alpha,\xi\right) - \pi\left(q\right)\right)\cdot \sigma_r / \left(q\cdot \sigma_S\right). \end{aligned} \tag{15}$$

To ensure this condition (and that $\rho \leq 1$), the correlation must be either rather large (and positive, implying a good diversification between assets and liabilities), or the standard deviation of assets (in the nominator) should be rather small as compared to the standard deviation of liabilities, which can thus be rather restrictive. If $k \leq 0$ (risk-neutrality or risk-seeking behavior of shareholders), which is generally in line with the theoretical results in Gollier et al. (1997),[8] there is either no restriction for the correlation (if $k = 0$) or in case of $k < 0$, the correlation must satisfy

$$\rho \leq \left(u + P(\alpha, \xi) - \pi(q)\right) \cdot \sigma_r / \left(q \cdot \sigma_S\right). \tag{16}$$

Condition (16) holds in the special case where the correlation between assets and claims ρ is zero or negative, thus implying an insufficient diversification in an asset-liability context (see Fischer and Schlütter, 2015). In particular, in this case low asset values are positively related to high liability values and hence risks are not well diversified, resulting in a risky asset-liability-profile.

In both cases, i.e., in case of risk-neutrality or if Equations (15) or (16) are satisfied depending on k and the respective correlations, we can see that the preference function Ψ^2 is not only monotonically increasing in μ_r but also in σ_r. Identical argumentation as in case of Ψ^1 with risk-neutrality and non-positive correlation leads to the result that the intersection point *IP* between the capital market line and the solvency line solves the optimization problem (6) and thus maximizes shareholder value while maintaining a minimum solvency status to protect the policyholders. We conduct further analyses on the other cases in the following numerical analyses section.

4. NUMERICAL ANALYSES

We now conduct numerical analyses for the special case in the Gaussian environment described in Section 3.4 in order to study the impact of various economic parameters on an insurer's optimal risk- and value-based management decisions and to derive respective key drivers.

4.1 Input parameters

Input parameters are summarized in Tables 1 to 3. Table 1 is thereby based on the parameters of a German non-life insurer estimated in Eling et al. (2009) (except for the newly introduced

8 Gollier et al. (1997) theoretically show that a risk-neutral or risk-averse attitude of shareholders in case of limited liability (see preference function Ψ^1) corresponds to a risk-seeking or risk-neutral attitude when considering the preference function Ψ^2 in terms of unlimited equity capital U_1. In particular, if shareholders are risk-neutral (or perfectly diversified) in case of limited liability, they will aim to maximize the expectation of a convex function of the equity capital U_1 (Gollier et al., 1997, p. 348), implying that shareholders exhibit risk-seeking behavior in investment decisions regarding the equity capital U_1 since they can only benefit from additional risk in U_1. The shareholders' risk-averse attitude in case of limited liability leads to less extreme but similar results. Here, the optimal risk exposure of U_1 is also always higher than under full liability and often results in maximum risk taking.

parameters asset-claims correlation, premium loadings, retention level of proportional reinsurance, which were subject to robustness tests). The parameters of the premium reduction function *PR* (in comparison to the default-free premium *p*) rely on the estimation by Lorson et al. (2012)[9]. Furthermore, the capital market line is calibrated based on monthly time series from January 2004 to November 2015 of benchmark indices from the Datastream database that illustrate the available investment opportunities. Each benchmark measures the total investment returns for its asset on a Euro basis including coupons and dividends where applicable. As is done in Eling et al. (2009), we consider 11 indices with different regional focus in the four asset classes stocks, bonds, real estate, and money market instruments where insurers typically invest in. The expected return of the JPM Euro Cash 3 Month from January 2004 to November 2015 (2.04%) is thereby used as a proxy for the risk-free rate as is done in Eling et al. (2009), and since a constant, maturity independent risk-free interest rate does not exist in practice, we later conduct sensitivity analyses in this regard. The empirical risk-return profiles for all considered assets are given in Table 2 and the associated variance-covariance matrix is displayed in Table 3. For the base case, the resulting capital market line is then given by

$$\mu_r = 0.0204 + 0.34 \cdot \sigma_r. \quad (17)$$

Table 1: Input parameters (base case)

Available equity capital at time 0	$U_0 = u$	175
Expected value of claims	μ_S	1,171
Standard deviation of claims	σ_S	66
Correlation between stochastic return of assets and claims	ρ	0
Premium loading for an insurer without default risk	δ	5%
Retention level of proportional reinsurance	q	1
Premium loading for proportional reinsurance	δ^{re}	5%
Policyholders' risk sensitivity	ξ	0, 0.3, 1
Parameters of the premium reduction function	a	0.0419
	b	0.3855
Maximum value of ruin probability (target ruin probability)	α	0.01%, 0.25%, 0.50%

9 In Lorson et al. (2012) a default-free insurer corresponds to a ruin probability of 0.01%. Since the estimation is based on very few data points taken from an empirical study in Zimmer et al. (2009), they also consider an upper and lower bound for the premium reduction to take into account the variability.

Table 2: Descriptive statistics (annualized) for monthly return time series from January 2004 to November 2015 from the Datastream database

Asset class	Index	Description	μ_i	σ_i
Money market	JPM Euro Cash 3 Months (1)	Money market in the EMU = r_f	2.04%	-
Stocks	MSCI World ex EMU (2)	Worldwide stocks without the EMU	8.11%	47.06%
	MSCI EMU ex Germany (3)	Stocks from the EMU without Germany	5.92%	61.71%
	MSCI Germany (4)	Stocks from Germany	8.55%	68.16%
Bonds	JPM GBI Global All Mats. (5)	Worldwide government bonds	4.58%	28.83%
	JPM GBI Europe All Mats. (6)	Government bonds from Europe	5.30%	14.46%
	JPM GBI Germany All Mats. (7)	Government bonds from Germany	4.74%	14.57%
	IBOXX Euro Corp. All Mats (8)	Corporate bonds from Europe	4.31%	13.71%
Real estate	GPR General World (9)	Real estate worldwide	9.11%	51.67%
	GPR General Europe (10)	Real estate in Europe	6.71%	36.53%
	GPR General Germany (11)	Real estate in Germany	2.60%	9.22%

Table 3: Variance-covariance matrix (annualized) for monthly return time series in Table 2

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
(2)		0.221	0.233	0.263	-0.018	-0.004	-0.015	0.022	0.189	0.110	0.021
(3)			0.381	0.383	-0.075	-0.010	-0.033	0.031	0.211	0.158	0.028
(4)				0.465	-0.078	-0.018	-0.036	0.029	0.228	0.156	0.031
(5)					0.083	0.029	0.032	0.004	0.001	-0.015	-0.001
(6)						0.021	0.019	0.010	0.014	0.007	0.002
(7)							0.021	0.008	0.003	-0.003	0.000
(8)								0.019	0.036	0.026	0.004
(9)									0.267	0.155	0.027
(10)										0.133	0.024
(11)											0.008

4.2 Maximizing shareholder value

The impact of the assets' risk-return combinations on the shareholder value is not straightforward in all cases as already laid out in Section 3.4. However, in case of risk-neutral attitude, both preference functions Ψ^1 (in case of non-positive correlation) and Ψ^2, i.e. $\mathbb{E}\left[\max\left(0,U_1\right)\right]$ and $\mathbb{E}\left[U_1\right]$, are monotonically increasing in μ_r and σ_r (see Equations (12) and (14)), implying that the intersection point between the capital market line and the solvency line *IP* solves the optimization problem (6) and thus maximizes the shareholders' expected utility as formally derived in Section 3.4. However, in case of risk-averse or risk-seeking attitude, further numerical analyses (that are available from the authors upon request) showed that the results strongly depend on the shareholders' risk attitude as well as on the correlation between assets and liabilities.

In particular, the impact of μ_r on Ψ^2 (full liability case) is positive in all considered examples, which implies that maximizing μ_r *ceteris paribus* also maximizes shareholder value. This is different for the preference function Ψ^1, which assumes limited liability. One can observe that in case of risk-averse attitude along with a non-positive asset-liability correlation, the preference function is first decreasing and then increasing again, which stems from offsetting effects arising from the variance and expected value of the shareholders' wealth (see Equation (8)).

Furthermore, the impact of σ_r on Ψ^1 and Ψ^2 is very similar for both preference functions, and it is exactly opposite for risk-averse and risk-seeking attitude. For non-positive asset-liability correlations, both Ψ^1 and Ψ^2 are monotonically increasing in σ_r for a risk-seeking attitude (k = -1) and decreasing in case of risk-aversion (k = 1). Hence, in this case the insurer *ceteris paribus* would have to choose the most (k = -1) or least (k = 1) risky asset allocations for maximizing shareholder value in the considered examples. For a positive correlation and thus in the presence of positive diversification benefits, the results are not as straightforward as in case of a non-positive correlation, which is due to the non-monotonic behavior of $\mathbb{V}\left[\max\left(0,U_1\right)\right]$ and $\mathbb{V}\left[U_1\right]$ regarding σ_r. Due to the positive asset-liability correlations ρ, both $\mathbb{V}\left[\max\left(0,U_1\right)\right]$ and $\mathbb{V}\left[U_1\right]$ are decreasing for low σ_r and increasing for high σ_r (in case of $\mathbb{V}\left[U_1\right]$ this could be analytically derived in Equation (14)).

In summary, we have formally shown in Section 3.4 that the preference function Ψ^2 is monotonically increasing in μ_r and σ_r in case of k = 0 (i.e. risk-neutral attitude), which is the same for Ψ^1 in case of k = 0 (risk-neutral attitude) and Ψ^2 in case of k < 0 (risk-seeking attitude) with non-positive correlations between assets and liabilities. This leads to the result that the intersection point between the capital market line and the solvency line *IP* solves the optimization problem for the present setting and thus maximizes the shareholders' preference functions under solvency constraints. The numerical analysis of Ψ^1 shows similar results, i.e. monotonically increasing behavior in μ_r and σ_r in case of non-positive correlation between assets and liabilities and k < 0, and hence implies that the intersection point also solves the shareholder maximization problem.[10]

[10] Moreover, Ψ^2 is convex in μ_r for arbitrary k and in σ_r for $k \leq 0$, as can be derived analytically. Further graphical analyses show that this is approximately the same for Ψ^1 (the analysis is available from the authors upon request).

4.3 The impact of policyholder risk sensitivity on admissible risk-return asset profiles

Figure 1 displays *attainable* risk-return combinations based on market restrictions concerning the asset allocation as reflected by the capital market line (CML, solid black line, see also Equation (17)). Furthermore, *admissible* combinations are presented as defined by the solvency lines in Equation (11). They represent the minimum expected rate of return μ_r that the insurer has to achieve for a given standard deviation σ_r in order to satisfy the intended safety level given by the target ruin probability α = 0.50% (as required by Solvency II). Therefore, for its investment strategy the insurer has to choose risk-return combinations above the respective solvency line and below the capital market line, which represent the set Υ of attainable and admissible risk-return combinations (Equation (7)) as highlighted by the grey areas in Figure 1. The intersection point *IP* is marked with an “x”, representing the maximum shareholder value in terms of the considered preference functions Ψ^1 and Ψ^2.

Figure 1: Attainable (capital market line CML) and admissible (“solvency line”) risk-return combinations given no (ξ = 0), medium (ξ = 0.3) and high (ξ = 1) policyholder risk sensitivity for a given target ruin probability α = 0.50%

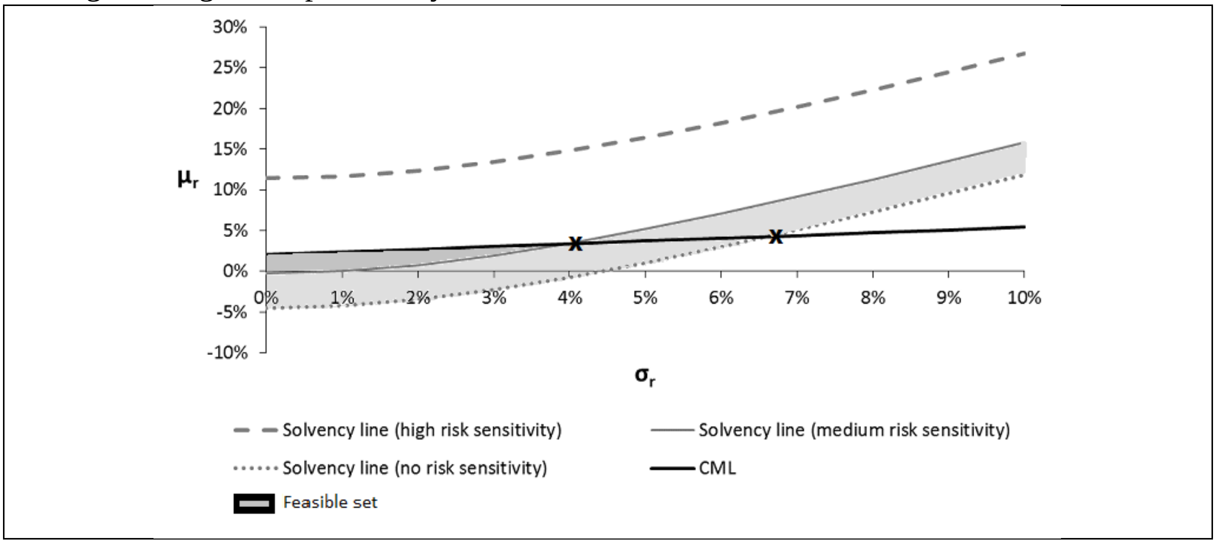


Figure 1 shows the case where the insurer faces fixed solvency constraints (ruin probability α = 0.50%) and emphasizes the impact of the default-risk-driven premium income on the maximum shareholder value. In particular, the admissible risk-return combinations (solvency lines) strongly depend on the policyholders’ risk sensitivity. For higher risk sensitivities (going from ξ = 0 to 1 in the considered example), the solvency lines are shifted upwards for a given ruin probability, until for ξ = 1 (high risk sensitivity) the solvency line lies above the CML, such that no possible allocation opportunities remain and the grey area disappears. The results emphasize that a more risk sensitive assessment of the solvency status reduces policyholder demand and hence the achievable premium income, implying that the insurer has considerably less flexibility for its asset allocation to fulfill the solvency requirements and thus a lower maximum shareholder value. We also observe in Figure 1 that in the case without policyholder risk sensitivity, expected asset returns may even be negative for low standard deviations, and the insurer would still satisfy the required safety level due to sufficient equity capital and premium loadings. Further sensitivity analysis emphasizes that reducing the initial equity capital or lowering the premium loading implies an upward shift of the solvency lines, such that negative expected returns are no longer permitted.

4.4 The impact of the insurer's safety level

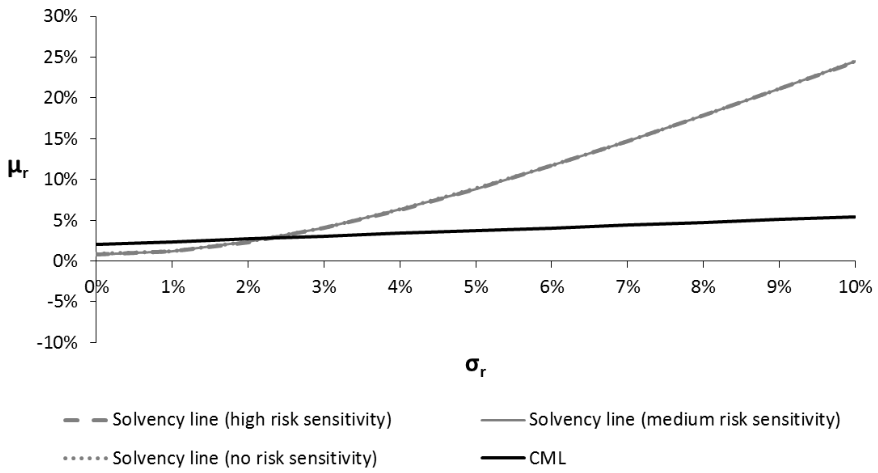
In the presence of market discipline, the insurer faces the challenge to balance the premium reduction driven by a higher default risk and the higher expected return on investment associated with a higher risk taking. We thus next address the situation where the insurer chooses the target ruin probability when maximizing shareholder value given a default-risk-driven premium income. Figure 2 shows the impact of different target ruin probabilities α = 0.01%, 0.50% given various levels of the policyholders' risk sensitivity and thus their willingness to pay depending on the reported ruin probability.

Figure 2a) displays the setting of a "default-free" insurer, as for α = 0.01%, the premium reduction estimated by Lorson et al. (2012) is zero, and the solvency lines for various levels of policyholders' risk sensitivity coincide. As can be seen when comparing Figures 4a) and b), the gap between the solvency lines for ξ = 0 and ξ = 1 increases considerably the higher the target ruin probability. In particular, the solvency line in the case without risk sensitivity ξ = 0 (lowest dotted line) shifts downward for a higher α, since it is easier for the insurer to fulfill the solvency requirements. In contrast, the solvency line for a high policyholder risk sensitivity (ξ = 1) shifts upward for a higher α, since the information of the higher ruin probability considerably reduces the achievable premium income (see Equation (4)). For higher reported target ruin probabilities, the willingness to pay by risk-sensitive policyholders declines and the premium income strongly decreases, which can lead to considerable difficulties in maintaining the desired solvency level and thus also in generating shareholder value.

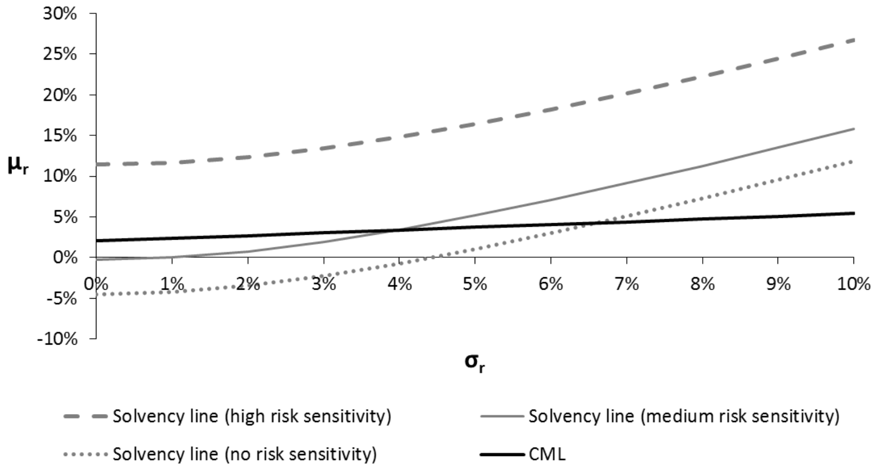
From Figure 2 we can see that the shareholder value is highest when choosing a target ruin probability corresponding to the absence of default risk in case of high policyholder risk sensitivity (ξ = 1), which is in line with Zimmer et al. (2014), who focus on the effects stemming from insurance demand on maximum shareholder value. However, for no (ξ = 0) and medium (ξ = 0.3) policyholder risk sensitivity, we obtain the opposite result. Overall, this strongly emphasizes that it is crucial to take into account the policyholders' risk sensitivity and thus the purchase behavior depending on the safety level, especially if insurers have to reveal their solvency status as required by Solvency II.

Figure 2: Attainable (capital market line CML) and admissible ("solvency line") risk-return combinations given no ($\xi = 0$), medium ($\xi = 0.3$) and high ($\xi = 1$) policyholder risk sensitivity for varying target ruin probabilities α

a) Risk-return asset combinations for $\alpha = 0.01\%$ ("default-free")



b) Risk-return asset combinations for $\alpha = 0.50\%$



4.5 The impact of dependencies between assets and liabilities

To investigate the impact of dependencies between assets and liabilities on the requirements for the investment strategy and hence for maximizing shareholder value, we compare the solvency lines for different correlations ρ = -0.5, -0.25, 0, 0.25, 0.5 and thus different diversification levels in Figure 3. In case of positive correlations, high asset values are positively related to high liability values and hence the risks are well diversified, resulting in a well-balanced asset-liability profile. Negative correlations, in contrast, represent an increasing riskiness of the asset-liability profile (in terms of the variance of equity capital) due to insufficient diversification, i.e. low asset values are positively related to high liability values (see Fischer and Schlütter, 2015).

Figure 3: Attainable (CML) and admissible (solvency lines) risk-return combinations for a given target ruin probability $\alpha = 0.50\%$ and medium ($\xi = 0.3$) policyholder risk sensitivity for various correlations ρ

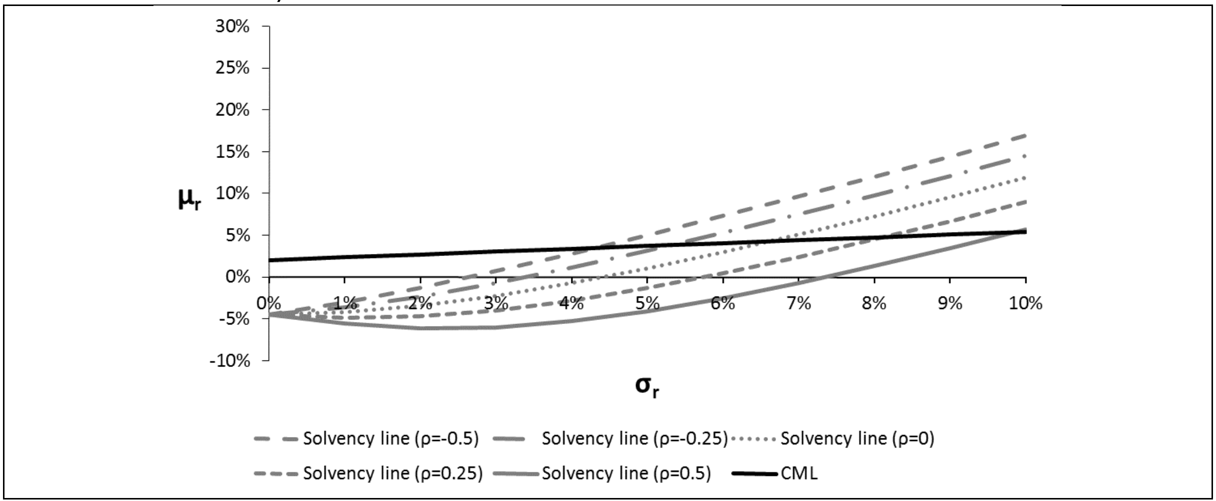


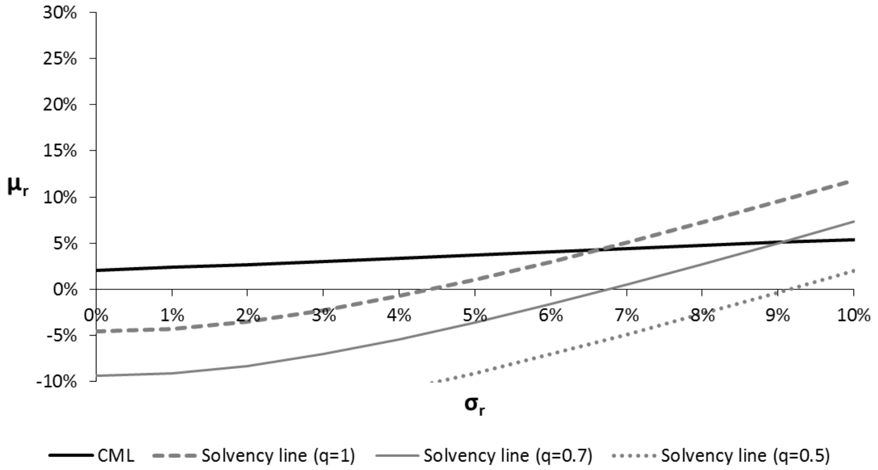
Figure 3 shows that the convexity of the solvency line increases for higher correlations ρ, while the intercept remains unchanged. A more negative linear dependence between assets and liabilities is thereby penalized by higher solvency capital requirements and reduces the area of acceptable and attainable risk-return combinations. Furthermore in case of a non-positive correlations we can conclude that an insufficient diversification in an asset-liability context in terms of a lower ρ has negative consequences in regard to maximizing shareholder value, as the intersection point moves to the lower left, leading to restrictions in capital investments and a lower maximum shareholder value. It is thus crucial that a risk management model takes into account the diversification between assets and liabilities. Otherwise, the model would incentivize insurers to take advantage of opportunities that realize a positive asset-liability relation, which is part of the criticism of Fischer and Schlütter (2015) in their analysis regarding the standard formula of Solvency II.

4.6 The impact of reinsurance decisions

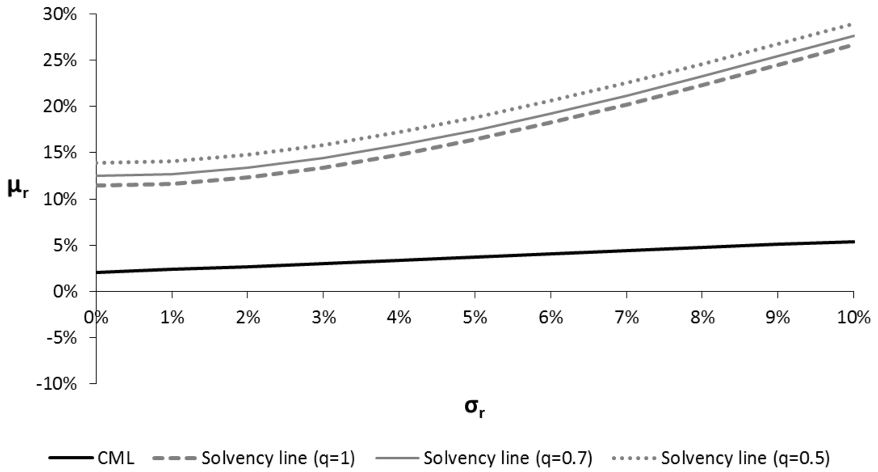
We next focus on decisions regarding the liability side by studying the impact of reinsurance contracts on (optimal) asset portfolio combinations. Figure 4 exhibits risk-return asset combinations for different retention levels of proportional reinsurance q = 0.5, 0.7, 1.0. The results show that the solvency lines shift upward for decreasing q (i.e., increasing reinsurance portions) given a high ($\xi = 1$) risk sensitivity (Figure 4b)), whereas in the case without ($\xi = 0$) risk sensitivity (Figure 4a)) the solvency lines shift downward, thus allowing the insurer more flexibility in the asset allocation and creating opportunities for enhancing shareholder value. This opposite behavior can be explained by the fact that the reinsurance premium is fixed, whereas the premiums of the insurer vary depending on the policyholders' risk sensitivity. In the case without ($\xi = 0$) risk sensitivity, premiums and reinsurance prices are calculated in the same manner by using the actuarial expected value principle with the same loading. Hence, such decisions regarding the liability side generate more flexibility on the asset side if policyholders exhibit no (or a low) risk sensitivity.

Figure 4: Attainable (CML) and admissible ("solvency lines") risk-return combinations for $\alpha = 0.50\%$ for different retention levels of proportional reinsurance q = 0.5, 0.7, 1.0

a) Risk-return asset combinations for no risk sensitivity ($\xi = 0$)



b) Risk-return asset combinations for high risk sensitivity ($\xi = 1$)

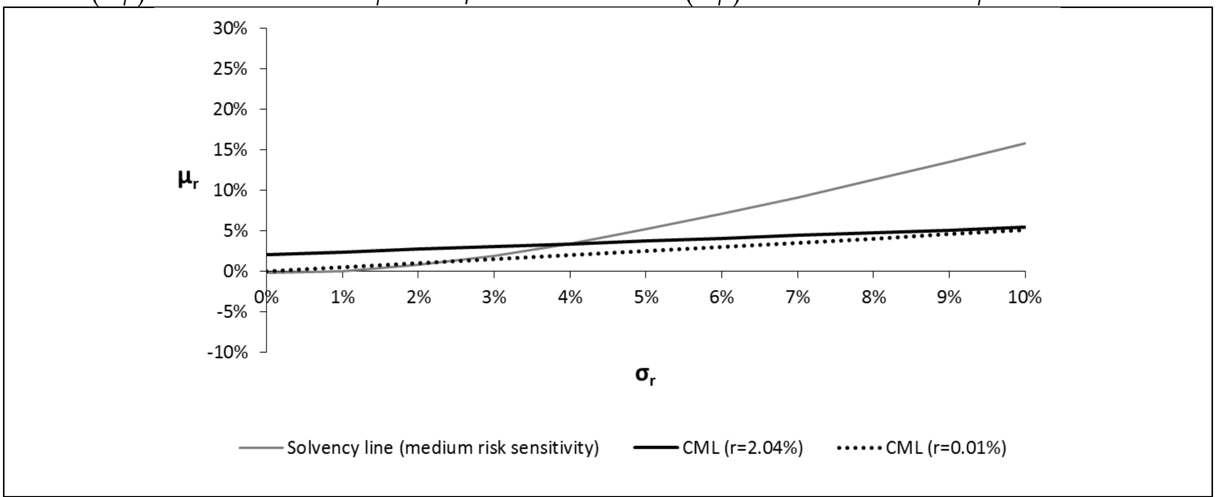


This is generally in line with Diasparra and Romera (2010) who study upper bounds for the ruin probability in an insurance model where the risk process can be controlled by proportional reinsurance. They find i.a. (also assuming a fixed premium income) that a decreasing retention level leads to decreasing upper bounds of the ruin probability. However, our results show that taking into account the policyholders' willingness to pay given a high ($\xi = 1$) risk sensitivity, this effect is reversed. In particular, as in this case there are no admissible and attainable asset allocation opportunities, shareholder value cannot be created (in terms of preference functions) as the insurer has to pay the fixed reinsurance premiums from a much lower premium income caused by the policyholders' premium reduction (see Equation (4)). The higher relative costs for reinsurance thus lead to stronger restrictions on the asset side, and a decreasing retention level of reinsurance q further intensifies this effect.[11] Overall, this again illustrates the strong interaction between decisions on the asset and liability side and it emphasizes the relevance of taking into account policyholders' willingness to pay for insurance products also in the context of reinsurance contracts, for instance, given that solvency levels have to be reported.

4.7 The impact of the risk-free interest rate

Lastly, since interest rates play an important role for solvency ratios, especially against the background of currently very low interest rate levels, Figure 5 illustrates attainable and admissible risk-return asset combinations for various levels of the risk-free rate. In the base case, we use the expected return of the JPM Euro Cash 3 Month from January 2004 to November 2015 (2.04%) as a proxy for a constant, maturity independent risk-free interest rate. As pointed out before, since such a risk-free interest rate does not exist in practice, the calibration only serves as a proxy and we additionally use the JPM Euro Cash 3 Month in November 2015 (0.01%) to illustrate the impact of the low interest rate level.

Figure 5: Attainable (capital market line CML) and admissible ("solvency line") risk-return combinations for a given target ruin probability α = 0.50% and given medium (ξ = 0.3) policyholder risk sensitivity for different levels of the risk-free interest rate r_f = 2.04%: $CML(\sigma_r) = 2.04\% + 0.34 \cdot \sigma_r$ and r_f = 0.01%: $CML(\sigma_r) = 0.01\% + 0.50 \cdot \sigma_r$



When comparing the capital market lines, it can be seen that the slope *ceteris paribus* increases for a decreasing risk-free interest rate from r_f = 2.04% (base case) to r_f = 0.01%, i.e. the risk premium per unit of standard deviation increases, whereas the intercept of the capital

[11] Further analyses showed that, as expected, the solvency lines shift upward for an increasing reinsurance premium loading δ^{re}. Therefore, the insurer generally has to take into account that higher loadings δ^{re} increase the costs and thus reduce the area of attainable and admissible risk-return combinations and therefore the maximum shareholder value.

market line decreases. Aggregating both offsetting effects leads to a reduction of the set of attainable and admissible risk-return combinations in the present setting and thus to a lower intersection point between solvency line and capital market line, implying a reduction of maximum shareholder value. These results emphasize that the current phase of low interest rates strongly restricts the insurer's range of admissible and attainable risk profiles regarding the asset investment, since the insurer cannot invest in riskier assets while maintaining the solvency level, resulting in a lower shareholder value.

5. SUMMARY

In this article, we study risk- and value-based management decisions regarding a non-life insurer's capital investment strategy by deriving minimum capital standards for the insurer's asset allocation based on a fixed solvency level, since adjusting the asset side to satisfy solvency capital requirements should generally be easier than short-term adaptions regarding equity capital or the liability side as pointed out by Eling et al. (2009). In this setting, we follow the latter and link the admissible (i.e. satisfying solvency requirements based on the ruin probability) risk-return combinations of the insurer's asset portfolio to actually attainable allocation opportunities at the capital market using Tobin's (1958) capital market line. We then study the optimal investment problem when maximizing shareholder value based on preference functions and simultaneously controlling for the ruin probability in order to protect the policyholders. We thereby extend previous work in several relevant ways: We use a more general model and explicitly include the policyholders' willingness to pay, which to the best of our knowledge has not been done so far in this context, taking into account the insurer's reported target safety level and the policyholders' risk sensitivity. We further investigate the impact of asset-liability dependencies, the influence of proportional reinsurance contracts and the impact of the risk-free interest rate, which considerably influence the set of attainable and admissible risk-return combinations as well as the maximum achievable shareholder value.

To study the impact of decisions with respect to assets and liabilities on shareholder value, we assume mean-variance preferences for shareholders and consider the cases with and without limited liability. To gain deeper insight, we conduct comprehensive analytical and numerical analyses for a special case in a Gaussian environment and formally show that the intersection point between the capital market line and the solvency line maximizes shareholder value in certain scenarios, while optimal solutions in other cases require numerical analyses.

Our numerical results show that the consideration of the policyholders' willingness to pay depending on their risk sensitivity based on the insurer's reported solvency status is crucial, since a more risk-sensitive assessment reduces the premium income and thus the flexibility of investments on the asset side as well as the resulting maximum shareholder value. This is especially relevant in the future in the presence of market discipline, when insurers have to reveal their solvency status according to Solvency II. Moreover, given a default-risk-driven premium income, the optimal reported target ruin probability differs for various levels of policyholders' risk sensitivity. In case of high risk sensitivity, a target ruin probability corresponding to the absence of default risk implies a higher shareholder value, while no and medium policyholder risk sensitivity can lead to contrary results. To satisfy the interests of both shareholders and policyholders, our approach can thus be used for balancing shareholder value and risk taking. In particular, depending on the policyholders' risk sensitivity, it is advisable for firms to closely monitor the reactions of policyholders to safety levels and to possibly enhance the solvency level in order to generate more flexibility for the investment strategy and to increase shareholder value.

In addition, we find that a negative correlation between assets and liabilities can considerably reduce the area of attainable and admissible risk-return combinations, emphasizing that diversification between assets and the underwriting portfolio can generate more flexibility regarding the risk profile of the capital investment, which also implies the potential of generating a higher shareholder value. We further observe that the set of attainable and admissible investment opportunities and thus the maximum shareholder value only increases for lower retention levels (i.e., higher portions) of reinsurance if policyholders are not risk sensitive. In particular, this effect is reversed and the shareholder value generally decreases when taking into account the policyholders' willingness to pay, as the insurer's premium income, which must be used to pay the fixed reinsurance premiums (not subject to risk sensitivity influences), is reduced, implying a reduction of the set of available risk-return asset combinations. Furthermore, an analysis of the impact of the risk-free interest rate shows that the current phase of low interest rates strongly restricts the insurer's investment opportunities and considerably reduces shareholder value.

Overall, our results strongly emphasize the strong interaction between decisions regarding the asset and the liability side, and they underline the importance of considering the policyholders' demand for insurance products given that solvency levels have to be reported, which should be taken into account by insurers in the context of their risk- and value-based management decisions.

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APPENDIX

Solution of the recurrence Equation (1):

$$U_n = B_n\left(U_{n-1} + p - \pi(q)\right) - q\left[S_n - S_{n-1}\right] = B_n\left(U_{n-1} + P\right) - C_n$$

$$= B_n\left(B_{n-1}\left(B_{n-2}\left(B_{n-3}\left(\ldots\left(B_1\left(U_0 + P\right) - C_1 + P\right)\ldots\right) - C_{n-3} + P\right) - C_{n-2} + P\right) - C_{n-1} + P\right) - C_n$$

$$= \left(U_0 + P\right)\prod_{i=1}^{n} B_i + B_n B_{n-1} B_{n-2}\cdots B_2\left(P - C_1\right) + \ldots + B_n B_{n-1} B_{n-2}\left(P - C_{n-3}\right) + B_n B_{n-1}\left(P - C_{n-2}\right) + B_n\left(P - C_{n-1}\right) -$$

$$= \left(U_0 + P\right)\prod_{i=1}^{n} B_i + \sum_{i=1}^{n-1}\left(P - C_i\right)\prod_{j=i+1}^{n} B_j - C_n = \left(U_0 + P\right)\prod_{i=1}^{n} B_i + \sum_{i=1}^{n-1} A_i \prod_{j=i+1}^{n} B_j - C_n$$

$$= \left(u + p - \pi(q)\right)\prod_{i=1}^{n} B_i + \sum_{i=1}^{n-1} A_i \prod_{j=i+1}^{n} B_j - q\left[S_n - S_{n-1}\right],$$

with $P = p - \pi(q)$, $C_n = q\left[S_n - S_{n-1}\right]$ and $A_n = P - C_n$.

Derivatives of $\mathbb{E}\left[\max(0, U_1)\right]$:

$$\frac{\partial}{\partial \mathbb{E}\left[U_1\right]}\mathbb{E}\left[\max(0, U_1)\right]$$

$$= 1 \cdot \Phi\left(\frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) + \mathbb{E}\left[U_1\right] \cdot \varphi\left(\frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) \cdot \frac{1}{\sqrt{\mathbb{V}\left[U_1\right]}} + \sqrt{\mathbb{V}\left[U_1\right]} \cdot \frac{\partial}{\partial \frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}}\varphi\left(\frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) \cdot \frac{1}{\sqrt{\mathbb{V}\left[U_1\right]}}$$

$$= \Phi\left(\frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) + \frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}} \cdot \varphi\left(\frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) - \frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}} \cdot \varphi\left(\frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right) = \Phi\left(\frac{\mathbb{E}\left[U_1\right]}{\sqrt{\mathbb{V}\left[U_1\right]}}\right),$$

and

$$\frac{\partial}{\partial \mathbb{V}[U_1]}\mathbb{E}[\max(0,U_1)]$$

$$=\mathbb{E}[U_1]\cdot\varphi\left(\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right)\cdot\left(-\frac{\mathbb{E}[U_1]}{2(\mathbb{V}[U_1])^{1.5}}\right)+\frac{1}{2\sqrt{\mathbb{V}[U_1]}}\cdot\varphi\left(\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right)$$

$$+\sqrt{\mathbb{V}[U_1]}\cdot\frac{\partial}{\partial x}\varphi(x)\Big|_{x=\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}}\cdot\left(-\frac{\mathbb{E}[U_1]}{2(\mathbb{V}[U_1])^{1.5}}\right)$$

$$=\mathbb{E}[U_1]\cdot\varphi\left(\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right)\cdot\left(-\frac{\mathbb{E}[U_1]}{2(\mathbb{V}[U_1])^{1.5}}\right)+\sqrt{\mathbb{V}[U_1]}\cdot\left(-\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right)\varphi\left(\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right)\left(-\frac{\mathbb{E}[U_1]}{2(\mathbb{V}[U_1])^{1.5}}\right)$$

$$+\frac{1}{2\sqrt{\mathbb{V}[U_1]}}\cdot\varphi\left(\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right)$$

$$=\frac{1}{2\sqrt{\mathbb{V}[U_1]}}\cdot\varphi\left(\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right),$$

where we use $\frac{\partial}{\partial x}\varphi(x)=-x\cdot\varphi(x), x\in\mathbb{R}$.