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Risk- and Value-Based Management for Non-Life Insurers under Solvency Constraints

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RISK- AND VALUE-BASED MANAGEMENT FOR NON-LIFE INSURERS UNDER SOLVENCY CONSTRAINTS

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ABSTRACT

The aim of this paper is to study optimal risk- and value-based management decisions regarding a non-life insurer's investment strategy by maximizing shareholder value based on preference functions, while simultaneously controlling for the ruin probability. We thereby extend previous work by explicitly accounting for the policyholders' willingness to pay depending on their risk sensitivity based on the insurer's reported solvency status, which will be of great relevance under Solvency II. We further investigate the impact of the risk-free interest rate, (non-linear) dependencies between assets and liabilities, distributional assumptions as well as reinsurance. One main finding is that the consideration of default-risk-driven premiums is vital for optimal management decisions, since, e.g., the target ruin probability implying a higher shareholder value differs for various risk sensitivities of the policyholders. Furthermore, in the present setting, proportional reinsurance increases shareholder value only for non-risk sensitive policyholders.

Keywords: Non-life insurance; Solvency II; shareholder value optimization; default-risk-driven premium; copulas

JEL Classification: G22; G28; G31

1. INTRODUCTION

An insurance company's equity capital can be exposed to large fluctuations, which may potentially result in severe solvency problems. These fluctuations can arise from both the investment side (due to an increasing volatility in the financial markets) as well as the underwriting side against the background of a rising frequency and severity of natural catastrophes in the last decades (Swiss Re, 2016). In this context, risk- and value-based management is essential for the long-term success of insurance companies, in that investment as well as underwriting decisions should take into account risk and return in order to ensure an efficient and profitable use of capital and to control for default risk.

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In order to protect the policyholders and the stability of the financial system, Solvency II as the new European regulatory framework for insurers[1] came into force on January 1st, 2016. Pillar 1 introduces risk-based capital requirements by demanding sufficient equity capital to fulfill insurance contracts also under adverse events, Pillar 2 specifies qualitative requirements such as a governance system and the Own Risk and Solvency Assessment (ORSA), and Pillar 3 comprises reporting requirements. The insurer thereby has to provide information not only to the supervisor but also to the public by means of the Solvency and Financial Condition Report (SFCR), which is intended to provide transparency and enforce market discipline. Pillar 1's solvency capital requirements are thereby based on the Value at Risk with a 99.5% confidence level, implying a one-year ruin probability that does not exceed 0.50%. It can be derived either by means of a standard model provided by the regulatory authorities or based on a company-specific internal model that adequately reflects the firm's risks (e.g., Eling et al., 2009). An internal model should also be used in Pillar 2 for the firm's ORSA and should thus represent an integral part of an insurer's risk- and value-based management, i.e., to be applied for corporate risk management and asset allocation decisions, for instance.[2]

Against this background, the aim of this paper is to study optimal risk- and value-based management decisions regarding the investment strategy for a non-life insurer, which contribute to increasing shareholder value in terms of preference functions.[3] In particular, by adjusting the asset allocation to satisfy solvency capital requirements, the approach should be less costly than raising equity capital or adjusting the liability side (Eling et al., 2009). Toward this end, we considerably extend the analyses and model frameworks in previous work (e.g., Eling et al., 2009; Zimmer et al., 2014; Braun et al., 2017) by using a more general model based on the literature on non-life insurance and ruin theory, thereby focusing on shareholder value based on preferences while simultaneously controlling for the insurer's ruin probability. In particular, we consider the impact of several new key features on risk- and value-based management regarding the investment strategy, including the policyholders' willingness to pay depending on the insurer's reported solvency status, which despite its great impact has not

[1] Furthermore, since many countries have a Solvency II equivalent (e.g., the Swiss Solvency Test or the China Risk Oriented Solvency System) or at least seek to establish one (Sub-Saharan African countries opt for regulation systems resembling a simplified form of Solvency II (EY, 2016)), these results are not limited to the impact of Solvency II but are generally of relevance.

[2] The quantitative approaches in Pillar 1 and 2 rely on a market consistent balance sheet of the insurer, which reflects an insurer's assets and liabilities and thus the shareholders' equity capital at a single point in time. For instance, Allianz Group as one of the most important insurance groups worldwide (and identified as a global systemically important insurer by regulators) exhibits a total value of assets of approximately 850 bn Euros and 66 bn Euros equity capital, implying an unweighted capital ratio of 7.8% (Allianz Group annual report 2015, also for remarks regarding their internal model).

[3] Note that for simplicity, we use the expression "shareholder value" or "shareholder preference value", while assuming preferences for the evaluation (instead of relative market valuation).

been studied to date in this context, the dependencies between assets and liabilities, the impact of reinsurance contracts as well as the risk-free interest rate.

In previous work, Eling et al. (2009) derive minimum requirements for a non-life insurer's capital investment strategy that satisfy solvency restrictions based on different risk measures. Using "solvency lines", i.e. isoquants of risk and return combinations of the asset allocation for a fixed safety level of the insurer, they determine admissible risk and return asset combinations given a certain liability structure, and then compare these to allocation opportunities actually available at the capital market based on portfolio theory. Similarly, but in a life insurance context, Braun et al. (2017) study optimal asset allocations taking into account restrictions from solvency capital requirements, thereby comparing the Solvency II standard formula with an internal model. In the context of a fixed investment decision and a default-risk-driven customer demand, Schlütter (2014) further studies an insurer who chooses insurance prices and an allowed solvency level when optimizing shareholder value given risk-based capital requirements or price regulation. In addition, experimental and empirical research (Wakker et al., 1997; Zimmer et al., 2009; Lorson et al., 2012; Zimmer et al., 2014) shows that an insurer's default risk can have a strong influence on customer demand, where lower safety levels can lead to a considerable reduction of achievable premiums. In this context, Zimmer et al. (2014) develop a risk management model assuming that the insurer's default risk is fully known to consumers, and based on this derive the solvency level that maximizes shareholder value, which is the case for a ruin probability of zero. These results emphasize that an insurer's safety level should be taken into account in risk- and value-based management as the reaction of customers to default risk (by way of the premium level) can considerably impact shareholder value. This will be even more relevant when insurers have to publicly report their solvency status under Solvency II. In the context of deriving minimum requirements for the investment strategy, Fischer and Schlütter (2015) further criticize that the standard model leads to an incentive to avoid diversification between assets and liabilities, as dependencies are not adequately taken into account in the standard model, which is in line with the results in Braun et al. (2017).

The ruin probability (as one basis of Solvency II) is also a classical topic of applied mathematics in non-life insurance as introduced by Lundberg (1903) and Cramér (1930), with the Cramér-Lundberg model being the classical model of risk theory in non-life insurance mathematics (Mikosch, 2009). Since then, various extensions have been developed, e.g., regarding the process of the total claims amount (e.g., the Sparre-Andersen (1957) model generalizes the total claims process to a renewal model, Albrecher and Teugels (2006) model the dependence between claim size and the inter-claim time) as well as the investment side of the insurer. Taking into account the possibility of investing in (risky) assets that influence the probability of ruin goes back to Segerdahl (1942) and Paulsen (1993) (see Paulsen, 2008). Some further recent works in this regard include Paulsen (2008), Klüppelberg and Kostadinova (2008), Heyde and Wang (2009), Hult and Lindskog (2011), Bankovsky et al. (2011), Hao and Tang (2012), and Ramsden and Papaioannou (2017). There are also a number of papers focusing on a discrete-time risk model where the insurer's surplus is controlled by i.i.d. discrete insurance and financial risk processes that are independent from each other (e.g., Nyrhinen, 1999; Tang and Tsitsiashvili, 2003, 2004; Yang and Zhang, 2006; Li and Tang, 2015). Moreover, the insurance risk models can be modified to a dual version that are suitable for the any company (Avanzi et al., 2007) where the ruin probability is also focus of research (e.g., Dimitrova et al., 2015).

In recent years, the applied math literature also focused on optimal investment and/or reinsurance strategies in the sense of minimizing the ruin probability or optimizing other objective functions. For example, Schmidli (2001, 2002) and Promislow and Young (2005) minimize the ruin probability in continuous time, and Schäl (2004), Diasparra and Romera (2009, 2010), Romera and Runggaldier (2012), and Lin et al. (2015) in discrete time. Another popular and relevant optimization criterion is the maximization of expected utility of the insurer's terminal wealth, e.g. in continuous time in general in Liang et al. (2011, 2012), Liang and Bayraktar (2014), and Huang et al. (2016), and in particular for mean-variance preferences in Bäuerle (2005), Bai and Zhang (2008), and Bi and Guo (2012). For discrete time, we further refer to Schäl (2004).

Another strand of the literature relates to the (frequent) assumption of independence between insurance and financial risk processes, which simplifies analytical solutions but is difficult for practical applications. Since discrete-time risk models create an efficient possibility to investigate the interplay of both risks, several papers drop this assumption, using, e.g., an insurance risk process and/or a financial risk process being a sequence of dependent random variables while keeping the independence between the two processes (e.g., Chen and Yuen, 2009; Collamore, 2009; Shen et al., 2009; Weng et al., 2009; Zhang et al., 2009; Yi et al., 2011). Alternatively, Chen (2011), Yang et al. (2012), Yang and Hashorva (2013), and Yang and Konstantinides (2015) allow for dependences between the insurance and financial risk processes assuming each process is i.i.d.

The purpose of this paper is to contribute to the existing literature in various relevant ways. First, in contrast to Braun et al. (2017), we study a non-life insurer instead of a life insurer and do not use the standard model. Moreover, we extend the analysis in Eling et al. (2009) and Braun et al. (2017) by focusing on the firm's shareholder value based on preferences under risk- and value-based management decisions and by considering the policyholders' willingness to pay when deriving admissible asset allocations under solvency constraints. The insurer's surplus process is modeled by a discrete time representation of the Sparre-Andersen

(1957) model in the presence of risky investments and reinsurance similar to Huang et al. (2016), Jin et al. (2016), and Yang and Zhang (2006) in order to derive the discrete one-year ruin probability as used in Solvency II. We thus combine approaches in the economic literature (e.g., Braun et al., 2017, Eling et al., 2009) and the applied math literature on ruin theory by generalizing the economic model for the total claims payments using a probabilistic framework with stochastic frequency and severity and by applying this model to a shareholder value optimization problem. Moreover, the dependence structure between the insurance and financial risk processes is modeled similar to Chen (2011) and Yang and Konstantinides (2015) by means of copulas and we consider the possibility of purchasing reinsurance. We further extend the economic and applied math literature by explicitly taking into account the policyholders' willingness to pay in our model framework when deriving admissible asset allocations under solvency constraints that are determined by the ruin probability. We thereby generalize the approach in Lorson et al. (2012) and Zimmer et al. (2014) and model the achievable premium in the presence of market discipline as a function of the insurer's safety level and the policyholders' risk assessment. In contrast to the mentioned applied math literature, we study the optimal investment problem regarding the shareholders' expected utility while simultaneously controlling for the ruin probability given several influencing factors. In this setting, we follow Eling et al. (2009) and link the admissible risk-return combinations of the insurer's asset portfolio (i.e. those that are allowed under solvency constraints) to allocation opportunities actually attainable at the capital market using Tobin (1958)'s capital market line.

In a numerical analysis, we first investigate the impact of a default-risk-driven premium income on the maximum shareholder value for an insurer facing solvency constraints and, second, the impact of changing the reported target solvency status on the shareholder value in the presence of market discipline. One main finding is that the consideration of policyholders' willingness to pay based on the ruin probability is of great relevance when deriving optimal risk-return asset allocations under solvency constraints (e.g., the target ruin probability implying a higher shareholder value differs for various risk sensitivities of the policyholders) and that reinsurance can considerably impact the results, depending on the level of the policyholders' risk sensitivity.

The remainder of this paper is structured as follows. Section 2 presents the model framework for a non-life insurer, while Section 3 focuses on the derivation of attainable and admissible risk-return combinations as well as shareholder value maximization. An application of the developed approach based on several underlying simplified assumptions with sensitivity analyses for various policyholders' risk sensitivities, proportional reinsurance, risk-free interest rate, (non-linear) asset-liability dependences, and claims distributions is provided in Section 4. The last section summarizes our main findings.

2. MODEL FRAMEWORK

2.1 Model foundations

We consider the problem of ruin in a risk model of a non-life insurer. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space equipped with a family of increasing (over time) σ-fields $(\mathcal{F}_t)_{t\geq 0}$, where $\mathcal{F}_t$ represents the information available at time t. Since Solvency II requires insurers to derive solvency capital requirements such that the one-year ruin probability does not exceed a certain threshold, we consider a discrete-time model for the insurer's surplus process $U_n, n \in \mathbb{N}$, accumulated until the end of year $(n-1, n]$ in the presence of risky investments with return r_n and with claims S_n as well as a constant premium intensity $p > 0$. The claims are subject to payments from reinsurance for a reinsurance premium Π_n, implying remaining claims payments $f(S_n, S_{n-1})$ (depending on the type of reinsurance), such that

$$U_n = (1+r_n)(U_{n-1} + p - \Pi_n) - f(S_n, S_{n-1}), \quad U_0 = u, \; n \in \mathbb{N}, \tag{1}$$

where $u > 0$ is the initial equity capital of the insurer. The following assumptions are made:

Assumption 1: *Total claims payments S_n are modeled by the discrete-time representation of a compound renewal process.*

Assumption 1 is motivated by the Sparre-Andersen (1957) model (see Andersen, 1957), which is widely used in risk theory due to its tractability, see, e.g., Li and Garrido (2004), Wu et al. (2007), Li (2012), and Jin et al. (2016).[4]

Assumption 2: *The financial process generating the return on investment from year n-1 to year n is defined as a sequence of normally distributed i.i.d. random variables $r_n, n \in \mathbb{N}$ with $r_n \sim N(\mu_r, \sigma_r^2)$.*

This distribution assumption is made in order to be consistent with the assumptions of Tobin (1958)'s capital market line and can be justified by the Euler discretization of the return process as used in Hult and Lindskog (2011) and Kabanov and Pergamenshchikov (2016).

Assumption 3: *The insurance process and the financial process (more precisely $S_n - S_{n-1}$ and r_n) form a sequence of i.i.d. random variables, and the dependence structure is given by a copula C.*

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[4] Regarding risk models used in non-life insurance, we further refer to Mikosch (2009), Asmussen and Albrecher (2010), and Beard et al. (2013) for an introduction to risk theory.

Typically, the insurance process and the financial process are assumed to be independent from each other as laid out in the introduction. To drop this assumption, we use the concept of copulas similar to Chen (2011) and Yang and Konstantinides (2015) as will be laid out in more detail later.

Assumption 4: *For each n, the insurer purchases a proportional (quota-share) reinsurance contract with retention level* $q_n \in [0,1]$*, i.e.,* $f(S_n, S_{n-1}) = q_n \cdot (S_n - S_{n-1})$ *with corresponding reinsurance premium rate* $\Pi_n = \pi(q_n)$.

This reinsurance contract provides coverage over the entire range of claims, which might be more expensive compared to contracts that reinsure losses above a certain threshold, e.g., an aggregate excess of loss reinsurance contract (e.g., Gatzert and Kellner, 2014), but is a common choice in actuarial literature (e.g., Diasparra and Romera, 2010; Liang et al., 2012; Huang et al., 2016). As a robustness test, we later also consider an aggregate excess of loss reinsurance contract, whose payment is given by $\min\left(\max\left((S_n - S_{n-1}) - d_n, 0\right), l_n\right)$ with the attachment of the company loss d_n and layer l_n, implying remaining claims payments $f(S_n, S_{n-1}) = S_n - S_{n-1} - \min\left(\max\left((S_n - S_{n-1}) - d_n, 0\right), l_n\right)$.

In addition, as most insurers issue financial reports once a year due to the one-year perspective of Solvency II, for instance, we focus on an annual frequency, leading to

$$U_1 = (1+r_1)\left(U_0 + p - \pi(q_1)\right) - q_1 S_1, \quad U_0 = u, \tag{2}$$

with a one-year ruin probability *RP* that should not exceed 0.50%,

$$RP = \mathbb{P}(U_1 < 0 \mid U_0 = u) = \mathbb{P}\left((1+r_1)\left(u + p - \pi(q_1)\right) - q_1 S_1 < 0\right) \leq 0.50\%.$$

2.2 Premium income and policyholders' risk sensitivity

In ruin theory, the premium income of the surplus process is mostly modeled by a constant rate $p > 0$ (Mikosch, 2009). In what follows, we use the expected value principle to determine the premium rate

$$p = (1+\delta) \cdot \mathbb{E}[S_1],$$

for some positive premium loading δ, which results in a premium income that on average exceeds the total claims payments and can thus absorb fluctuations of the claims amount. However, if the insurer imposes an overly large premium loading, it becomes less competitive compared to other premiums offered at the market. Thus, we assume a fixed premium loading that is exogenously given in the sense of a market standard in the insurance industry. In addition, for robustness test purposes we also consider the standard deviation principle, where the loading is determined by a portion η of the standard deviation of the claims payments

$$p = \mathbb{E}[S_1] + \eta \cdot \sqrt{\mathbb{V}[S_1]}, \quad \eta > 0.$$

Moreover, according to experimental and empirical research (Wakker et al., 1997; Zimmer et al., 2009; Zimmer et al., 2014), an insurer's default risk can strongly impact customer demand, with lower safety levels leading to a considerable reduction of the achievable premiums below the actuarially fair premium. Whereas expected utility theory suggests that a small increase in the ruin probability should only reduce the policyholders' willingness to pay in a marginal way, Wakker et al. (1997) observe that the actual willingness to pay decreases sharply in the context of "probabilistic insurance", i.e. insurance contracts with a small non-zero ruin probability. They explain this phenomenon based on Kahneman and Tversky's (1979) prospect theory, according to which individuals tend to overweigh small probability events. Hence, we adapt our model framework in order to take into account the policyholders' willingness to pay as a reaction to the reported solvency status depending on their risk sensitivity, which is especially relevant under Solvency II's Pillar 3. In the presence of market discipline, customers could be influenced by the reported solvency status when comparing it to other insurers. A potentially induced change of customer demand could thereby incentivize insurers to achieve a higher solvency status than the required regulatory minimum. To take this aspect into account, we use and extend the approach in Lorson et al. (2012), who calculate the premium reduction compared to the premium offered by a default-free insurer as a log-linear function of the reported one-year ruin probability *RP*, given by

$$PR(RP) = a \cdot \ln(RP) + b, \quad RP \in (0,1]. \tag{3}$$

However, since the policyholders' risk assessment is not known, i.e. how well-informed the policyholders are and if they can assess the numerical ruin probability correctly, the actual premium reduction might differ. To take into account the policyholders' risk sensitivity, we thus extend this model and not only link the premiums paid in $t = 0$ to the insurer's reported ruin probability $RP = \alpha$, but also to the policyholders' risk sensitivity represented by a scaling parameter ξ similar to Gatzert and Kellner (2014). In case of the expected value principle, for instance, the premium payments in $t = 0$, $P(RP,\xi)$, are then given by

$$P(RP,\xi) = p \cdot \max\left(1 - \xi \cdot PR(RP), 0\right), \quad \xi \geq 0,\ RP = \alpha \in (0,1], \tag{4}$$

where $RP = \alpha$ represents the reported one-year ruin probability and *PR* the premium reduction function in Equation (3). Note that the functional form of the latter is chosen for illustration purposes and can also be adjusted (e.g. depending on the type of contract). For $\xi = 1$, we obtain the risk sensitivity modeled in Lorson et al. (2012).

We further assume that the reinsurance premium is also calculated according to the expected value or the standard deviation principle, respectively, resulting in

$$\pi(q_1) = (1+\delta_{re}) \cdot (1-q_1) \cdot \mathbb{E}[S_1] \text{ or } \pi(q_1) = (1-q_1) \cdot \mathbb{E}[S_1] + \eta \cdot (1-q_1) \cdot \sqrt{V[S_1]}.$$

3. Optimal Attainable and Admissible Risk-Return Combinations

To derive optimal risk- and value-based management decisions, we next consider minimum solvency requirements in a risk-return (asset) context (here: expected return and standard deviation of assets that are compatible with the solvency requirements) and link these "admissible" risk-return combinations of the insurer's asset portfolio to allocation opportunities actually "attainable" at the capital market as is done in Eling et al. (2009). In contrast to previous work, however, we generalize this model by explicitly taking into account the policyholders' willingness to pay depending on the insurer's solvency level, dependencies between assets and liabilities as well as the effect of reinsurance contracts.

3.1 Capital market line: "Attainable" risk-return combinations

To identify the risk-return profiles (σ_r, μ_r) that are actually attainable at the market, we follow the classical approach of Tobin (1958) and derive the capital market line with the risk-free interest rate r_f and the Sharpe ratio of the market portfolio m:

$$CML: \begin{cases} \mathbb{R}_0^+ \to \mathbb{R} \\ \sigma_r \mapsto \mu_r = r_f + m \cdot \sigma_r, \end{cases} \tag{5}$$

representing the set of efficient risk–return combinations (σ_r, μ_r) given risk-free lending and borrowing. Note that a derivation of efficient risk-return combinations that takes into account the insurer's liabilities can be found in Brito (1977) and Mayers and Smith (1981).

3.2 Solvency lines: "Admissible" risk-return combinations

We next fix the insurer's desired ruin probability RP at time t = 1 to a prescribed maximum value α denoted "target ruin probability" (note that the risk measure can as well be changed as is done in Eling et al. (2009), but closed-form solutions may not be derivable). At time t = 0 the insurer communicates the maximum value α to the policyholders, who in turn adapt their willingness to pay based on this information. Based on the resulting amount of premium income (Equation (4)), the insurer makes the actual investment decision (i.e., chooses a risk-return asset combination), which must be compatible with the announced target level α to preserve the policyholders' trust and confidence. Thus, the real ruin probability *RP* must not exceed the reported target ruin probability α, i.e.

$$RP = \mathbb{P}\Big(\big(1+r_1\big)\big(u + P\big(\alpha,\xi\big) - \pi\big(q_1\big)\big) - q_1 S_1 < 0\Big) \overset{!}{\leq} \alpha. \tag{6}$$

For a given σ_r, we can solve for μ_r and obtain the so-called "solvency lines" *SolvL*.[5] Thus, the solvency lines are (σ_r, μ_r) -combinations that satisfy $RP \overset{!}{=} \alpha$, which implies that for a given risk the expected return μ_r needs to be at least as high to ensure that the ruin probability does not exceed the given target level α.

3.3 Maximizing shareholder value given attainable and admissible risk-return combinations

Among the typical firm objectives is the creation of shareholder value through risk- and value-based decision making regarding assets and liabilities. Toward this end, our model can be used for deriving the shareholders' maximum expected utility while maintaining a minimum (typically regulatory required) solvency level in order to protect the policyholders. Let Ψ denote the shareholders' preference function to determine their expected utility dependent on the (σ_r, μ_r)-combination of the insurer's asset allocation. In its decisions regarding the investment portfolio, the insurer can consider the shareholders' preferences within the limits of attainable and admissible investment opportunities, leading to the optimization problem

$$\max\left\{\Psi\left(\sigma_r,\mu_r\right):\left(\sigma_r,\mu_r\right)\in\Upsilon\right\}, \tag{7}$$

with (σ_r, μ_r) element of the feasible set

$$\Upsilon = \left\{\left(\sigma,\mu(\sigma)\right):\sigma\in\left[0,\infty\right),\mu(\sigma)\in\left[SolvL(\sigma),CML(\sigma)\right],SolvL(\sigma)\leq CML(\sigma)\right\}. \tag{8}$$

The constraints ensure that only risk-return combinations are taken into consideration that are attainable at the market (i.e. on or below the *CML*) and that are admissible according to solvency restrictions (i.e. on or above the *SolvL*). The optimal (σ_r, μ_r)-combination that solves the optimization problem (7) depends on the actual preference function Ψ. Since decisions based on any utility function can be well approximated by assuming mean-variance preferences[6] (Kroll et al. (1984) and Markowitz (2014) in general, and Bäuerle, 2005; Bai and

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5 The notion follows Eling et al. (2009) who derived closed-form solutions for a one-year period model using a normal power approximation for the difference between assets and liabilities.

6 Full compatibility of mean-variance analysis and expected utility theory is given in case of a normal distribution or a quadratic utility function. Even if these conditions do not hold, decisions of any utility function can be well approximated by those based on mean-variance preferences as shown in Kroll et al. (1984) or Markowitz (2014), where the necessary and sufficient condition for the practical use of mean–variance analysis is

Zhang, 2008; Bi and Guo, 2012 in ruin theory), we assume that Ψ is based on the expected value and variance of the shareholders' wealth at the end of the period (e.g., Gatzert et al., 2012; Braun et al., 2017)

$$\Psi : \begin{cases} \mathbb{R}_0^+ \times \mathbb{R} \to \mathbb{R} \\ (\sigma_r, \mu_r) \mapsto \mathbb{E}(U_1) - \frac{k}{2} \cdot \mathbb{V}(U_1), \end{cases} \tag{9}$$

where k represents the risk aversion coefficient, with $k > 0$ implying a risk-averse attitude.[7] Let r_1 and S_1 have mean and variance μ_r and σ_r^2 and μ_S and σ_S^2, respectively, with Pearson's correlation coefficient $Cor(r_1, S_1)$. For the surplus U_1 (i.e. equity capital) at time $t = 1$, the expected value and the variance are given by

$$\mathbb{E}[U_1] = (1+\mu_r)\big(u + P(\alpha,\xi) - \pi(q_1)\big) - q_1\mu_S\,, \tag{10}$$

$$\begin{aligned} \mathbb{V}[U_1] = {} & \big(u + P(\alpha,\xi) - \pi(q_1)\big)^2 \cdot \sigma_r^2 + q_1^{\,2} \cdot \sigma_S^2 \\ & -2 \cdot \big(u + P(\alpha,\xi) - \pi(q_1)\big) \cdot q_1 \cdot \sigma_r \cdot \sigma_s \cdot Cor(r_1, S_1). \end{aligned} \tag{11}$$

Lemma 1: *Under the rational assumption that the insurer does only buy a reinsurance contract with reinsurance premiums being less than the insurer's premium income and initial equity capital (referred to as Assumption 5), the maximization problem* $\max_{(\sigma_r,\mu_r)} \Psi$ *in* (7) *is solved by the point on the capital market line*

$$\big(\tilde{\sigma}_r, CML(\tilde{\sigma}_r)\big), \tag{12}$$

with $\tilde{\sigma}_r = \max\Big(\min\{\sigma_r : (\sigma_r,\mu_r) \in \Upsilon\}, \min\big(\sigma_r^*, \max\{\sigma_r : (\sigma_r,\mu_r) \in \Upsilon\}\big)\Big)$ *and*

$\sigma_r^* = \big(m + k \cdot q_1 \cdot \sigma_S \cdot Cor(r_1, S_1)\big) \big/ \Big(k \cdot \big(u + P(\alpha,\xi) - \pi(q_1)\big)\Big)$ (for the proof, see Appendix).

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that "a careful choice from a mean–variance efficient frontier will approximately maximize expected utility for a wide variety of concave (risk-averse) utility functions". For further discussion, see also Loistl (2014) and Markowitz (2015).

7 Note that the case of shareholders' limited liability can also be considered for the preference function, which implies asymmetric expressions. Derivations of closed-form expressions and further analyses in the present setting under limited liability can be obtained from the authors upon request. In this case, the premium level should correspond to the value of indemnity payments less the default put option arising from the shareholders' limited liability, which we have not taken into account in pricing as we assume (in the sense of a behavioral-type approach) that policyholders are willing to pay a premium that depends on their risk sensitivity and that may thus exceed the expected payoff. These assumptions can also be changed, but closed-form solutions for the optimization problem as well as the solvency lines may no longer be possible, such that one has to revert to numerical simulation approaches.

Note that at the end of the year, the insurer again faces the same decision problem of an optimal investment strategy and can re-apply the model. Given the new equity capital $U_1 = u$ and the remaining input parameters, which do not necessarily have to be identical to the previous year values, there might be changes in the feasible set and the solution of the shareholder value maximization problem. However, as the asset composition is more easily adjusted than the liability side or raising additional equity capital, solving the optimization problem and making the necessary adjustments appears doable. More insight in regard to the impact of the initial equity capital u can be obtained when considering the following: If $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right) \in \Upsilon$, shareholder value is maximized by $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$ and the derivative of $\Psi\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$ with respect to u (see Appendix) is

$$\frac{\partial \Psi\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)}{\partial u} = 1 + r_f ,$$

implying that the maximum shareholder value is linearly increasing in u with slope $1+r_f$. If $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right) \notin \Upsilon$, the boundary point of the feasible set next to $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$ maximizes shareholder value. A graphical contour plot analysis of the solvency lines indicates that a higher initial equity capital u implies a downward shift of the solvency lines and thus the boundary point comes closer to $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$, therefore also implying a higher maximum shareholder value, since $\Psi\left(\sigma_r, CML\left(\sigma_r\right)\right)$ is increasing towards $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$.

The effect of further parameter changes on the feasible set and the resulting maximum shareholder value can also be assessed by sensitivity analyses as shown in Section 4.

3.4 Special case: The model in a Gaussian environment

We now consider a special case that bridges the gap to the simpler but more tractable model frameworks in economics that do not differentiate between the number and size of claims, as is done in, e.g., Eling et al (2009) and Braun et al. (2017), which allows to calculate the solvency lines and the shareholder preference function Ψ (Equations (6) and (9)) analytically.

The one-period model of the non-life insurer's surplus in $t = 1$ is given by Equation (2) taking into account the policyholders' willingness to pay, i.e.

$$U_1 = \left(1 + r_1\right)\left(U_0 + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right) - q_1 S_1, \quad U_0 = u .$$

We make use of the central limit theorem to approximate the total claims amount using the normal distribution for the Sparre-Andersen model with finite variance of the inter-arrival times and claims sizes (e.g., Embrechts et al., 2013; Mikosch, 2009), i.e.,

$$S(t) \overset{approx.}{\sim} N\left(\mathbb{E}\left[S(t)\right],\mathbb{V}\left[S(t)\right]\right).$$

The approximation by a normal distribution based on the central limit theorem is quite good in the center of the distribution, but not as good in case of tail probabilities. Nevertheless, the Gaussian dynamic allows obtaining first insight due to its simple calculation.

The bivariate distribution function of the asset return r_1 and the total claims amount S_1 is fully captured by their marginal distribution functions that are both normally distributed with $r_1 \sim N\left(\mu_r,\sigma_r^2\right)$ and $S_1 \overset{app}{\sim} N\left(\mu_S,\sigma_S^2\right)$ and their copula C (Assumption 3). In particular, according to Sklar's Theorem (Sklar, 1959), for any multivariate distribution function F on $\mathbb{R}^d$ with univariate margins F_i, a function $C:[0,1]^d \to [0,1]$ exists, such that $F(x)=C\left(F_1(x_1),...,F_d(x_d)\right) \forall x \in \mathbb{R}^d$. To model the dependence structure, one can use elliptical copulas such as the Gaussian copula or the t-copula, which only capture elliptical symmetry, or Archimedean and Hierarchical Archimedean copulas to obtain asymmetric dependencies, for instance (McNeil et al., 2005). In this paper, we first focus on the (elliptical) 2-dimensional Gaussian copula (before changing this assumption, see next section) given by

$$C_\rho^{\text{Gauss}}(u_1,u_2)=\Phi_\rho\left(\Phi^{-1}(u_1),\Phi^{-1}(u_2)\right),$$

where $\Phi_\rho(\cdot)$ stands for the distribution function of the bivariate standard normal distribution with linear correlation ρ and Φ for a univariate normal distribution function (McNeil et al., 2005).

Note that in case of normally distributed marginals, (r_1,S_1) are bivariate normally distributed and the parameter of the Gaussian copula equals the linear correlation, i.e., $Cor(r_1,S_1)=\rho$. Therefore, $\mathbb{E}(U_1)$ and $\mathbb{V}(U_1)$ and thus the shareholder preference function Ψ (Equations (9)–(11)) can be calculated analytically, and the solvency lines can be derived as follows:

Lemma 2: *If r_1 and S_1 are normally distributed with $r_1 \sim N\left(\mu_r,\sigma_r^2\right)$ and $S_1 \sim N\left(\mu_S,\sigma_S^2\right)$ with a Gaussian copula C_ρ^{Gauss} with parameter ρ, the solvency line SolvL with target ruin probability α is given by*

$$\mu_r = SolvL(\sigma_r)=\frac{q_1\mu_S - N_\alpha \cdot \sqrt{\mathbb{V}[U_1]}}{u+P(\alpha,\xi)-\pi(q_1)}-1. \tag{13}$$

Proof: Given the assumptions above, the resulting surplus (i.e. equity capital) at time $t = 1$ is also normally distributed with

$$U_1 = (1+r_1)\left(u + P(\alpha,\xi) - \pi(q_1)\right) - q_1 S_1 \sim N\left(\mathbb{E}[U_1], \mathbb{V}[U_1]\right),$$

with parameters given by Equations (10) and (11) (and $Cor(r_1, S_1) = \rho$). The real ruin probability *RP* not exceeding the reported target ruin probability α can thereby be expressed as

$$RP = \mathbb{P}(U_1 < 0) = \Phi\left(-\frac{\mathbb{E}[U_1]}{\sqrt{\mathbb{V}[U_1]}}\right) \overset{!}{\leq} \alpha.$$

With N_α denoting the α-quantile of the standard normal distribution and for a given σ_r (reflected in the variance of equity capital $\mathbb{V}[U_1]$), we can solve for μ_r and obtain the solvency line *SolvL* in Equation (13). □

3.5 Further robustness tests regarding distributional assumptions and dependencies

For robustness test purposes, we also model the total claims by a lognormal distribution to study a skewed distribution setting, and we additionally vary the dependence assumption by using a t-copula as an alternative to the Gaussian copula, which allows taking into account tail dependencies. The t-copula is determined by the bivariate t-distribution

$$C^t_{\nu,R}(u_1, u_2) = t_{\nu,R}\left(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2)\right),$$

where t_ν is the distribution function of a standard univariate t-distribution with degrees of freedom *v*, and $t_{\nu,R}$ is the distribution function of a bivariate standard t-distribution with degrees of freedom *v* and linear correlation R. In contrast to the Gaussian copula that does not allow for tail dependence and might thus underestimate joint risks, the t-copula exhibits flexible tail dependence. All four tail dependence coefficients of the t-copula are non-zero for any correlation (Kurowicka and Joe, 2011): The upper tail and lower tail dependence measuring the strength of dependence in the upper and lower tail of a bivariate distribution, respectively, i.e., in the corner of the lower-left quadrant or upper-right quadrant; but also the negative lower-upper tail dependence coefficient and negative upper-lower tail dependence coefficient ("negative" meaning extremes in opposite directions, i.e., dependence in the corner of the upper-left quadrant or lower-right quadrant) are non-zero, which generally describe the limiting probability that a random variable falls below (exceeds) extremely low (large) values given that another random variable exceeds (falls below) extremely large (low) values.[8]

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[8] Due to reflection symmetry, the first two (upper and lower tail dependence coefficient) and the second two (negative lower-upper tail and negative upper-lower tail dependence coefficient) are equal to each other. If more flexibility is needed, Kurowicka and Joe (2011) recommend Vine copulas.

To ensure comparability with the Gaussian setting in Section 3.4, we calibrate the lognormally distributed claims to the same mean μ_S and standard deviation σ_S; for the copulas we fix Kendall's tau τ and calculate the parameters ρ and R using $\tau_{Kendall} = 2 \cdot \arcsin(\rho)/\pi$ for the Gaussian and $\tau_{Kendall} = 2 \cdot \arcsin(R)/\pi$ for the t-copula. Since the closed-form solution of the solvency line in Equation (13) relies on a Gaussian setting, the solvency line (Equation (6)) must be derived numerically in these cases. Moreover, Pearson's correlation coefficient is not invariant under non-linear strictly increasing transformations of the margins, so the correlation $Cor(r_1,S_1)$ in Equation (11) does not equal the parameter ρ of the Gaussian copula in case of lognormally distributed claims and R of the t-copula, respectively, implying that $Cor(r_1,S_1)$ as input of the preference function Ψ (Equation (9)) is calculated by Monte Carlo Simulation.

4. NUMERICAL ANALYSES

We now conduct numerical analyses in order to study the impact of various economic parameters on an insurer's optimal risk- and value-based management decisions and to derive respective key drivers. The base case relies on the Gaussian setting in Section 3.4, where the solvency line, the preference function Ψ and the optimal solution (Equations (6), (9), (12) and (13)) are derived analytically. For the robustness tests in Section 3.5, the solvency lines are determined by Monte Carlo simulation with 10^8 sample paths and Brent's (1973) method as a root searching algorithm. $Cor(r_1,S_1)$ as input of the preference function Ψ and the optimal solution (Equations (9) and (12)) is also derived by Monte Carlo simulation with 10^8 sample paths. For the simulation of the copulas, we use the "copula" R-package of Hofert et al. (2017).[9]

4.1 Input parameters

Input parameters of the base case are summarized in Tables 1 to 3. Table 1 is thereby based on the parameters of a German non-life insurer estimated in Eling et al. (2009) (except for the newly introduced parameters asset-claims correlation, premium loadings, retention level of proportional reinsurance, and risk aversion parameter, which were subject to robustness tests). The parameters of the premium reduction function *PR* (in comparison to the default-free premium *p*) rely on the estimation by Lorson et al. (2012)[10].

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9 For Monte Carlo simulation, we used a sufficiently high number of sample paths and ensured that the results remain stable for different sets of random numbers. Moreover, the analytic solution of shareholder value optimization problem (Equation (12)) is also checked by the Differential Evolution algorithm in the "DEoptim" R-package of Ardia et al. (2016).

10 In Lorson et al. (2012) a default-free insurer corresponds to a ruin probability of 0.01%. Since the estimation is based on very few data points taken from an empirical study in Zimmer et al. (2009), they also consider an

Table 1: Input parameters (base case)

Available equity capital at time 0	$U_0 = u$	175
Expected value of claims	μ_S	1,171
Standard deviation of claims	σ_S	66
Parameter of Gaussian copula of stochastic return of assets and claims	ρ	0
Premium loading for an insurer without default risk	δ	5%
Retention level of proportional reinsurance	q_1	1
Premium loading for proportional reinsurance	δ^{re}	5%
Policyholders' risk sensitivity	ξ	0, 0.3, 1
Parameters of the premium reduction function	a	0.0419
	b	0.3855
Maximum value of ruin probability (target ruin probability)	α	0.50%
Shareholders' risk aversion coefficient	k	0.005

Table 2: Descriptive statistics (annualized) for monthly return time series from January 2004 to November 2015 from the Datastream database

Asset class	Index	Description	μ_i	σ_i
Money market	JPM Euro Cash 3 Months (1)	Money market in the EMU = r_f	2.04%	-
Stocks	MSCI World ex EMU (2)	Worldwide stocks without the EMU	8.11%	47.06%
	MSCI EMU ex Germany (3)	Stocks from the EMU without Germany	5.92%	61.71%
	MSCI Germany (4)	Stocks from Germany	8.55%	68.16%
Bonds	JPM GBI Global All Mats. (5)	Worldwide government bonds	4.58%	28.83%
	JPM GBI Europe All Mats. (6)	Government bonds from Europe	5.30%	14.46%
	JPM GBI Germany All Mats. (7)	Government bonds from Germany	4.74%	14.57%
	IBOXX Euro Corp. All Mats (8)	Corporate bonds from Europe	4.31%	13.71%
Real estate	GPR General World (9)	Real estate worldwide	9.11%	51.67%
	GPR General Europe (10)	Real estate in Europe	6.71%	36.53%
	GPR General Germany (11)	Real estate in Germany	2.60%	9.22%

Furthermore, the capital market line is calibrated based on monthly time series from January 2004 to November 2015 of benchmark indices from the Datastream database that illustrate the available investment opportunities. Each benchmark measures the total investment returns for its asset on a Euro basis including coupons and dividends where applicable. As is done in Eling et al. (2009), we consider 11 indices with different regional focus in the four asset classes stocks, bonds, real estate, and money market instruments where insurers typically invest in. The expected return of the JPM Euro Cash 3 Month from January 2004 to November 2015 (2.04%) is thereby used as a proxy for the risk-free rate as is done in Eling et al. (2009), and since a constant, maturity independent risk-free interest rate does not exist in practice, we

upper and lower bound for the premium reduction to take into account the variability, and we further conduct sensitivity analyses, using these parameters as a starting point.

later conduct sensitivity analyses in this regard. The empirical risk-return profiles for all considered assets are given in Table 2 and the associated variance-covariance matrix is displayed in Table 3. For the base case, the resulting capital market line is then given by

$$\mu_r = 0.0204 + 0.34 \cdot \sigma_r. \tag{14}$$

Table 3: Variance-covariance matrix (annualized) for monthly return time series in Table 2

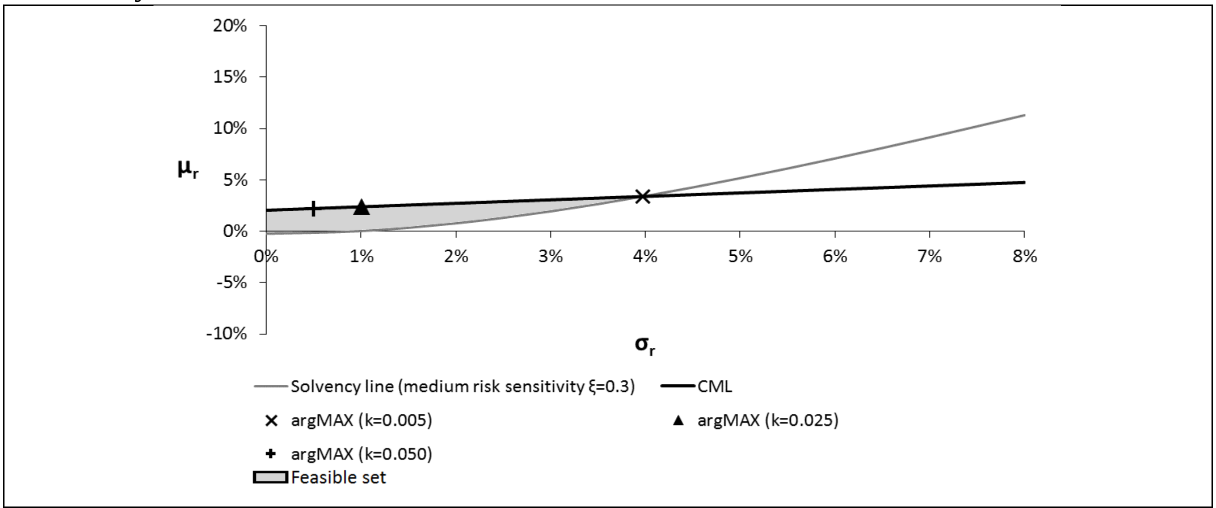
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
(2)		0.221	0.233	0.263	-0.018	-0.004	-0.015	0.022	0.189	0.110	0.021
(3)			0.381	0.383	-0.075	-0.010	-0.033	0.031	0.211	0.158	0.028
(4)				0.465	-0.078	-0.018	-0.036	0.029	0.228	0.156	0.031
(5)					0.083	0.029	0.032	0.004	0.001	-0.015	-0.001
(6)						0.021	0.019	0.010	0.014	0.007	0.002
(7)							0.021	0.008	0.003	-0.003	0.000
(8)								0.019	0.036	0.026	0.004
(9)									0.267	0.155	0.027
(10)										0.133	0.024
(11)											0.008

4.2 The impact of the shareholders' risk aversion

Figure 1 displays *attainable* risk-return combinations based on market restrictions concerning the asset allocation as reflected by the capital market line (*CML*, solid black line, Equation (14)). Moreover, *admissible* combinations are presented as defined by the solvency line in Equations (6) and (13) for the target ruin probability α = 0.50% as required by Solvency II. Hence, for its investment strategy the insurer must choose risk-return combinations on or above the respective solvency line and on or below the capital market line, which represent the set ϒ of attainable and admissible risk-return combinations (Equation (8)) highlighted by the grey area.

Figure 1 shows that the optimal asset allocation solving the shareholder maximization problem (Equation (7)) strongly varies when changing the shareholders' risk aversion coefficient k and emphasizes that a higher risk aversion coefficient implies a less risky asset allocation, associated with a lower maximum shareholder value in terms of the considered preference function: Ψ(4.0%, 3.4%) = 201 in case of k = 0.005 (optimal risk-return combinations marked by **X**), Ψ(1.0%, 2.4%) = 148 in case of k = 0.025 (marked by ▲) and Ψ(0.5%, 2.2%) = 93 in case of k = 0.050 (marked by +). Moreover, for k = 0.025 and k = 0.050, the global maximum is given by the local maxima $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$ in the feasible set, whereas for k = 0.005 the optimal point $\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$ is outside (on the right side) of the feasible set, such that the right boundary point (intersection point of capital market line and solvency line) maximizes shareholder value (Lemma 1).

Figure 1: Attainable (capital market line *CML*) and admissible ("solvency line") risk-return combinations for target ruin probability α = 0.50% and medium (ξ = 0.3) policyholder risk sensitivity for various shareholder risk aversion coefficients k = 0.005, 0.025, 0.050

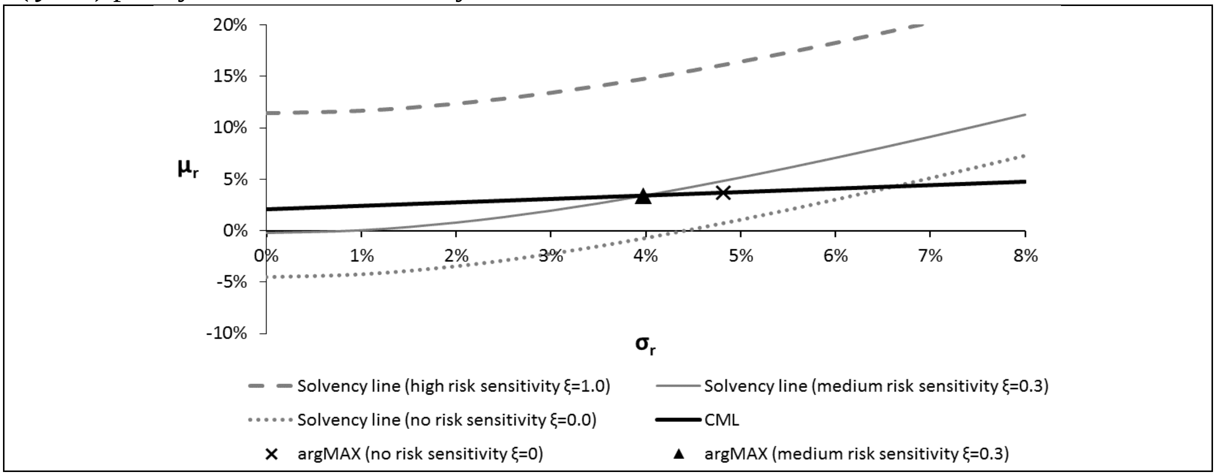


4.3 The impact of policyholder risk sensitivity and the insurer's safety level on admissible risk-return asset profiles

Figure 2 shows the case where the insurer faces fixed solvency constraints (ruin probability α = 0.50%) and emphasizes the impact of the default-risk-driven premium income on the maximum shareholder preference value. In particular, the admissible risk-return combinations (solvency lines) strongly depend on the policyholders' risk sensitivity. For higher risk sensitivities (going from ξ = 0 to 1 in the considered example), the solvency lines are shifted upwards for a given ruin probability, until for ξ = 1 (high risk sensitivity) the solvency line lies above the *CML*, such that no possible allocation opportunities remain. The results emphasize that a more risk sensitive assessment of the solvency status reduces policyholder demand and hence the achievable premium income, implying that the insurer has considerably less flexibility for its asset allocation to fulfill the solvency requirements. Moreover, the maximum shareholder value in terms of the considered preference function Ψ is given by Ψ(4.8%, 3.7%) = 263 in case of no risk sensitivity (marked by **X**) and Ψ(4.0%, 3.4%) = 201 in case of medium risk sensitivity (marked by ▲). Thus, a higher risk sensitivity implies a less risky investment and a lower maximum shareholder preference value.

We also observe in Figure 2 that in the case without policyholder risk sensitivity, expected asset returns may even be negative for low standard deviations, and the insurer would still satisfy the required safety level due to sufficient equity capital and premium loadings. Further sensitivity analysis emphasizes that reducing the initial equity capital or lowering the premium loading implies an upward shift of the solvency lines, such that negative expected returns are no longer permitted.

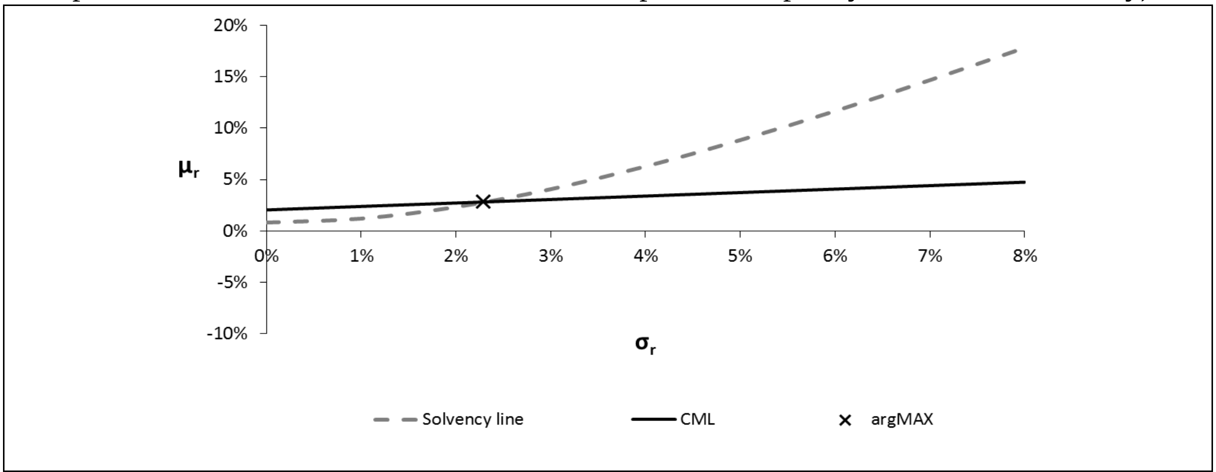
Figure 2: Attainable (capital market line *CML*) and admissible (“solvency line”) risk-return combinations for target ruin probability α = 0.50% and no (ξ = 0), medium (ξ = 0.3) and high (ξ = 1) policyholder risk sensitivity



In the presence of market discipline, the insurer faces the challenge to balance the premium reduction driven by a higher default risk and the higher expected return on investment associated with a higher risk taking. We thus next address the situation where the insurer chooses the target ruin probability when maximizing shareholder preference value given a default-risk-driven premium income. While Figure 2 assumed a target ruin probability of α = 0.50%, Figure 3 displays the setting of a “default-free” insurer, as for α = 0.01% the premium reduction estimated by Lorson et al. (2012) is zero, and the solvency lines for various levels of policyholders’ risk sensitivity coincide. As can be seen when comparing Figures 3 and 2, there is a considerable gap between the solvency lines depending on the target ruin probability. In particular, the solvency line in Figure 2 in the case without risk sensitivity ξ = 0 (lowest dotted line) shifts upward for the lower α in Figure 3, since it is easier for the insurer to fulfill the solvency requirements. In contrast, the solvency line for a high policyholder risk sensitivity (ξ = 1) shifts downward, since the information of the lower ruin probability considerably increases the willingness to pay by risk-sensitive policyholders and the premium income strongly increases (Equation (4)). Thus, a higher ruin probability along with high risk sensitivity can lead to considerable difficulties in maintaining the desired solvency level and thus also in generating shareholder preference value.

The maximum shareholder value in case of α = 0.01% (and thus independent of the policyholders’ risk sensitivity) is given by Ψ(2.3%, 2.8%) = 260 (marked by **X**). When comparing this to the values for α = 0.05% in Figure 2, we can see that in case of medium (ξ = 0.3) risk sensitivity (Ψ(4.0%, 3.4%) = 201) and high (ξ = 1) risk sensitivity (no feasible set) the shareholder preference value is higher when choosing a target ruin probability corresponding to the "absence" of default risk, which is in line with Zimmer et al. (2014), who focus on the effects stemming from insurance demand on maximum shareholder value. However, if policyholders are not risk sensitive ($\xi = 0$), one obtains the opposite result. In particular, the shareholder preference value can be slightly increased to $\Psi(4.8\%, 3.7\%) = 263$ when choosing a higher target ruin probability of $\alpha = 0.05\%$ that allows riskier investment strategies, since the policyholders do not sanction the lower solvency level. Overall, this strongly emphasizes that it is crucial to take into account the policyholders' risk sensitivity and thus the purchase behavior depending on the safety level, especially if insurers have to reveal their solvency status as required by Solvency II.

Figure 3: Attainable (capital market line *CML*) and admissible ("solvency line") risk-return combinations for target ruin probability $\alpha = 0.01\%$ (corresponds to a "default-free" insurer and premium reduction of zero; results thus independent of policyholder risk sensitivity)

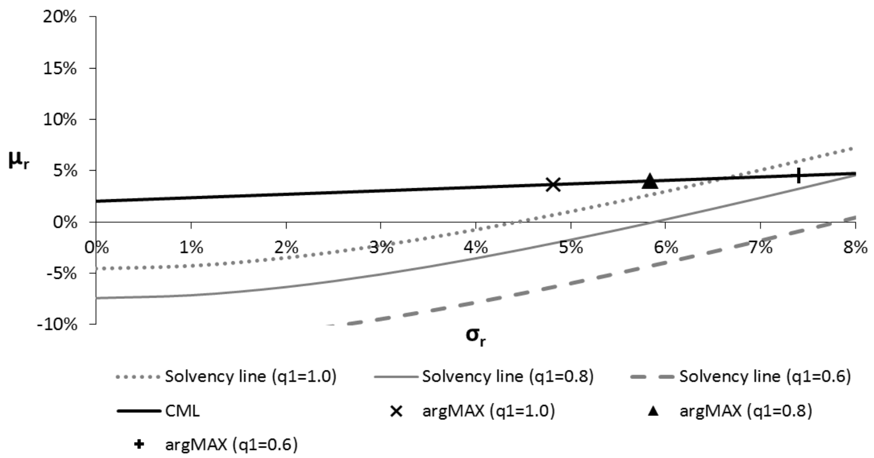


4.4 The impact of reinsurance decisions

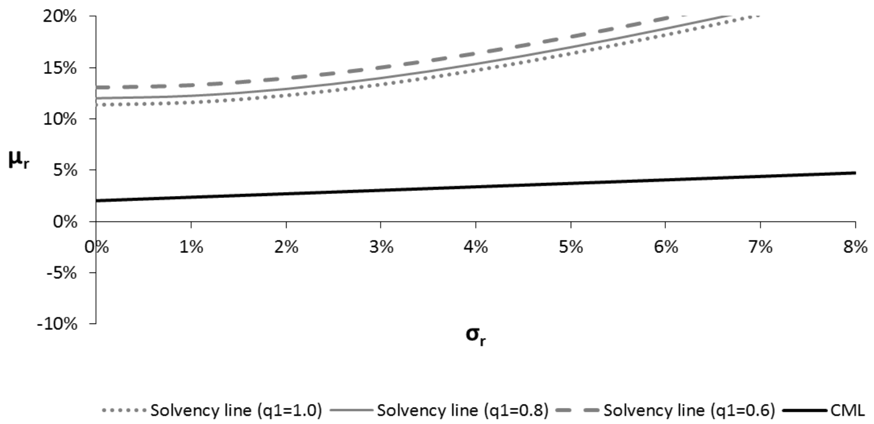
We next focus on decisions regarding the liability side by studying the impact of reinsurance contracts on (optimal) asset portfolio combinations. Figure 4 exhibits risk-return asset combinations for different retention levels of proportional reinsurance q_1 = 0.6, 0.8, 1.0, where $q_1 = 1.0$ corresponds to the base case without reinsurance. The results show that the solvency lines shift upward when decreasing q_1 (i.e., for increasing reinsurance portions) given a high ($\xi = 1$) risk sensitivity (Figure 4b)), whereas in the case without ($\xi = 0$) risk sensitivity (Figure 4a)) the solvency lines shift downward, thus allowing the insurer more flexibility in the asset allocation. This opposite behavior can be explained by the fact that the reinsurance premium is fixed, whereas the premiums of the insurer vary depending on the policyholders' risk sensitivity. In the case without ($\xi = 0$) risk sensitivity, premiums and reinsurance prices are calculated in the same manner by using the actuarial expected value principle with the same loading. Hence, such decisions regarding the liability side generate more flexibility on the asset side if policyholders exhibit no (or a low) risk sensitivity. This is generally in line with Diasparra and Romera (2010) who study upper bounds for the ruin probability in an insurance model where the risk process can be controlled by proportional reinsurance. They find i.a. (also assuming a fixed premium income) that a decreasing retention level leads to decreasing upper bounds of the ruin probability. However, our results show that considering the policyholders' willingness to pay given a high ($\xi = 1$) risk sensitivity, this effect is reversed.

Figure 4: Attainable (*CML*) and admissible ("solvency lines") risk-return combinations for $\alpha = 0.50\%$ for different retention levels of proportional reinsurance $q_1 = 0.6, 0.8, 1.0$

a) Risk-return asset combinations for no risk sensitivity ($\xi = 0$)



b) Risk-return asset combinations for high risk sensitivity ($\xi = 1$)



In particular, as for high risk sensitivity there are no admissible and attainable asset allocation opportunities, shareholder value cannot be created (in terms of the preference function) since the insurer has to pay the fixed reinsurance premiums from a much lower premium income caused by the policyholders' premium reduction (Equation (4)).[11] One can further observe

[11] Further analyses (available upon request) showed that, as expected, the solvency lines shift upward for an increasing reinsurance premium loading δ^{re}. Therefore, the insurer generally has to take into account that

that in the case of no risk sensitivity, the shareholder preference value decreases from $\Psi(4.8\%, 3.7\%) = 263$ for $q_1 = 1.0$ (no reinsurance, marked by **X**) to $\Psi(7.4\%, 4.5\%) = 236$ in case of $q_1 = 0.6$ (marked by **+**). Overall, this again illustrates the strong interaction between decisions on the asset and liability side and it emphasizes the relevance of taking into account policyholders' willingness to pay also in the context of reinsurance contracts, for instance, given that solvency levels have to be reported.

4.5 The impact of the risk-free interest rate

Since interest rates play an important role for solvency ratios, especially against the background of currently very low interest rate levels, Figure 5 illustrates risk-return asset combinations for various levels of the risk-free rate. In the base case, we use the expected return of the JPM Euro Cash 3 Month from January 2004 to November 2015 (2.04%) as a proxy for a constant, maturity independent risk-free interest rate. As pointed out before, since such a risk-free interest rate does not exist in practice, the calibration only serves as a proxy and we additionally use the JPM Euro Cash 3 Month in November 2015 (0.01%) to illustrate the impact of the low interest rate level.

When comparing the capital market lines, it can be seen that the slope *ceteris paribus* increases for a decreasing risk-free interest rate from $r_f = 2.04\%$ (base case) to $r_f = 0.01\%$, i.e. the risk premium per unit of standard deviation increases, whereas the intercept of the capital market line decreases. Aggregating both offsetting effects leads to a reduction of the set of attainable and admissible risk-return combinations in the present setting and thus to a considerable reduction of maximum shareholder value ($\Psi(4.0\%, 3.4\%) = 201$ in case of $r_f = 2.04\%$ (marked by **X**) compared to $\Psi(2.4\%, 1.2\%) = 176$ in case of $r_f = 0.01\%$ (marked by ▲). These results emphasize that the current phase of low interest rates strongly restricts the insurer's range of admissible and attainable risk profiles regarding the asset investment, since the insurer cannot invest in riskier assets while maintaining the solvency level, resulting in a lower shareholder preference value.

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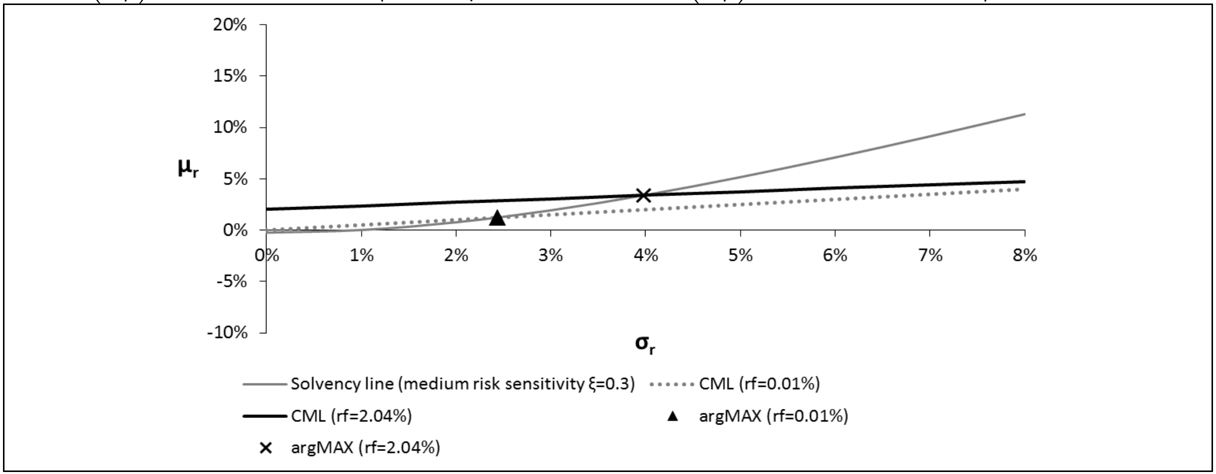
higher loadings δ^{re} increase the costs and thus reduce the area of attainable and admissible risk-return combinations and therefore the maximum shareholder value. In addition, the use of the standard deviation principle as a common risk-sensitive pricing principle in actuarial practice for insurance and reinsurance leads to similar results. Finally, the use of a non-proportional aggregate excess of loss reinsurance contract shows different results, as the impact of a higher attachment point (i.e. in tendency lower coverage given an unlimited layer) on the maximum shareholder value is ambiguous (depending on the respective attachment point). This arises due to asymmetries in the mean and the variance of the resulting equity capital U_1, since the non-proportional reinsurance contract only covers one tail of the claims distribution. Moreover, in case of high risk sensitivity, a higher attachment point shifts the solvency lines upwards (instead of downwards). Further results can be obtained from the authors upon request.

Figure 5: Attainable (capital market line *CML*) and admissible ("solvency line") risk-return combinations for a target ruin probability $\alpha = 0.50\%$ and given medium ($\xi = 0.3$) policyholder risk sensitivity for different levels of the risk-free interest rate $r_f = 2.04\%$: $CML(\sigma_r) = 2.04\% + 0.34 \cdot \sigma_r$ and $r_f = 0.01\%$: $CML(\sigma_r) = 0.01\% + 0.50 \cdot \sigma_r$

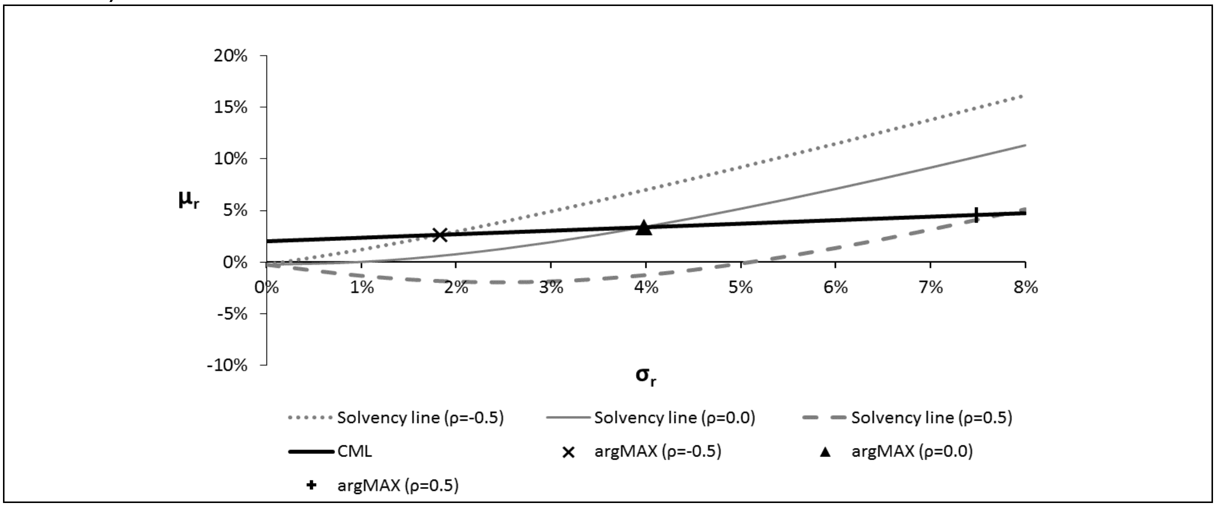


4.6 The impact of dependencies between assets and liabilities

To investigate the impact of the dependence structure between assets and liabilities on the requirements for the investment strategy and hence for maximizing shareholder value, we first compare the solvency lines for different parameters ρ = -0.5, 0.0, 0.5 of the Gaussian copula (equal to Pearson's correlation coefficient in the Gaussian setting) and thus different diversification levels in Figure 6. In case of a positive ρ, high asset values are positively related to high liability values and hence the risks are well diversified, resulting in a well-balanced asset-liability profile. A negative ρ, in contrast, represents an increasing riskiness of the asset-liability profile (in terms of the variance of equity capital) due to insufficient diversification, i.e. low asset values are positively related to high liability values (Fischer and Schlütter, 2015).

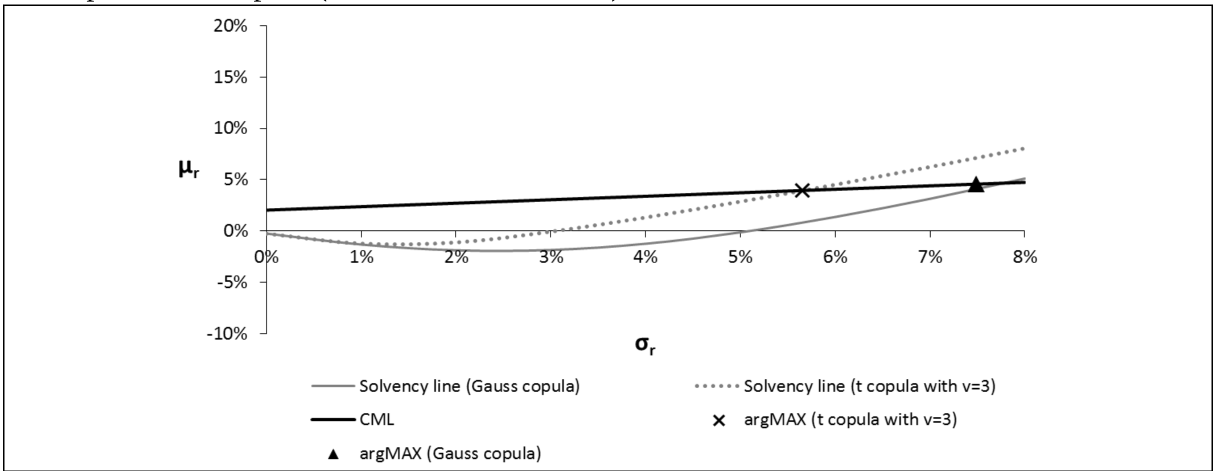
Figure 6 shows that the convexity of the solvency line increases for higher ρ, while the intercept remains unchanged. A lower linear dependence between assets and liabilities is thereby penalized by higher solvency capital requirements and reduces the area of admissible and attainable risk-return combinations. Furthermore, an insufficient diversification in an asset-liability context in terms of a lower ρ has negative consequences in regard to maximizing shareholder value, as Ψ declines from Ψ(7.5%, 4.6%) = 215 in case of ρ = 0.5 (marked by **+**) to Ψ(1.8%, 2.7%) = 192 in case of ρ = -0.5 (marked by **X**). It is thus crucial that a risk management model takes into account the diversification between assets and liabilities to avoid inadequate incentives for insurers, which is part of the criticism of Fischer and Schlütter (2015) in their analysis regarding the standard formula of Solvency II.

Figure 6: Attainable (*CML*) and admissible (solvency lines) risk-return combinations for a target ruin probability α = 0.50% and medium (ξ = 0.3) policyholder risk sensitivity for correlations ρ = -0.5, 0.0, 0.5



To study the impact of non-linear dependence structures, we now drop the Gaussian setting (i.e. Gaussian copula with Gaussian marginals) and consider a t-copula with the same marginal distributions. Figure 7 exhibits results for a Gaussian copula C_{ρ}^{Gauss} with parameter ρ = 0.5, implying a Kendall's tau τ = 0.33, and a t-copula $C_{\upsilon,R}^{t}$ with degrees of freedom υ = 3 and parameter R = 0.5 calibrated to the identical Kendall's tau to ensure comparability.

Figure 7: Attainable (*CML*) and admissible (solvency lines) risk-return combinations for target ruin probability α = 0.50% and medium (ξ = 0.3) policyholder risk sensitivity for Gaussian copula and t-copula (Kendall's tau τ = 0.33)

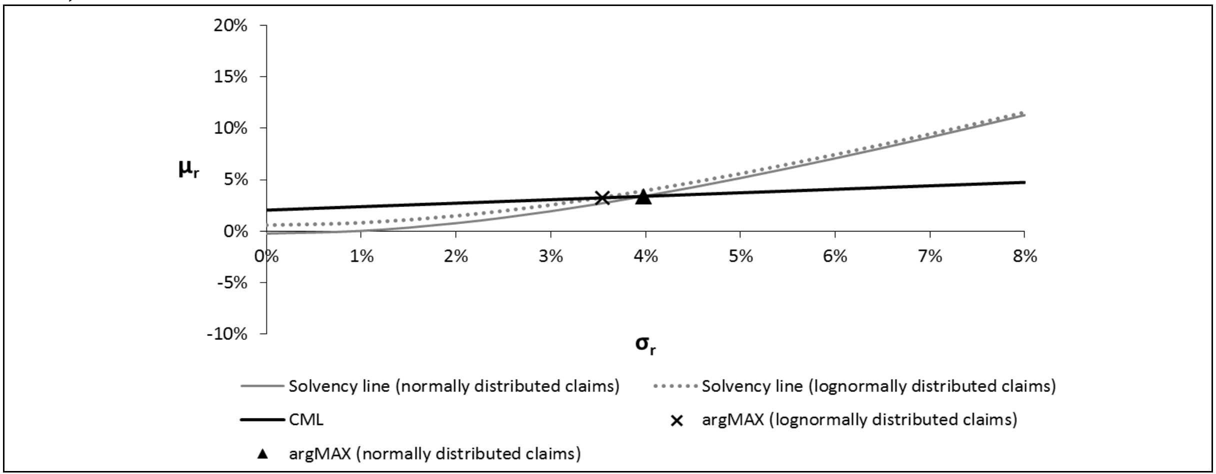


As can be seen from Figure 7, even though fixing Kendall's tau ensures a constant degree of general dependence between assets and liabilities, the solvency lines differ considerably. For the t-copula, the solvency line lies above the one of the Gaussian copula, implying stronger restrictions and less risky asset allocations along with a slightly lower maximum shareholder value: Ψ(5.7%, 4.0%) = 213 in case of the t-copula compared to Ψ(7.5%, 4.6%) = 215 in case of the Gaussian copula. While the Gaussian copula exhibits no tail dependence, the t-copula demands a higher expected return due to (negative lower-upper) tail dependence, i.e. the limiting probability of extremely low asset returns given extremely high claims is non-zero. This emphasizes that even though many risk models such as the standard formula of Solvency II focus on linear correlations, one should take into account that non-linear dependencies may lead to decisively different results and might be more appropriate to model dependences depending on the insurer's individual risk situation.

4.7 The impact of the claims distribution

Lastly, we also deviate from the purely Gaussian setting by using a lognormal distribution for the claims that ensures non-negative values and better reflects empirically observed properties (e.g., right-skewness, heavy tail) than a normal distribution. Figure 8 compares the solvency line derived based on normally distributed claims with that based on lognormally distributed claims calibrated to the same mean μ_S and standard deviation σ_S. While both solvency lines show similar results, the lognormal distribution with heavy tails still implies higher solvency capital requirements, leading to an almost equal maximum shareholder value but with a different optimal asset allocation: Ψ(3.5%, 3.2%) = 200 in case of the lognormal distribution (marked by **X**) compared to Ψ(4.0%, 3.4%) = 201 for the normal distribution (marked by ▲).

Figure 8: Attainable (capital market line *CML*) and admissible ("solvency line") risk-return combinations for target ruin probability α = 0.50% and medium (ξ = 0.3) policyholder risk sensitivity for normally and lognormally distributed claims (same mean and standard deviation)



5. SUMMARY

In this article, we study risk- and value-based management decisions regarding a non-life insurer's capital investment strategy by deriving minimum capital standards for the insurer's asset allocation based on a fixed solvency level, since adjusting the asset side to satisfy solvency capital requirements should generally be easier than short-term adaptions regarding equity capital or the liability side as pointed out by Eling et al. (2009). In this setting, we follow the latter and link the admissible (i.e. satisfying solvency requirements based on the ruin probability) risk-return combinations of the insurer's asset portfolio to actually attainable allocation opportunities at the capital market using Tobin's (1958) capital market line. We then study the optimal investment problem when maximizing shareholder value based on mean-variance preference functions and simultaneously controlling for the ruin probability in order to protect the policyholders. We thereby extend previous work in several relevant ways: We use a more general model and explicitly include the policyholders' willingness to pay, which to the best of our knowledge has not been done so far in this context, taking into account the insurer's reported target safety level and the policyholders' risk sensitivity. We further investigate the impact of reinsurance contracts, the risk-free interest rate, (non-linear) asset-liability dependencies as well as distributional assumptions, which considerably influence the set of attainable and admissible risk-return combinations as well as the maximum achievable shareholder preference value. This is done based on comprehensive analytical and numerical analyses, whereby we formally derive the global maximum of the shareholder preference function.

Our results show that the consideration of the policyholders' willingness to pay depending on their risk sensitivity based on the insurer's reported solvency status is crucial, since a more risk-sensitive assessment reduces the premium income and thus the flexibility of investments on the asset side as well as the resulting maximum shareholder preference value. This is especially relevant in the future in the presence of market discipline, when insurers have to regularly reveal their solvency status according to Solvency II. Moreover, given a default-risk-driven premium income, the optimal reported target ruin probability differs for various levels of policyholders' risk sensitivity. In case of high and medium risk sensitivity, a target ruin probability corresponding to the absence of default risk implies a higher shareholder value, while no policyholder risk sensitivity leads to contrary results. To satisfy the interests of both shareholders and policyholders, our approach can thus be used for balancing shareholder value and risk taking. In particular, depending on the policyholders' risk sensitivity, it is advisable for firms to closely monitor the reactions of policyholders to safety levels and to possibly enhance the solvency level in order to generate more flexibility for the investment strategy and to increase shareholder value.

We further observe that the set of attainable and admissible investment opportunities and thus the maximum shareholder preference value only increases for lower retention levels (i.e., higher portions) of proportional reinsurance if policyholders are not risk sensitive. In particular, this effect is reversed when taking into account the policyholders' willingness to pay, as the insurer's premium income, which must be used to pay the fixed reinsurance premiums (not subject to risk sensitivity influences), is reduced, implying a reduction of the set of available risk-return asset combinations. Furthermore, an analysis of the impact of the risk-free interest rate shows that the current phase of low interest rates strongly restricts the insurer's investment opportunities and considerably reduces shareholder preference value. In addition, we find that lower linear dependencies between assets and liabilities can considerably reduce the area of attainable and admissible risk-return combinations, emphasizing that diversification between assets and the underwriting portfolio can generate more flexibility regarding the risk profile of the capital investment, which also implies the potential of generating a higher shareholder value. Moreover, robustness tests emphasize that non-linear (tail) dependencies and heavy tail claims distributions can lead to even stronger solvency restrictions and lower maximum shareholder preference value.

Overall, our results strongly emphasize the strong interaction between decisions regarding the asset and the liability side, and they underline the importance of considering the policyholders' demand for insurance products given that solvency levels have to be reported, which should be taken into account by insurers in the context of their risk- and value-based management decisions.

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APPENDIX

Proof of Lemma 1: The preference function Ψ is strictly increasing in μ_r under Assumption 5, since it is differentiable and

$$\frac{\partial \Psi\left(\sigma_r, \mu_r\right)}{\partial \mu_r} = u + P\left(\alpha, \xi\right) - \pi\left(q_1\right) > 0\,.$$

In addition, since the capital market line CML (Equation (5)) is the upper bound of the feasible set of risk-return combinations, the following holds for the optimization problem in Equation (7)

$$\max\left\{\Psi\left(\sigma_r, \mu_r\right) : \left(\sigma_r, \mu_r\right) \in \Upsilon\right\} = \max\left\{\Psi\left(\sigma_r, CML\left(\sigma_r\right)\right) : \left(\sigma_r, CML\left(\sigma_r\right)\right) \in \Upsilon\right\}$$

$$= \max\left\{\Psi^*\left(\sigma_r\right) : \sigma_r \in I := \left[\min\left\{\sigma_r : \left(\sigma_r, CML\left(\sigma_r\right)\right) \in \Upsilon\right\}, \max\left\{\sigma_r : \left(\sigma_r, CML\left(\sigma_r\right)\right) \in \Upsilon\right\}\right]\right\},$$

with

$$\Psi^* : \begin{cases} \mathbb{R}_0^+ \to \mathbb{R} \\ \sigma_r \mapsto \Psi\left(\sigma_r, CML\left(\sigma_r\right)\right), \end{cases}$$

i.e. with $\mu_r = r_f + m \cdot \sigma_r$ from Equations (5), (10), and (11), one obtains under Assumption 5

$$\Psi^*\left(\sigma_r\right) = \mathbb{E}\left(U_1\right) - \frac{k}{2} \cdot \mathbb{V}\left(U_1\right)$$

$$= \left(1 + r_f + m \cdot \sigma_r\right)\left(u + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right) - q\mu_S$$

$$-\frac{k}{2} \cdot \left(\left(u + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right)^2 \cdot \sigma_r^2 + {q_1}^2 \cdot \sigma_S^2 - 2 \cdot \left(u + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right) \cdot q_1 \cdot \sigma_r \cdot \sigma_s \cdot Cor\left(r_1, S_1\right)\right),$$

$$\frac{\partial \Psi^*\left(\sigma_r\right)}{\partial \sigma_r} = m\left(u + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right) - k\left(u + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right)^2 \sigma_r$$

$$+k\left(u + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right) q_1 \sigma_s Cor\left(r_1, S_1\right),$$

and

$$\frac{\partial^2 \Psi^*\left(\sigma_r\right)}{\partial {\sigma_r}^2} = -k\left(u + P\left(\alpha, \xi\right) - \pi\left(q_1\right)\right)^2 < 0$$ (with $k > 0$ and Assumption 5).

Therefore, Ψ^* is strictly concave on $\mathbb{R}_0^+$ with unique global maximum ($\frac{\partial \Psi^*(\sigma_r)}{\partial \sigma_r} = 0$) in

$$\sigma_r^* = \frac{m + k \cdot q_1 \cdot \sigma_S \cdot Cor(r_1, S_1)}{k \cdot \left(u + P(\alpha, \xi) - \pi(q_1)\right)}.$$

If $\sigma_r^* \in I$, $\max\left\{\Psi^*(\sigma_r) : \sigma_r \in I\right\}$ is solved by σ_r^*. If $\sigma_r^* \notin I$, it holds that $\sigma_r^* < \min\left\{\sigma_r : \left(\sigma_r, CML(\sigma_r)\right) \in \Upsilon\right\}$ or $\sigma_r^* > \max\left\{\sigma_r : \left(\sigma_r, CML(\sigma_r)\right) \in \Upsilon\right\}$. In the first case, Ψ^* is strictly decreasing on *I* and $\max\left\{\Psi^*(\sigma_r) : \sigma_r \in I\right\}$ is solved by the left boundary point of *I*. In the second case, Ψ^* is strictly increasing on *I* and $\max\left\{\Psi^*(\sigma_r) : \sigma_r \in I\right\}$ is solved by the right boundary point of *I*. □

Derivative of $\Psi\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)$ with respect to *u*:

$$\Psi\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right) =$$

$$= \left(1 + r_f\right) \cdot \left(u + P(\alpha, \xi) - \pi(q_1)\right) + m \cdot \frac{m + k \cdot q_1 \cdot \sigma_S \cdot Cor(r_1, S_1)}{k} - q\mu_S$$

$$-\frac{k}{2} \cdot \left(\left(\frac{m + k \cdot q_1 \cdot \sigma_S \cdot Cor(r_1, S_1)}{k} \right)^2 + q_1^{\,2} \cdot \sigma_S^2 - 2 \cdot q_1 \cdot \frac{m + k \cdot q_1 \cdot \sigma_S \cdot Cor(r_1, S_1)}{k} \cdot \sigma_s \cdot Cor(r_1, S_1) \right).$$

Hence one obtains:

$$\frac{\partial \Psi\left(\sigma_r^*, CML\left(\sigma_r^*\right)\right)}{\partial u} = 1 + r_f.$$