Estate fiscal policies, long-term care insurance and informal care†

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Abstract: This paper studies the effects of estate recovery, estate taxation and long-term care (LTC) subsidies on the incentives both to buy LTC insurance and to transfer wealth by elderly parents and to provide informal care by children to their dependent elderly parents. We also investigate the effect of these three fiscal policies on public budgets allocated to finance LTC needs. Estate recovery is found to dominate the two other fiscal policies as it is more likely to incentivise both LTC insurance purchase and informal care supply while impacting positively government budget.

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1. Introduction

Most industrialised countries are confronted with a rapid ageing of their population which creates increasing needs for long-term care\(^1\) (LTC) and questions the financial coverage of LTC risks. LTC is mainly financed by governments through different types of public programs (Colombo, 2012). As a way to improve public budgets allocated to finance these growing LTC needs, governments can implement various fiscal policies, and in particular estate recovery, estate taxation and LTC co-payment. The first policy consists of recovering public LTC expenses from the estates of deceased beneficiaries. These policies exist in the U.S., existed in France until recently and their implementation is currently under discussion in Switzerland, England and Wales\(^2\). A second policy to improve LTC public budget is to tax the estate received by children. The use of earmarking revenues from inheritance tax to finance public LTC expenses is supported by some European countries (Rodrigues, 2014). Inheritance taxation has also been proposed as a possible source of funding for the Spanish public LTC system (Costa-Font and García González, 2007). A third and obvious way to improve public budgets allocated to finance LTC is simply to reduce public subsidies to the purchase of formal LTC or equivalently increase co-payments on individual publicly financed LTC expenses. These three policies are linked to each other as, directly or indirectly, they use the proceeds of estate taxation to finance public LTC. Given the growing share of inherited wealth in overall capital accumulation (Piketty and Zucman, 2014), linking LTC public budget to estate taxation seems rather natural (Cremer et al., 2016).

Two other actors, private insurance and the family, play a role in the LTC funding. Since some decades, LTC insurance markets, covering the financial risks linked to LTC needs, have developed. Yet, despite the significance of LTC risks, these markets experienced very limited success and insurance struggles to play an important role in the coverage and funding of LTC risk, compared to other insurance business lines. Various arguments have been put forward in the literature to explain the relatively low development of this market including the issue of long term risks insurability, asymmetric information, LTC risk pricing, bias in risk perception, and crowding out effects (see Brown and Finkelstein, 2009).

Finally, the family plays an important role in LTC funding. Not only the family often contributes financially to help their dependent relative, but also a large part of LTC needs is met through family members, and in particular children (Laferrière and Wolff, 2006). Several factors can influence the amount and the organization of informal help, such as family disintegration, geographical remoteness, women’s work, professional activity, fertility rates, the amount of inheritance and the availability of LTC public support as well as private insurance benefits (Van Houtven and Norton, 2008; Klimaviciute, 2017).

In this article, we study the effects of the three above mentioned fiscal policies, i.e. estate recovery, estate taxation and lower LTC subsidies, on the incentives to purchase LTC insurance and supply informal care. The policies we consider might strongly influence these decisions via a bequest motive. Indeed, leaving and receiving a bequest are major motives for the purchase of LTC insurance and the supply of informal care respectively, as they protect bequests against LTC expenses. Thus estate fiscal policies should influence LTC insurance purchase and informal care supply. We also study the impact of each of these three fiscal policies on public budgets. Our aim is to investigate which of these tools would benefit most public budgets to

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\(^1\) LTC is defined as “a range of services required by persons with a reduced degree of functional capacity, physical or cognitive, and who are consequently dependent for an extended period of time on help with basic activities of daily living” (Colombo et al., 2011).

\(^2\) See respectively Greenhalgh-Stanley (2012) and Thiébaut et al. (2012) for an overview of estate recovery programmes in the U.S. and France. See ATS (2018) and Cremer et al. (2016) for more details about the discussion in Switzerland, and England and Wales respectively.
finance LTC taking into account the impact of a change in these policies on inheritance transfers and informal care which respectively modify the inheritance tax revenue and the expense in subsidized formal care.

Very little research has investigated these issues. The only works we are aware of are Thiébaut et al. (2012) and Cremer et al. (2016). Thiébaut et al. (2012) theoretically analyse the effect of a hypothetical estate recovery program financing the APA, the French main public LTC benefit, on the supply of informal care. Cremer et al. (2016) address the effects of children’s earning tax, inheritance tax and social public LTC insurance on informal care supply and welfare. None of these papers studies the implications of such policies on private LTC insurance demand. By doing so, our paper aims to investigate how to improve the three main sources of funding LTC coming respectively from the government, private insurers and the family.

In order to carry out our analysis, we develop a theoretical model based on Courbage and Eeckhoudt (2012) and Cremer et al. (2016). We introduce a bequest motive by assuming the parent to be altruistic in the sense of Becker (1974), i.e. the parent extracts a positive amount of utility from transferring financial resources to his offspring. We also introduce an estate recovery program partially financing the amount of subsidised public help in the same spirit of Thiébaut et al. (2012) and an inheritance tax scheme. Finally, regarding the child as potential informal caregiver, we introduce the classical arbitrage between working and informal care supply. We assume the child to bear an opportunity cost of caring for his parent while having at the same time incentives to provide informal care in order to reduce formal care expenses and increase the amount of inheritance.

We show that the optimal level of informal care from the child depends on the amount of inheritance transfers only when the child is altruistic or dislikes providing care. As for the parent, we show that the optimal levels of insurance and transfer, as well as how they are impacted by LTC fiscal policies, depend on whether or not the elderly parent anticipates the reaction of the child to a change in the transfer or in fiscal policies.

Our results also show that estate recovery dominates the two other fiscal policies as it is more likely found to incentivise both LTC insurance purchase and informal care supply while impacting positively government budget. Such results offer new insights in terms of LTC financing policies.

The paper is organised as follows. In the next section, we introduce the benchmark model and the hypothesis used. In section 3, we present the optimal behaviour of the child in terms of informal care provided to his dependent parent. In section 4, we address the joint optimal conditions of the parent in terms of financial transfer to his child and LTC insurance purchase. In section 5, we present the results of the comparative statics for the child, and in section 6 for the parent. Section 7 studies the impact of the three fiscal policies on the government’s budget. Finally, a short conclusion is provided in the last section.

2. The model

The model set-up mainly stems from Courbage and Eeckhoudt (2012) and Cremer et al. (2016). We consider a parent and a child. The parent is characterised by a state-dependent utility function. He faces a probability \( p \) of being dependent and needing LTC at home. According to whether the parent is dependent or not, his utility functions are respectively \( u(x, H) \) or \( v(x, H) \), with \( u(x, H) < v(x, H) \). The first argument \( x \) of the utility functions represents the parent’s final wealth and the second argument \( H \) represents the bequest to his child. The parent is altruistic in the sense of Becker (1974) as he cares about the financial transfers made to his child. The utility functions are increasing and concave both in wealth and the level of transfer.
The bequest received by the child is multiplied by a constant \( \theta = \theta_0(1 - \tau) \) in the parent's utility function, the term \( \theta_0 \) quantifying the degree of parental altruism and the term \( \tau \) with \( 0 < \tau < 1 \) being equal to the inheritance tax rate. Following Cremer et al. (2016), we make the standard simplifying assumption that the cross derivatives with respect to wealth and bequest, \( u_{xH} \) and \( v_{xH} \), are negligible.

We first assume that the child is only interested in his wealth. He is characterised by a utility function \( V = \bar{u}(c) \) increasing and concave in his wealth \( c \), which is composed by an exogenous pre-bequest wealth \( z_0 \), his working income, with \( \omega \) being his hourly wage, and the bequest received from his parent. In a second step, we assume that the child is also concerned by the amount of informal care \( e \) provided to his parent through the function \( b(e) \) which is added to his utility function, such that \( \bar{V} = \bar{u}(c) + b(e) \).

The parent is retired and has accumulated an amount of wealth equal to \( w_0 \). In case of becoming dependent, he incurs formal LTC expenses for an amount of \( N \). The parent can receive informal care \( e \) provided by his child. Informal care has the benefit of reducing the cost of LTC at a decreasing rate. Hence, \( N \) depends on \( e \), and \( N(e) \) is such that \( N'(e) < 0 \) and \( N''(e) > 0 \). The State subsidises a proportion \( \beta \) of the parent’s LTC expenses during his life. However, a proportion \( \psi \) of this subsidy is recovered by the State after the parent’s death from the bequest transferred to his child. Note that \( \tau, \beta \) and \( \psi \) are assumed to be independent of the parent’s wealth.

The parent can purchase a LTC insurance policy offering a cash benefit equal to \( l \) in case of dependency. \( \mu l \) is the insurance premium corresponding to this contract. If \( \mu = p \) the premium is actuarially fair and if \( \mu > p \) the premium is loaded. The parent also decides the amount of bequest, \( T \), to be transferred to his child in the state of dependency. In case of autonomy, the amount transferred to his offspring is fixed and equals to \( \tilde{T} \). In that way, we can implicitly assume that the parent could control the level of informal care through the level of the transfer in case of dependency.

We assume the parent and the child to interact in the guise of a non-cooperative game. As in Cremer et al. (2016) the timing of the model is as follows: at \( t = 0 \), the government announces its policies. At \( t = 1 \), the parent simultaneously chooses the optimal amount of transfer \( T^* \) and the optimal level of LTC insurance \( l^* \) if he were to become dependent. As both decisions are taken simultaneously, each choice anticipates the optimal levels of the other. At \( t = 2 \), the state of nature is revealed and the child decides on the optimal quantity of informal care \( e^* \) to provide if his parent is dependent.

3. The optimal behaviour of the child

As the model is solved by backward induction, we start by looking at the optimal caregiving choice of the child. Following Courbage and Eeckhoudt (2012) and Klimaviciute (2017), we study three different cases separately. First, we consider the case where the child is “selfish” and only cares about his wealth. Second, we assume the child to be altruistic. Finally, we assume the child to dislike providing informal care.

3.1. The child only cares about his wealth

At stage two, if the parent is autonomous, the child does not have to make any decision. He just consumes his wealth, labour income and bequest. If the parent is dependent, the child faces an arbitrage between working and caring for his parent and solves the following optimization problem:
The first order condition (FOC) with respect to $e$ is:

$$V_e = \frac{\partial V}{\partial e} = \left( -\omega - (1 - \tau)\psi\beta N'(e) \right)\bar{u}'(c) = 0$$

with $c = z_0 + \omega(1 - e) + (1 - \tau)[T - \psi\beta N(e)]$

In Appendix 1, we show that the second order condition (SOC) for a maximum is satisfied.

From the FOC, we see that the optimal level of informal care is given by

$$\omega = -(1 - \tau)\psi\beta N'(e^*)$$

Optimally, the child supplies informal care until the point where the marginal economic benefit of providing care, i.e. the gain on inheritance due to the parent consuming less subsidised formal care, is equal to its opportunity cost, i.e. the salary $\omega$. For that level of effort, the child’s wealth is maximized. Unlike in Becker’s (1974) “rotten kid” theorem, the child is only interested in maximising his own wealth and not his family wealth. This happens as the child cannot modify his parent’s behaviour since he plays at $t = 2$ after his parent’s decisions are already taken place.

The optimal level of informal care is independent of insurance and of the amount of the parent’s transfer $T$, as none of the two affects either the marginal costs or the marginal benefits of providing informal care. Hence, in that case, the transfer cannot be used by the parent to influence the amount of care provided by his child.

### 3.2. The child is altruistic

Previously, we assumed that the child was only concerned by his wealth and in particular the bequest he would receive from his parent. However, the child could also derive satisfaction from providing informal care to his dependent parent, expressing some form of altruism. To define altruism, we follow Courbage and Eeckhoudt (2012) and assume that the child positively values the supply of informal care to his elderly parent when he is dependent. Providing informal care entails satisfaction to the child at a decreasing rate via the function $b(e)$ which is such that $b'(e) > 0$ and $b''(e) < 0$. The child’s optimisation problem becomes:

$$\max_e \bar{V} = \bar{u}(z_0 + \omega(1 - e) + (1 - \tau)[T - \psi\beta N(e)]) + b(e)$$

The FOC with respect to $e$ is

$$\bar{V}_e = \frac{\partial \bar{V}}{\partial e} = -(\omega + (1 - \tau)\psi\beta N'(e^*))\bar{u}'(c) + b'(e^*) = 0$$

In Appendix 1, we show that the SOC for a maximum is satisfied.

As $b'(e) > 0$, Eq. (4) implies $\omega > -(1 - \tau)\psi\beta N'(e^*)$ at the optimal level of effort. The child’s opportunity cost of providing informal care $\omega$ is now superior to the gain on inheritance due to the parent spending less on formal care.

As in Courbage and Eeckhoudt (2012), we can easily show that the level of informal care is higher under child altruism. Indeed, by evaluating the FOC of Eq. (4) at the optimal level of informal care when the child is “selfish” we have that:
\[
\frac{\partial \bar{q}}{\partial e_{e^*}} = b'(e) > 0
\]  

(5)

Hence the child provides a larger amount of informal care when he is altruistic compared to the case where he only cares about his level of wealth.

Interestingly, as in Pestieau and Sato (2008) and Cremer et al. (2016), the optimal level of informal care depends positively on the amount of the parent’s transfer \( T \) if the child is altruistic. By differentiating the FOC with respect to \( T \), we find that:

\[
\hat{v}_{eT} = \frac{\partial \bar{q}}{\partial e_{e^*} \partial T} = -[\omega + (1 - \tau)\psi \beta N'(e^*)](1 - \tau)\bar{u}''(c) > 0
\]

(6)

Intuitively, as the bequest in the state of dependency is larger after an increase in \( T \), \( \bar{u}'(\cdot) \) in Eq. (4) is reduced since the child is risk averse and therefore the altruistic component of \( e \) becomes relatively more attractive. To compensate, the child provides a larger amount of informal care.

Hence, when the child is altruistic, the parent can influence, by using the transfer, the amount of informal care provided by his child if he correctly anticipates his offspring’s optimal reaction to an increase in the bequest.

As before, the child’s behaviour is not affected by insurance in our setting.

3.3. The child dislikes providing informal care

Given that informal care has been shown to be detrimental for the caregiver’s physical and mental health (Schulz and Beach, 1999), we also consider the case where the child suffers some disutility when providing LTC\(^3\). The child’s preferences are modelled as in the previous subsection, the only difference being that now \( b(e) \) is such that \( b'(e) < 0 \) and \( b''(e) < 0 \). As in Klimaviciute (2017), we assume that providing informal care entails dissatisfaction to the child at an increasing rate.

We show in Appendix 1 that the SOC also holds in this case. When the child dislikes providing care, the FOC is the same as in Eq. (4) but now, given that \( b'(e) < 0 \), the optimality condition implies \( \omega < -(1 - \tau)\psi \beta N'(e^*) \). At the optimal level of care, the child’s opportunity cost is lower than the economic gain of providing informal care.

If we evaluate the FOC of at the optimal level of informal care when the child is “selfish” (see Eq. 5) we find, quite naturally, that the child’s provision of informal care is lower when he dislikes providing care since \( b'(e) < 0 \).

Finally, according to Eq. (6), the child’s optimal informal care supply depends negatively on the transfer \( T \) as \( \omega < -(1 - \tau)\psi \beta N'(e^*) \). In other words, a larger bequest discourages the child to provide informal care. Intuitively, as in the previous case, the child’s marginal utility of wealth decreases when \( T \) increases, hence, the child trades off less hours of informal care provision, which is an undesirable activity for him, against a lower amount of bequest. Hence, the possibility for the parent to influence the supply of informal care through the level of transfer depends on whether the child likes or dislikes providing care. Thus, Pestieau and Sato (2008) and Cremer et al. (2016) results stating that bequests have a stimulating effect on informal care strongly rely on the assumption of child’s altruism.

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\(^3\) Klimaviciute (2017) stresses that caregiving might be associated simultaneously with both a certain degree of disutility and a certain degree of utility coming, for example, from altruistic feelings. According to her, the case “the child dislikes (likes) providing care” can be seen as a shortcut that reflects the situation when the costs (benefits) of informal care offset the utility (disutility) of caregiving.
4. The optimal behaviour of the parent

Moving now to the parent’s behaviour, he chooses simultaneously the optimal amounts of transfer and of insurance at \( t = 1 \) before knowing whether he is dependent or not. For simplicity, we assume the transfer to be made in the state of autonomy \( \hat{T} \) to be fixed. \( \hat{T} \) is seen as an objective of transfer for the parent. In the state of dependency, the parent’s wealth is reduced because of formal care expenses and he may not be able to transfer the desired quantity \( \hat{T} \) to his child. In this case, as he is altruistic but cares at the same time about his wealth, he faces a trade-off between transferring resources to his child and maximizing his own wealth. The parent can also use the amount of the transfer in the state of dependency to influence the supply of informal care by his child.

4.1. The parent does not anticipate the reaction of the child

We start by assuming the parent to not anticipate the optimal behaviour of the child. We then make the additional assumption that \( T \leq \hat{T} \) as the parent’s wealth is lower in the state of dependency and \( \hat{T} \) is an objective of transfer. The expected utility optimization problem of the parent can be written as:

\[
\max_{\hat{T}, I} W = pu(w_0 - (1 - \beta)N(e) + (1 - \mu)I - T, \theta[T - \psi\beta N(e)]) + (1 - p)v(w_0 - \mu I - \hat{T}, \theta\hat{T}) \tag{7}
\]

The FOC with respect to \( T \) is given by:

\[
W_T = \frac{\partial W}{\partial T} = p(-u_x(x_1, H_1) + \theta u_H(x_1, H_1)) = 0 \tag{8}
\]

with \( x_1 = w_0 - (1 - \beta)N(e) + (1 - \mu)I - T \) and \( H_1 = \theta[T - \psi\beta N(e)] \).

In Appendix 2, we show that the SOC for a maximum is satisfied.

In the state of dependency, the parent transfers the optimal amount \( T^* \) such that \( \theta u_H(x_1, H_1) = u_x(x_1, H_1) \), i.e. such that the marginal benefit of the transfer, expressed by the additional utility from the transfer, equals the marginal cost of the transfer, given by the decrease of utility due to lower wealth. This can be referred as an “intra-state” optimality condition. This is related to Becker (1972) who shows for an altruistic parent that the optimal transfer is driven by both the child and parent’s marginal utility of consumption.

We can now analyse how this optimal transfer reacts to a change in insurance. This relationship can be obtained by differentiating the FOC with respect to \( T \) and \( I \). We have thus:

\[
W_{TI} = \frac{\partial W}{\partial T \partial I} = -p(1 - \mu)u_{xx} > 0
\]

which indicates a positive relationship between the transfer and insurance. This happens as when insurance increases, the parent is richer in the bad state of nature ceteris paribus, which makes the level of the transfer rise as he values both wealth and inheritance.

Moving to the optimal purchase of insurance, the FOC with respect to \( I \) is the following:

\[
W_I = \frac{\partial W}{\partial I} = p(1 - \mu)u_x(x_1, H_1) - (1 - p)\mu v_x(x_2, H_2) = 0 \tag{9}
\]

with \( x_2 = w_0 - \mu I - \hat{T} \) and \( H_2 = \theta\hat{T} \).

In Appendix 2, we show that the SOC for a maximum is satisfied.
If the premium is actuarially fair ($p = \mu$), we can provide an explicit solution of the optimal level of insurance $I^*$. Indeed, in that case the FOC becomes:

$$u_x(x_1, H_1) = v_x(x_2, H_2)$$

As the cross-derivatives $u_{xH}$ and $v_{xH}$ are assumed to be nil, the optimal level of insurance then depends on the comparison between the wealth levels $x_1$ and $x_2$. We can thus have two cases satisfying the FOC depending on whether the marginal utility of wealth is state-dependent or not.

If $u_x(\ldots) = v_x(\ldots)$ then $x_1 = x_2$. In that case, the optimal level insurance is such that

$$I^* = (1 - \beta)N(e) + T - \hat{T}$$

Depending on the comparison between $T$ and $\hat{T}$, optimal insurance is full or partial. If $T = \hat{T}$, then $I^* = (1 - \beta)N(e)$, which corresponds to full insurance i.e. such as the insurance indemnity covers the full financial loss (the cost of formal care not covered by the public subsidy). If $T < \hat{T}$, $I^* < (1 - \beta)N(e)$ which corresponds to partial insurance, i.e. the indemnity is lower than the full financial loss in case of dependency.

If the marginal utility of wealth in case of dependency is lower than the marginal utility of wealth in case of good health, i.e. $u_x(\ldots) < v_x(\ldots)$ then $x_1 < x_2$ and therefore optimal insurance is partial whatever the comparison between $T$ and $\hat{T}$. Note that Evans and Viscusi (1991) and Finkelstein et al. (2009) showed that the marginal utility of wealth in case of bad health is usually lower than the marginal utility of wealth in case of good health, therefore supporting that optimal LTC insurance purchase is partial.

To investigate how the optimal level of insurance reacts to a change in the transfer, we differentiate the FOC with respect to $I$ and $T$ which gives:

$$W_{IT} = \frac{\partial W^2}{\partial I \partial T} = -p(1 - \mu)u_{xx} > 0$$

The optimal amount of insurance depends positively on $T$. This occurs as an increase of the transfer in the state of dependency implies automatically a reduction in wealth in that state. To compensate for a lower wealth in the bad state of nature, the parent buys more insurance.

As the parent takes two simultaneous decisions interacting with each other, it is useful to represent together the optimality conditions on $T$ and $I$ on Fig. 1. The $T^*(I)$ curve indicates how the optimal level of the bequest reacts to an exogenous change in insurance coverage. The $I^*(T)$ curve expresses how the optimal insurance coverage adjusts to an exogenous change in $T$. The joint optimum is obtained when both curves intersect.\(^4\)

\(^4\) As the determinant of the Hessian matrix is positive (see Appendix 1) the slope of the curve $T^*(I)$ is greater than the one of the curve $I^*(T)$.
4.2 The parent anticipates the reaction of the child to a change in transfer

In sections 3.2 and 3.3, it has been shown that a larger transfer stimulates informal care if the child is altruistic while discourages it if the child dislikes providing LTC. Therefore, the parent’s optimal transfer in the state of dependency is likely to change if he anticipates the influence of the transfer on his descendant’s reaction, i.e. if \( e \) becomes a function of \( T \) in the form \( e(T) \). As stressed in Pestieau and Sato (2008), this additional component in the parent’s bequest motive is not strategic. In the case of an altruistic child, the bequest acts an insurance premium for LTC within the family. In the case of a child disliking to provide care, the transfer represents an additional cost for parental altruism.

Even if his wealth is lower in the state of dependency, the parent might have incentives to transfer a larger amount of wealth in this state of nature to encourage informal care supply if the child is altruistic. For this reason, we now relax the assumption of \( T \leq \hat{T} \). Under this framework, the parent’s optimization problem becomes:

\[
\max_{T,I} W = pu(w_0 - (1-\beta)N(e^*(T)) + (1-\mu)l - T; \theta[T - \psi\beta N(e^*(T))] + (1-p)v(w_0 - \mu l - \hat{T}, \theta \hat{T})
\]

The FOC with respect to \( T \) associated to Eq. (11) is:

\[
W_T = \frac{\partial W}{\partial T} = p(-Bu_x(x_1, H_1) + A\theta u_H(x_1, H_1)) = 0
\]

with \( A = [1 - \psi \beta e_T^o N'(e)] \) and \( B = [1 + (1-\beta)e_T^r N'(e)] \).

In Appendix 2, we show that the SOC for a maximum is satisfied if \( e_T^{tr} < 0 \).

The optimality condition implies that the parent transfers the amount \( T^* \) such that \( A\theta u_H(x_1, H_1) = Bu_x(x_1, H_1) \). Since now both \( A \) and \( B \) depend on the sign of \( e_T^r \) which is driven by whether the child is selfish, altruistic or dislikes providing care, so is the optimal transfer. Indeed, it is easily shown that \( T^* \) is relatively large when the child is altruistic. In that case the transfer generates a positive externality as it encourages informal care, a desirable behaviour for the parent. When the child dislikes providing care, the transfer is relatively low as it...
generates a negative externality (informal care is reduced). When the child is selfish, the transfer lies between the two because no externalities are generated.

Mathematically, this is shown by evaluating the FOC of Eq. (12) at the optimal transfer when the child is selfish, or equivalently, when the parent does not anticipate the reaction of the child, i.e., according to Eq. (8), for which \( u_H(x_1, H_1) = u_x(x_2, H_2) \). This gives:

\[
\frac{\partial w}{\partial \tau} \bigg|_{\tau^*} = -Bu_x + Au_x = -(1 - \beta(1 - \psi)) e_T^\prime N'(e) u_x
\]  

(13)

If the child is altruistic \( e_T^\prime N'(e) < 0 \), then Eq. (13) is positive. Thus the parent’s transfer in the case where he does not anticipate the reaction of the child is lower than the optimal transfer. The opposite happens when the child dislikes providing care as \( e_T^\prime N'(e) > 0 \). The parent’s transfer without anticipation is optimal when the child is selfish as \( T \) does not affect informal care supply in this case (i.e., \( e_T^\prime N'(e) = 0 \)). The optimal transfer still depends positively on insurance. Indeed,

\[
W_{TI} = \frac{\partial^2 w}{\partial \tau \partial I} = -pB(1 - \mu)u_{xx} > 0
\]

as \( B > 0 \) by the FOC.

Looking at the optimal purchase of insurance, the first order condition with respect to \( I \) is

\[
W_I = \frac{\partial w}{\partial I} = p(1 - \mu)u_x(x_1, H_1) - (1 - p)\mu v_x(x_2, H_2) = 0
\]

which is equivalent to Eq. (9). However, the optimal level of insurance is different as \( T^* \) depends now on the child’s preferences. If the child is altruistic, \( T^* \) is relatively high (see Eq. (13)) and the level of insurance is higher. More than full insurance could even be optimal in this case if \( T^* > T \) (see Eq. (10)). Following the same logic, \( I^* \) is relative low if the child dislikes providing informal care and \( I^* \) lies in between if the child is selfish or if the parent does not anticipate his offspring’s behaviour (see Eq. (10)).

4.3. The parent anticipates the reaction of the child to changes in fiscal policies

The parent could also anticipate that the optimal behaviour of the child might be modified after a change on the government’s fiscal policy. In this case, the parent’s problem writes as:

\[
\max_{\tau,I} W = pu(w_0 - (1 - \beta)N(e^*(\psi, \beta, \tau)) + (1 - \mu)I - T, \theta[T - \psi\beta N(e^*(\psi, \beta, \tau))] + (1 - p)\nu(w_0 - \mu I - T, \theta T)
\]  

(14)

As in this case the effort does not depend on neither the transfer nor insurance, the first and second order conditions, including the Hessian matrix, remain the same as in section 4.1. The main changes with respect to the model of section 4.1 relate to the comparative statics as the parent anticipates the changes on informal care supply driven by the modification of the different fiscal policies, i.e. \( e^*(\psi, \beta, \tau) \). This is discussed in the following section.

Ideally, it could also have been interesting to study the case when the parent anticipates the reaction of the child following both an increase in the transfer \( T \) and a change on the government’s fiscal policy. Unfortunately, the model becomes too complex to extract interpretable results.
5. The child’s comparative statics

In the following two sections we investigate how exogenous shocks in $\psi$, $\beta$ and $\tau$ affect the optimal level of informal care, the transfer and insurance. We consider these shocks as we are interested in how these three different fiscal public LTC policies affect the incentives of both the child and the parent. More specifically, we wonder whether governmental intervention to finance public LTC expenses creates incentives or disincentives for informal care supply and private LTC insurance demand.

We first look at how informal care supply is affected by governmental intervention. If $\alpha$ is an exogenous shock and the optimal utility of the child in the state of dependency is given by $V(e^{*}(\alpha))$, by applying the implicit function theorem, we have that $sgn(\partial e^{*}/\partial \alpha) = sgn(V_{e^{*}})$ given that the SOCs hold (see Appendix 1). The details of the computations are shown in Appendixes 3 and 4 and the results below, in Table 1. We also indicate the effect of the transfer on optimal effort for the three cases in the same table.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Comparative statics for the child</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Selfish child</td>
</tr>
<tr>
<td>$e^{*}$</td>
<td>$e^{*}$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>iif $-\frac{N'(e)}{N(e)} &gt; \Gamma_{c}A_{c}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>iif $-\frac{N'(e)}{N(e)} &gt; \Gamma_{c}A_{c}$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>-</td>
</tr>
<tr>
<td>$T$</td>
<td>0</td>
</tr>
</tbody>
</table>

where $\Gamma_{c} = \omega + (1 - \tau)\psi\beta N'(e^{*})$ and $A_{c} = -\tilde{u}'/\tilde{u}'$ is the child’s absolute risk aversion coefficient.

Starting with the case where the child is selfish, we first show that a higher percentage $\psi$ of subsidised care recovered from the bequest increases the amount of informal care supplied. This result is similar to the one of Thiébaut et al. (2012) who argue that when $\psi$ is high, the child has more incentives to provide informal care in order to reduce the amount of formal care purchased by the parent, partially recovered by the government from his inheritance. Second, we find that the effect of an increase in the subvention rate $\beta$ also increases the amount of informal care provided by the child because the amount levied from his bequest is higher. Finally, an increase in the inheritance tax rate reduces informal care supply. This happens as the child prefers working rather than caring for his parent since the marginal benefit of informal care provision is reduced.

If the child is altruistic, the effects of the three fiscal policies on informal care are slightly different. The effect of an increase in $\psi$ or $\beta$ on informal care is ambiguous and is positive only if $-N'(e)/N(e) > (\omega + (1 - \tau)\psi\beta N'(e^{*}))A_{c}$. As before, a rise in $\psi$ or $\beta$ increases the child marginal cost of formal care which tends to increase informal care supply. However, because the child is altruistic, he provides more informal care than before (see Eq. 5). As a consequence, hourly wages $\omega$ are relatively high and the child has incentives to compensate the negative shock in his bequest arising from an increase in $\psi$ or $\beta$ by working a larger amount of hours in the labour market. Therefore, for a given level of absolute risk aversion, the higher the wages, the higher the likelihood that the child reduces informal care to compensate for an increase in
ψ or β if he is altruistic. Concerning the effect of τ, as in Cremer et al. (2016), when the child is altruistic he lowers informal care supply if the inheritance tax rate increases.

If the child dislikes providing LTC, the effect of an increase in ψ or β on e∗ is positive, as before. Finally, an increase in τ has an ambiguous effect on e∗ as even if the marginal benefit of informal care is reduced after the tax increase, the child’s wage is already relatively low (he works a large amount of hours) and thus it might be detrimental to him to further increase the amount of time working in a remunerated job.

6. The parent’s comparative statics

Moving to the parent’s comparative statics, we look at how the optimal transfer in the state of dependency and optimal insurance purchase are affected by the government fiscal policies. By proceeding as before and applying the implicit function theorem, we have

$$sgn(\partial T^*/\partial \alpha) = sgn(-W_{\tau a}W_{li} + W_{la}W_{lT})$$

and

$$sgn(\partial I^*/\partial \alpha) = sgn(-W_{lT}W_{TT} + W_{Ta}W_{IT})$$

with

$$W(T^*(\alpha), I^*(\alpha))$$ and $$\alpha = \psi, \beta, \tau$$, given that the determinant of the Hessian matrix is positive (see Appendix 2).

6.1. The parent does not anticipate the reaction of the child

We start by studying the case where the parent does not anticipate the optimal behaviour of the child. The details of the computations are provided in Appendix 5 and the results are presented below, in Table 2.

Table 2
Comparative statics for the parent (no anticipation)

<table>
<thead>
<tr>
<th></th>
<th>T∗</th>
<th>I∗</th>
</tr>
</thead>
<tbody>
<tr>
<td>ψ</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>β</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>τ</td>
<td>+ if $$\theta_0(1 - \tau)\gamma_H &gt; 1$$</td>
<td>+ if $$\theta_0(1 - \tau)\gamma_H &gt; 1$$</td>
</tr>
</tbody>
</table>

where $$\gamma_H = -H u_{HH}/u_{H}$$ is the relative risk aversion coefficient (w.r.t. bequest).

A higher ψ leads to an increase in both the transfer and insurance purchase. This is rather intuitive as an increase in ψ reduces the amount of the child’s inheritance. Since the parent is altruistic, it reduces as well his utility. To compensate for that disutility, the parent increases the transfer to his child. At the same time, as the increase in the transfer reduces the parent’s wealth in the state of dependency, he has incentives to purchase more insurance. Hence, the altruistic component in the parent’s preferences makes him purchase more LTC insurance when his child’s wealth, i.e. the bequest, is reduced.

Moving to the effect of an increase in β, since it increases the parent’s wealth in case of dependency, the transfer to the child raises consequently. Given that the parent’s wealth increases in the bad state even if the transfer is larger, he purchases less insurance. In that case, public support substitutes private insurance as the parent purchases less insurance because his own wealth increases.

Finally, the effect of an increase in τ on T∗ and I∗ is ambiguous and depends on the parameters $$\theta_0, \gamma_H$$ and τ. More altruistic parents with high $$\theta_0$$, who value their child’s wealth relatively more than their own wealth, tend naturally to transfer more resources to their children.
if \( \tau \) increases. Similarly, more risk averse parents with respect to bequest with high \( \gamma_H \), who are highly averse to fluctuations on their child’s inheritance, increase as well \( T^* \) when \( \tau \) is higher. Finally, the higher the level of \( \tau \) the more relatively expensive the bequest is (i.e. the parent’s marginal utility of bequeathing to his child is relatively low). If the relative cost of inheritance is high, the parent tends to reduce the optimal transfer after an increase in the tax rate \( \tau \). This is the classical substitution effect arising from proportional taxation. So if \( \theta_0 \) or \( \gamma_H \) are high or \( \tau \) is low (equivalently, \( 1 - \tau \) is high), the parent tends to increase the optimal transfer after an increase in \( \tau \). At the same time, the effect of \( \tau \) on \( I^* \) has the same sign as the effect on \( T^* \). As for an increase in \( \psi \), if \( T^* \) increases the parent buys more insurance as a higher transfer reduces his own wealth in the state of dependency.

6.2. The parent anticipates the reaction of the child to a change in the transfer

We now assume that the parent anticipates the effect of the optimal transfer \( T^* \) on the child’s behaviour. When the child is selfish results are equivalent to the ones of the previous subsection as the effort of the child is not influenced by the transfer from the parent. However, results might differ when the child is altruistic or dislikes providing care as in these cases the transfer influences informal care supply. We present separately the results of the comparative statics when the parent has an altruistic child, who provides a larger amount of informal care if the transfer increases, and the results when the child dislikes taking care of his parent. In this last case, informal care supply diminishes when the transfer increases. The details of the computations are shown in Appendix 6. The results when the child is altruistic are first presented in Table 3 below.

**Table 3**
Comparative statics for the parent (altruistic child)

<table>
<thead>
<tr>
<th></th>
<th>( T^* )</th>
<th>( I^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi )</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \beta )</td>
<td>+ if ( - \xi \frac{e_T N'(e)}{n(e)} &lt; \theta \psi B\gamma H )</td>
<td>–</td>
</tr>
<tr>
<td>( \tau )</td>
<td>+ iif ( \theta_0 (1 - \tau) \gamma_H &gt; 1 )</td>
<td>+ iif ( \theta_0 (1 - \tau) \gamma_H &gt; 1 )</td>
</tr>
</tbody>
</table>

where \( \xi = 1 - \psi (1 + e_T' N'(e)) \geq 0 \) and \( \gamma_H = -u_{\theta H}/u_{\theta} \) is the absolute risk aversion coefficient w.r.t. bequest

The parent increases the transfer when \( \psi \) increases if his child is altruistic. First because as before, the child’s bequest is reduced and the parent is altruistic. Second, the parent has an additional incentive to increase \( T^* \) as it has a positive effect on informal care supply. As a consequence, \( I^* \) increases as well because the parent’s wealth in the state of dependency is lower if \( T^* \) raises.

The effect of an increase in \( \beta \) is more complex because the levels of informal care and the child’s bequest are already relatively high (see Eqs. (5) and (13)). As a consequence, the parent has less incentives to encourage, with a larger transfer, his child to supply additional informal care and the positive effect of \( \beta \) on \( T^* \) is weakened and could even be negative. As in the previous section, the effect of an increase in \( \beta \) on LTC insurance is negative.

The effect of \( \tau \) on \( T^* \) and \( I^* \) is the same as in the case where the parent does not anticipate his child’s behaviour. Table 4 below presents the results of the comparative statics when the child dislikes to provide care.
Table 4
Comparative statics for the parent (child who dislikes providing care)

<table>
<thead>
<tr>
<th></th>
<th>( T^* )</th>
<th>( I^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi ) + iif ( \frac{e'_N(e)}{N(e)} &lt; A\theta_AH )</td>
<td>+ iif ( \frac{e'_N(e)}{N(e)} &lt; A\theta_AH )</td>
<td></td>
</tr>
<tr>
<td>( \beta ) +</td>
<td>- iif ( \xi \frac{e'_N(e)}{N(e)} &lt; C\alpha + \xi \theta AAH )</td>
<td></td>
</tr>
<tr>
<td>( \tau ) + iif ( \theta_0(1-\tau)\gamma_H &gt; 1 )</td>
<td>+ iif ( \theta_0(1-\tau)\gamma_H &gt; 1 )</td>
<td></td>
</tr>
</tbody>
</table>

where \( \Gamma = e'_\tau N'(e) + (e'_\tau)^2 N''(e) > 0 \) and \( \alpha = \psi \beta + (1-\beta)\frac{A}{B} > 0 \)

When the child dislikes to provide care, an increase in \( \psi \) leads the parent to increase the transfer only if \( e'_\tau N'(e)/N(e) < A\theta_AH \). This happens as now the transfer creates disincentives to informal care supply, which affects negatively the parent’s wealth and the child’s bequest. Therefore, the parent increases \( T^* \) only if the transfer does not discourage informal care too much (i.e. if \( e'_\tau N'(e) \) is relatively low). The effect of \( \psi \) on \( I^* \) is the same as the one on \( T^* \) as the parent’s wealth in the state of dependency is lower (higher) if \( T^* \) increases (decreases).

An increase in \( \beta \) makes the transfer increase. This is the case because the parent’s wealth increases, the child’s bequest is reduced and additionally, the level of \( T^* \) before the fiscal policy measure is already relatively low (see Eq. (13)). The effect of \( \beta \) on optimal insurance is negative if the transfer does not discourage too much informal care. Contrarily, if the transfer strongly discourages informal care supply (i.e. if \( e'_\tau N'(e) \) is relatively large), the parent is forced to buy more insurance substituting the child’s informal LTC. Finally, the effect of \( \tau \) remains unchanged.

6.3. The parent anticipates the reaction of the child to changes in fiscal policies.

The last situation we consider is when the parent anticipates the child’s optimal behaviour after a change in the government’s fiscal policies.

We first consider the case where the child just cares about his wealth. In that case, we know that \( e'_\psi > 0 \), \( e'_\beta > 0 \) and \( e'_\tau < 0 \). These assumptions are also valid when the child is altruistic and \( -N'(e)/N(e) > \Gamma_c A_C \), and when the child dislikes providing care and \( -(\psi \beta N'(e))/(T - \psi \beta N(e)) > -\Gamma_c A_C \) as indicated in Table 1. The results of the comparative statics are displayed below, in Table 5, while the details of the calculations are given in Appendix 7. To sign the comparative statics, we make the additional assumption that \( N(e) + e'_\alpha N'(e) > 0 \). This is a reasonable assumption meaning that formal care expenses can never be negative\(^5\).

Table 5
Comparative statics for the parent (anticipation of fiscal policies) when \( e'_\psi > 0 \), \( e'_\beta > 0 \) and \( e'_\tau < 0 \).

<table>
<thead>
<tr>
<th></th>
<th>( T^* )</th>
<th>( I^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi ) +</td>
<td>+ iif ( \beta &gt; -\left(1 - \beta(1 - \psi)\right)\frac{e'_\psi N'(e)}{N(e)} )</td>
<td></td>
</tr>
<tr>
<td>( \beta ) +</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( \tau ) + iif ( \theta_0(1-\tau)\gamma_H &gt; 1 ) and ( \theta_0(1-\tau)\psi \beta A_H &gt; (1-\beta)A_X )</td>
<td>+ iif ( \theta_0(1-\tau)\gamma_H &gt; 1 ) and ( \theta_0(1-\tau)\psi \beta A_H &gt; (1-\beta)A_X )</td>
<td></td>
</tr>
</tbody>
</table>

\(^5\) By differentiating formal care expenses, we have \( N(e + d\ell da) = N(e) + e'_\alpha N'(e) da \).
where \( \gamma_H = -H u_{HH}/u_H \) is the relative risk aversion coefficient (w.r.t. bequest), \( A_x = -u_{xx}/u_x \) the absolute risk aversion coefficient w.r.t. wealth and \( A_H = -u_{HH}/u_H \) the absolute risk aversion coefficient w.r.t. bequest.

The results of the comparative statics of Table 5 are rather similar to the previous ones. Starting with an increase in \( \psi \), the effect on \( T^* \) remains positive while the effect on \( I^* \) is uncertain. Intuitively, as now the parent anticipates the effect on informal care of the tax increase and given that a higher \( \psi \) incentivizes informal care (i.e. \( e^\psi > 0 \)), the parent, even if he increases the transfer, might reduce his demand of insurance if the substitution of formal care by informal care supplied by the child makes his wealth increase. This occurs if informal care is a very strong substitute of formal care or where informal care supply is very sensitive to changes in \( \psi \) (i.e. \( \tilde{N}'(e) \) or \( e^\psi \) are very large in absolute value, respectively). In this case, informal care substitutes both public support and private insurance.

The effect of an increase in \( \beta \) on \( T^* \) is the same as in section 6.1, i.e. a decrease in the transfer as wealth is lower and an increase in insurance to protect assets.

Finally, the effect of \( \tau \) on \( T^* \) and \( I^* \) is still ambiguous. Altruism, risk aversion with respect to the bequest and the inheritance tax rate play the same role as before and even amplify the effects obtained in section 6.1. Furthermore, risk aversion with respect to wealth affects negatively the transfer and insurance when \( \tau \) increases. This happens as now, the inheritance tax reduces indirectly the parent’s wealth by discouraging informal care (i.e. \( e^\tau < 0 \)), making the parent transferring less resources to his child and reducing insurance demand if he is highly risk averse with respect to wealth.

Second, we consider the case where the child is altruistic and \( e^\psi > 0 \), \( e^\beta > 0 \) and \( e^\tau < 0 \). The results of the comparative static are displayed below in Table 6.

### Table 6
Comparative statics for the parent (anticipation of fiscal policies) when \( e^\psi > 0 \), \( e^\beta > 0 \) and \( e^\tau < 0 \).

<table>
<thead>
<tr>
<th></th>
<th>( T^* )</th>
<th>( I^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi )</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>( \beta )</td>
<td>+</td>
<td>(-) if ( \psi &lt; 1 - (1 - \beta (1 - \psi)) \frac{e^\psi N'(e)}{N(e)} )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>(+) if ( \theta_0 (1 - \tau) \gamma_H &gt; 1 ) and ( \theta_0 (1 - \tau) \psi \beta A_H &gt; (1 - \beta) A_x )</td>
<td>(+) if ( \theta_0 (1 - \tau) \gamma_H &gt; 1 ) and ( \theta_0 (1 - \tau) \psi \beta A_H &gt; (1 - \beta) A_x )</td>
</tr>
</tbody>
</table>

where \( \gamma_H = -H u_{HH}/u_H \) is the relative risk aversion coefficient (w.r.t. bequest), \( A_x = -u_{xx}/u_x \) the absolute risk aversion coefficient w.r.t. wealth and \( A_H = -u_{HH}/u_H \) the absolute risk aversion coefficient w.r.t. bequest.

The effect of an increase in \( \psi \) on \( T^* \) is uncertain if \( e^\psi < 0 \). This happens as, on one hand, the child’s bequest is reduced and the parent wants to compensate the negative shock on the child’s future wealth, given that he is altruistic. On the other hand, given that the child reduces informal care supply when \( \psi \) rises, the parent might reduce the transfer to preserve his wealth. Thus, more altruistic parents would tend to increase \( T^* \) anyway while less altruistic parents would tend to preserve their wealth. The effect of a change in \( \psi \) on \( I^* \) has now an unambiguous positive effect on insurance as the child’s reduction in informal care supply affects negatively the parent’s wealth in the state of dependency. Hence, insurance substitutes informal care in this case. The effect of \( \beta \) on \( T^* \) is positive as before. However, the effect on insurance could be positive if \( \beta \) has a large negative impact on informal care supply or if \( \psi \) is relatively high. Finally, as \( e^\tau < 0 \), the effect of the inheritance tax remains unchanged.
Third, we consider the case when the child dislikes providing care and \(e'_{\psi} > 0\), \(e'_{\beta} > 0\) and \(e'_{\tau} > 0\). The results of the comparative statics are displayed in Table 7 below.

**Table 7**
Comparative statics for the parent (anticipation of fiscal policies) when \(e'_{\psi} > 0\), \(e'_{\beta} > 0\) and \(e'_{\tau} > 0\)

<table>
<thead>
<tr>
<th></th>
<th>(T^*)</th>
<th>(I^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi)</td>
<td>+</td>
<td>+ if (\beta &gt; -(1 - \beta(1 - \psi))\frac{e'_{\psi}N'(e)}{N(e)})</td>
</tr>
<tr>
<td>(\beta)</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(\tau)</td>
<td>+ if (\theta_0(1 - \tau)\gamma_H &gt; 1) and (\theta_0(1 - \tau)\psi \beta A_H &lt; (1 - \beta)A_x)</td>
<td>+ if (\theta_0(1 - \tau)\gamma_H &gt; 1) and (\theta_0(1 - \tau)\psi \beta A_H &lt; (1 - \beta)A_x)</td>
</tr>
</tbody>
</table>

where \(\gamma_H = -H u_{HH}/u_H\) is the relative risk aversion coefficient (w.r.t. bequest), \(A_x = -u_{xx}/u_x\) the absolute risk aversion coefficient w.r.t. wealth and \(A_H = -u_{HH}/u_H\) the absolute risk aversion coefficient w.r.t. bequest.

The effects of \(\psi\) and \(\beta\) on informal care are the same as in Table 5. The effect of \(\tau\) is even more complex and ambiguous than before if \(e'_{\tau} > 0\) as parental altruism, the inheritance tax and risk aversion with respect to the bequest have contradictory effects.

**7. The government**

In this section, we study the impact of each of the three fiscal policies previously analysed on the government budget. The aim is to investigate which of these tools would benefit most public budgets to finance LTC.

In our model, the government budget for financing LTC is the difference between revenues received from the estate recovery program \((p\psi \beta N(e^*))\) and the inheritance tax \((p\tau(T^* - \psi \beta N(e^*)) + (1 - p)\tau \hat{T})\) minus the expenses in subsidised formal care \((p \beta N(e^*))\), i.e.:

\[
G = p\psi \beta N(e^*) + p\tau(T^* - \psi \beta N(e^*)) + (1 - p)\tau \hat{T} - p\beta N(e^*)
\]

(15)

Rearranging Eq. (15), we obtain:

\[
G = p(\tau T^* - [1 - \psi (1 - \tau)]\beta N(e^*)) + (1 - p)\tau \hat{T}
\]

(16)

We know from the comparative statics of the previous sections that the optimal amounts of transfer and informal care supply, \(T^*\) and \(e^*\) respectively, depend on the three parameters \(\psi\), \(\beta\) and \(\tau\) decided by the government.

To investigate how public budgets to finance LTC react to a change in one of the three fiscal policies, we simply differentiate Eq. (16) with respect to the parameter of interest. This gives:

\[
G'_{\psi} = \frac{\partial G}{\partial \psi} = p \left( \beta \left((1 - \tau)N(e^*) - [1 - \psi (1 - \tau)]e'_{\psi}N'(e^*)\right) + \tau T'_{\psi}\right)
\]

(17)

\[
G'_{\beta} = \frac{\partial G}{\partial \beta} = -p \left([1 - \psi (1 - \tau)] \left(N(e^*) + \beta e'_{\beta}N'(e^*)\right) - \tau T'_{\beta}\right)
\]

(18)

\[
G'_{\tau} = \frac{\partial G}{\partial \tau} = (1 - p)\hat{T} + p(\tau T^* - \psi \beta N(e^*)) - [1 - \psi (1 - \tau)]\beta e'_{\tau}N'(e^*) + \tau T'_{\tau})
\]

(19)
The impact of a change in each fiscal policy on the government budget is the sum of three effects. The first effect is simply the direct effect of a change in the policy on the public budget everything else held constant. The two other effects are indirect effects due to the impact of a change in the policy on the optimal quantities $e^*$ and $T^*$ which respectively modify the expense in subsidized formal care and the inheritance tax revenue. The effects of the different fiscal policies on the public budget are studied in greater details in the next subsections.

7.1. An increase in $\psi$

The total impact of a change in the estate recovery tax $\psi$ on the public budget is given by Eq. (17). The direct effect is positive and equals $p\beta(1 - \tau)N(e^*)$. Regarding the indirect effects, as shown in section 5, an increase in $\psi$ induces the child to provide more informal care (i.e. $e^*_\psi > 0$), which reduces subsidised formal care and therefore public spending, except when the child is selfish or does not like providing care. The only case where the child provides less informal care following an increase in $\psi$ is when he is altruistic and $-N'(e)/N(e) < \omega + (1 - \tau)\psi \beta N'(e^*)A_c$ as indicated in Table 1. However, the negative effect on public budget of $e^*_\psi$ in this case is relatively small as informal care is a weak substitute of subsidised formal care (i.e. $-N'(e)/N(e)$ is relatively low).

Considering the indirect effect of $\psi$ on $T^*$, the different models’ results are summarised in Table 8 below.

Table 8
Effects of an increase in $\psi$ on $T^*$

<table>
<thead>
<tr>
<th>$T^* \psi$</th>
<th>$T^*_\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No anticipation / anticipation of $e^*_\psi = 0$ (selfish child)</td>
<td>+</td>
</tr>
<tr>
<td>Anticipation of $e^*_\psi &gt; 0$ (altruistic child)</td>
<td>+</td>
</tr>
<tr>
<td>Anticipation of $e^*_\psi &lt; 0$ (child disliking provide care)</td>
<td>$\text{iff } e^*_\psi &lt; \frac{\psi N'(e)}{N(e)} &lt; A\theta A_H$</td>
</tr>
<tr>
<td>Anticipation of $e^*_\psi &gt; 0$</td>
<td>+</td>
</tr>
<tr>
<td>Anticipation of $e^*_\psi &lt; 0$</td>
<td>?</td>
</tr>
</tbody>
</table>

An increase in $\psi$ induces the parent to make a larger transfer (i.e. $T^*_\psi > 0$) in most scenarios. In the cases where the child dislikes providing care and where the child is altruistic and the parent anticipates $e^*_\psi < 0$, a larger recovery rate has an ambiguous effect and might reduce the parent’s transfer, negatively affecting the inheritance tax revenue.

Hence a higher estate recovery tax improves LTC public budgets in most of our models, i.e. for those where $T^*_\psi > 0$ and $e^*_\psi > 0$.

7.2. A decrease in $\beta$

The effect on the public budget of a decrease in the subvention rate is equal to $-G'_{\beta}$ and given by Eq. (18). Hence, in the case of a decrease in $\beta$, the direct effect on the government’s budget is positive and equals $p[1 - \psi(1 - \tau)]N(e^*)$. Moving to the indirect effects, the effect of a reduction in $\beta$ on informal care supply is equal to $-e^*_\beta$. As shown before, a decrease in the subsidy generally has a negative impact on informal care, i.e. $e^*_\beta > 0$ as indicated in Table 1. The only situation where a lower subvention increases informal care is when the child is
altruistic and $-N'(e)/N(e) < \omega + (1 - \tau)\psi\beta N'(e^*)A_c$ as indicated in Table 1. In this case, similarly to the case of an increase in $\psi$ the positive effect of a reduction in $\beta$ is limited as $-N'(e)/N(e)$ is relatively low.

The effect of a decrease in the subvention rate on the optimal transfer equals $-T'_\beta$. In Table 9 below, we summarise the different models’ results obtained previously.

<table>
<thead>
<tr>
<th>Table 9</th>
<th>Effects of a decrease in $\beta$ on $T^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No anticipation / anticipation of $e_T' = 0$ (selfish child)</td>
<td>$-T'_\beta$</td>
</tr>
<tr>
<td>Anticipation of $e_T' &gt; 0$ (altruistic child)</td>
<td>$-\text{if } -\xi \frac{e_T'N'(e)}{N(e)} &lt; \theta^2\psi BAA_H$</td>
</tr>
<tr>
<td>Anticipation of $e_T' &lt; 0$ (child disliking provide care)</td>
<td>$-$</td>
</tr>
<tr>
<td>Anticipation of $e_\psi' &gt; 0$</td>
<td>$-$</td>
</tr>
<tr>
<td>Anticipation of $e_\psi' &lt; 0$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

In four of the five models studied, a lower subvention rate reduces the parent’s transfer. The only situation where the parent increases the transfer is when the child is altruistic and the transfer considerably encourages informal care supply (i.e. $-\xi e_T'N'(e)/N(e) > \theta^2\psi BAA_H$).

Therefore, in the context of estate recovery, a policy aiming at reducing public expenses through a reduction in the subvention rate $\beta$, or equivalently, through an increase in the copayment rate, might not be very effective in terms of public budget. Indeed, the two indirect effects of this measure are generally negative and tend to reduce the government budget.

7.3. An increase in $\tau$

The total impact of a change in the inheritance tax rate $\tau$ on the public budget is given by Eq. (19). The direct effect is positive and equals $(1 - p)\hat{T} + p[T^* - \psi\beta N'(e^*)]$. Since $\hat{T}$ is also taxed, there is a redistribution of wealth from the autonomous to the dependent parents through inheritance taxation as the transfers of the first also participate to the financing of public support to dependent individuals.

Concerning the indirect effects, $e_T'$ is generally negative. This effect is found to be positive only when the child dislikes providing care and $-\psi\beta N'(e)/(T - \psi\beta N(e)) < -\Gamma_c A_c$ as indicated in Table 1. In Table 10 below, we display the sign of the indirect effect of $\tau$ on $T^*$ previously obtained.

<table>
<thead>
<tr>
<th>Table 10</th>
<th>Effects of an increase in $\tau$ on $T^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No anticipation / anticipation of $e_T' = 0$ (selfish child)</td>
<td>$+$ if $\theta_0(1 - \tau)\gamma_H &gt; 1$</td>
</tr>
<tr>
<td>Anticipation of $e_T' &gt; 0$ (altruistic child)</td>
<td>$+$ if $\theta_0(1 - \tau)\gamma_H &gt; 1$</td>
</tr>
<tr>
<td>Anticipation of $e_T' &lt; 0$ (child disliking provide care)</td>
<td>$+$ if $\theta_0(1 - \tau)\gamma_H &gt; 1$</td>
</tr>
<tr>
<td>Anticipation of $e_\psi' &gt; 0$</td>
<td>$+$ if $\theta_0(1 - \tau)\gamma_H &gt; 1$ and $\theta_0(1 - \tau)\psi\beta A_H &gt; (1 - \beta)A_x$</td>
</tr>
</tbody>
</table>
The indirect effect of $\tau$ on $T^*$ is complex. It could be either positive or negative as is $T_\tau'$. More altruistic or risk averse individuals tend to increase $T^*$ after a rise in the tax rate $\tau$ while the opposite happens with less altruistic or risk averse parents.

Hence the total effect of an increase in the estate tax rate is ambiguous as the direct effect is positive, the indirect effect on informal care is usually negative and the indirect effect on the transfer is ambiguous.

8. Conclusion

As a way to improve public budgets allocated to finance growing LTC needs, governments can implement various fiscal policies, and in particular estate recovery, estate taxation and lower LTC subsidies. In this article, we study how these three fiscal policies affect the elderly parents’ incentives to purchase private LTC insurance and to transfer wealth and the children’s incentives to supply informal care to their elderly dependent family members. We also analyse the impact of these fiscal policies on government budgets to finance LTC. The aim being to investigate how to improve the three main sources of funding LTC coming respectively from the government, private insurers and the family.

We first consider the optimal behaviours of the child and the parent. We show that the child’s behaviour can be influenced by the transfer made by his parent only when he is not selfish. In particular, a larger transfer stimulates informal care if the child is altruistic while discourages it if the child dislikes providing informal care. We also show that the child’s optimal effort reacts to changes in the three fiscal policies. As for the parent’s optimal behaviour in terms of transfer and insurance, it depends on whether he anticipates or not the reaction of the child to a change in the transfer or the fiscal policies, and in case of anticipation, on whether the child is selfish, altruistic or dislikes providing care.

If we assume that the parent does not anticipate his child’s reaction if he were to become dependent, we show that an increase in estate recovery leads to an increase in both LTC insurance purchase and informal care supply. Interestingly, insurance is purchased for altruistic reasons in this case as estate recovery does not affect directly the parent’s wealth but his child’s bequest. An increase in the inheritance tax rate is found to increase insurance purchase only if the parent is sufficiently altruistic or highly averse to fluctuations in his child’s bequest. It also discourages informal care supply. Finally, a reduction in the public subsidy encourages the parent to purchase a larger amount of insurance while discourages informal care supply.

In a second step, we consider that the parent anticipates, if he were to become dependent, his child’s reaction to a change in the transfer. Results are rather similar than with no anticipation except mainly for the case when the children dislikes providing informal care. In that case, an increase in the estate recovery rate or a decrease in the public subsidy could disincentive LTC insurance purchase if the transfer discourages informal care too much. In a third step, we assume the parent anticipates his child’s reaction to a change in the fiscal policies. Results, in general, are similar. However, in some scenarios, a higher estate recovery rate or a lower co-payment lead to a reduction in informal care supply and LTC insurance purchase.

Regarding the impact of these various policies on the government budget to finance LTC, it can be decomposed in three effects. The first effect is simply the direct effect of a change in the policy on the public budget everything else held constant. The two other effects are indirect...
effects due to the impact of a change in the policy on the optimal levels of the inheritance transfer and informal care which respectively modify the estate tax revenue and the expense in subsidised formal care. Amongst the three fiscal policies, estate recovery seems to provide adequate incentives to parents and children as it is more likely to encourage both informal care supply and inheritance transfers respectively reducing the amount of publicly subsidised formal care expenses and increasing the revenue of the inheritance tax. The first policy is, therefore, more likely to improve LTC public budget.

A reduction in the subvention rate or similarly, an increase in the individual’s co-payment, does not have the desired effects on the child’s and the parent’s behaviour from the government’s point of view as it discourages both informal care supply and inheritance transfers. Thus, the final effect of this policy on LTC public budget is ambiguous.

Finally, an increase in the inheritance tax rate increases the amount of subsidised formal care expenses as it discourages informal care supply. However, it could encourage or discourage the parent to leave a larger bequest depending on his level of altruism, increasing or decreasing the government’s revenue from inheritance taxation. Thus, the effect on LTC public budget of this measure is also ambiguous.

Our results show that estate recovery programmes dominate the two other fiscal policies as they are more likely to provide incentives to private LTC insurance purchase and to informal care supply while impacting positively public LTC budget. They would then offer an efficient tool to finance LTC needs as they would improve simultaneously the three main sources of LTC funding coming respectively from the government, private insurers and the family.

A limit of our model is not to consider the indirect consequences of the detrimental effects of providing informal care on the informal care giver’s health and income. Indeed, providing informal care can be painful for the caregiver’s health (Schulz and Beach, 1999) and tends to be correlated with less earnings (Carmichael and Charles, 2003). This might represent an indirect additional financial cost to governments through higher public health care expenses and lower labour tax and social contributions’ income. Another limit of our model is to assume that the three fiscal policies considered are fixed. One can easily imagine that estate recovery and inheritance taxation rates depend on the parent’s wealth as is the co-payment rate. Extending our results towards these directions would be an interesting topic for future research.
Appendix 1: Second order conditions of the child

A.1.1. The child only cares about his wealth

\[ V_{ee} = \frac{\partial V^2}{\partial e^2} = (1 - \tau)\psi\beta N''(e)\bar{u}' + \left( -\omega - (1 - \tau)\psi\beta N'(e) \right)^2 \bar{u}'' \]
\[ = (1 - \tau)\psi\beta N''(e)\bar{u}' < 0 \text{ using } \omega = -(1 - \tau)\psi\beta N'(e^*) \]

A.1.2. The child is altruistic and the child dislikes providing care

\[ \hat{V}_{ee} = \frac{\partial \hat{V}}{\partial e^2} = -\left( (1 - \tau)\psi\beta N''(e)\bar{u}' - \left( \omega + (1 - \tau)\psi\beta N'(e^*) \right)^2 \bar{u}'' \right) + b''(e) < 0 \]

Appendix 2: Second order conditions of the parent

A.2.1. The parent does not anticipate the reaction of the child

\[ W_{TT} = \frac{\partial W^2}{\partial T^2} = p(u_{xx} + \theta^2 u_{HH}) < 0 \]
\[ W_{II} = \frac{\partial W^2}{\partial I^2} = p(1 - \mu)^2u_{xx} + (1 - p)\mu^2 v_{\hat{x}\hat{x}} < 0 \]
\[ W_{TI} = \frac{\partial W^2}{\partial T \partial I} = -p(1 - \mu)u_{xx} > 0 \]
\[ W_{IT} = \frac{\partial W^2}{\partial I \partial T} = -p(1 - \mu)u_{xx} > 0 \]

\[ |H| = W_{TT}W_{II} - W_{TI}W_{IT} \]
\[ |H| = p^2(1 - \mu)^2\theta^2 u_{xx}u_{HH} + p(1 - p)\mu^2 v_{xx}v_{xx} + p(1 - p)\theta^2 \mu^2 u_{HH}v_{xx} > 0 \]

As \(|H|\) is positive, the Hessian is negative definite given that \(W_{TT} < 0\) and \(W_{II} < 0\). Thus, the second order condition for a maximum in \((T^*, I^*)\) is satisfied.

A.2.2. The parent anticipates the reaction of the child to a change in the transfer

\[ W_{TT} = \frac{\partial W^2}{\partial T^2} = p(\theta[A^2 u_{HH} - \psi\beta C u_H] + B^2 u_{xx} - (1 - \beta) C u_{\hat{x}}) < 0 \]

Where \( C = e''_{TT}N'(e) + (e''_{TT})^2 N''(e) > 0 \) and assuming \(e''_{TT} < 0 \)

\[ W_{II} = \frac{\partial W^2}{\partial I^2} = p(1 - \mu)^2u_{xx} + (1 - p)\mu^2 v_{\hat{x}\hat{x}} < 0 \]

\[ \frac{\partial W^2}{\partial T \partial I} = W_{TI} = -p(1 - \mu)B u_{xx} > 0 \]
\[ \frac{\partial W^2}{\partial I} \frac{\partial T}{\partial T} = W_{IT} = -p(1 - \mu)Bu_{xx} > 0 \]

\[ |H| = p\Gamma_HW_{II} + p(1 - p)\mu^2 B^2 u_{xx}v_{xx} - p(1 - \beta)[e''_T N'(e) + (e'')^2 N''(e)]u_xW_{II} > 0 \]

where \( \Gamma_H = \theta[A^2 u_{HH} - \psi \beta e''_T N'(e) + (e')^2 N''(e)]u_{HH} \) < 0 and assuming \( e''_T < 0 \)

Appendix 3: Comparative statics of the child when he only cares about his wealth

**Effect of** \( T \)

\[ V_{eT} = -[\omega + (1 - \tau)\psi \beta N'(e)](1 - \tau) \bar{u}'(.) = 0 \text{ as } \omega = -(1 - \tau)\psi \beta N'(e^*) \]

\[ \frac{\partial e^*}{\partial T} = -\frac{V_{eT}}{V_{ee}} = 0 \]

**Effect of** \( \psi \)

\[ V_{e\psi} = -(1 - \tau)\beta N'(e) \bar{u}'(.) > 0 \]

\[ \frac{\partial e^*}{\partial \psi} = -\frac{V_{e\psi}}{V_{ee}} = -\frac{N'(e)}{\psi N''(e)} > 0 \]

**Effect of** \( \beta \)

\[ V_{e\beta} = -(1 - \tau)\psi N'(e) \bar{u}'(.) > 0 \]

\[ \frac{\partial e^*}{\partial \beta} = -\frac{V_{e\beta}}{V_{ee}} = -\frac{N'(e)}{\beta N''(e)} > 0 \]

**Effect of** \( \tau \)

\[ V_{e\tau} = \psi \beta N'(e) \bar{u}'(.) < 0 \]

\[ \frac{\partial e^*}{\partial \tau} = -\frac{V_{e\tau}}{V_{ee}} = \frac{N'(e)}{(1 - \tau)N''(e)} < 0 \]

Appendix 4: Comparative statics of the child with altruism and when he dislikes providing care

**Effect of** \( T \)

\[ \hat{V}_{eT} = -[\omega + (1 - \tau)\psi \beta N'(e^*)](1 - \tau)\bar{u}''(.) \]

**Effect of** \( \psi \)
\[ \hat{V}_{e\psi} = -K_1 \left[ \frac{N'(e)}{N(e)} + (\omega + (1 - \tau)\psi \beta N'(e))A_c \right] \]

where \( K_1 = (1 - \tau)\beta u'(e)N(e) > 0 \)
and \( A_c = -\frac{\beta''}{\beta'} (\text{Absolute Risk Aversion}) \)

**Effect of \( \beta \)**

\[ \hat{V}_{e\beta} = -K_2 \left[ \frac{N'(e)}{N(e)} + (\omega + (1 - \tau)\psi \beta N'(e))A_c \right] \]

where \( K_2 = (1 - \tau)\psi u'(e)N(e) > 0 \)
and \( A_c = -\frac{\psi''}{\psi'} (\text{Absolute Risk Aversion}) \)

**Effect of \( \tau \)**

\[ \hat{V}_{e\tau} = K_3 \left[ \psi \beta \frac{N'(e)}{(T - \psi \beta N(e))} - (\omega + (1 - \tau)\psi \beta N'(e))A_c \right] \]

where \( K_3 = (T - \psi \beta N(e))\ddot{u}(e) > 0 \)
and \( A_c = -\frac{\psi''}{\psi'} (\text{Absolute Risk Aversion}) \)

**Appendix 5: Comparative statics of the parent when he does not anticipate the reaction of the child**

**Effect of \( \psi \)**

\[ W_{T\psi} = -p \beta N(e)\theta^2 u_{HH} > 0 \]
\[ W_{I\psi} = 0 \]
\[ \text{sgn}(\partial T^*/\partial \psi) = \text{sgn}(-W_{T\psi}W_{II}) = p\beta N(e)\theta^2 u_{HH}W_{II} > 0 \]
\[ \text{sgn}(\partial I^*/\partial \psi) = \text{sgn}(W_{T\psi}W_{IT}) = p^2(1 - \mu)\beta N(e)\theta^2 u_{HH}u_{xx} > 0 \]

**Effect of \( \beta \)**

\[ W_{T\beta} = -p N(e)(u_{xx} + \theta^2 \psi u_{HH}) > 0 \]
\[ W_{I\beta} = p(1 - \mu)N(e)u_{xx} < 0 \]
\[ \text{sgn}(\partial T^*/\partial \beta) = \text{sgn}(-W_{T\beta}W_{II} + W_{I\beta}W_{IT}) \]
\[ = pN(e)\theta^2 \psi u_{HH}W_{II} + p(1 - p)N(e)\mu^2 u_{xx}v_{xx} > 0 \]
\[ \text{sgn}(\partial I^*/\partial \beta) = \text{sgn}(-W_{I\beta}W_{IT} + W_{T\beta}W_{IT}) = p^2(1 - \mu)N(e)(\psi - 1)\theta^2 u_{xx}u_{HH} < 0 \]

**Effect of \( \tau \)**

\[ W_{T\tau} = -p\theta_0(1 - \theta y_H)u_H = -p\theta_0(1 - \theta_0(1 - \tau)Y_H)u_H \geq 0 \]

where \( Y_H \equiv -H \frac{u_{HH}}{u_H} \) is the RRA coefficient (w.r.t. bequest).

\[ W_{I\tau} = 0 \]
\[ \text{sgn}(\partial T^*/\partial \tau) = \text{sgn}(-W_{T\tau}W_{II}) = p\theta(1 - Y_H\theta)u_HW_{II} \geq 0 \]
\[ \text{sgn}(\partial I^*/\partial \tau) = \text{sgn}(W_{T\tau}W_{IT}) = -p^2(1 - \mu)\theta(1 - \theta Y_H)u_Hu_{xx} \geq 0 \]
Appendix 6: Comparative statics of the parent when he anticipates the reaction of the child to a change in the transfer

Effect of $\psi$

$W_{\psi} = -p\theta\beta u_H (e'_\tau N'(e) - A\theta N(e)A_H)$

$W_{I\psi} = 0$

$\text{sgn}(\partial T^*/\partial \psi) = \text{sgn}(-W_{T\psi} W_{II}) = p\theta\beta (e'_\tau N'(e) - A\theta N(e)A_H) u_H W_{II}$

$\text{sgn}(\partial I^*/\partial \psi) = \text{sgn}(W_{T\psi} W_{IT}) = p^2(1 - \mu)\theta B\beta u_H (e'_\tau N'(e) - A\theta N(e)A_H) u_{xx}$

Effect of $\beta$

$W_{T\beta} = -p \left[ \theta^2 \psi AN(e) u_{HH} + BN(e) u_{xx} + e'_\tau N'(e) \left( \psi - \frac{A}{B} \right) u_H \right]$  

$W_{I\beta} = p(1 - \mu) N(e) u_{xx} < 0$

$\text{sgn}(\partial T^*/\partial \beta) = \text{sgn}(-W_{T\beta} W_{H} + W_{I\beta} W_{TI})$

$\text{sgn}(\partial I^*/\partial \beta) = \text{sgn}(-W_{I\beta} W_{TT} + W_{T\beta} W_{IT})$

where $C = e''_{\tau\tau} N'(e) + (e'_\tau)^2 N''(e) > 0$, $\alpha = \psi\beta + (1 - \beta) \frac{A}{B} > 0$,

$\xi = 1 - \psi \left( 1 + e'_\tau N'(e) \right) > 0$ if altruistic and assumed to be positive if disliking providing care and assuming $e''_{\tau\tau} < 0$.

Effect of $\tau$

$W_{T\tau} = -p\theta_0 (1 - \theta\gamma_H) u_H = -p\theta_0 (1 - \theta_0 (1 - \tau)\gamma_H) u_H \geq 0$

$W_{I\tau} = 0$

$\text{sgn}(\partial T^*/\partial \tau) = \text{sgn}(-W_{T\tau} W_{II}) = p\theta u_H (1 - \gamma_H \theta) W_{II} \geq 0$

$\text{sgn}(\partial I^*/\partial \tau) = \text{sgn}(W_{T\tau} W_{IT}) = -p^2(1 - \mu)\theta (1 - \theta\gamma_H) u_H u_{xx} \geq 0$

Appendix 7: Comparative statics of the parent when he anticipates the reaction of the child to changes in fiscal policies

Effect of $\psi$

$W_{\psi} = p \left( (1 - \beta) e'_\psi N'(e) u_{xx} - \theta^2 \beta \left( N(e) + \psi\beta e'_\psi N'(e) \right) u_{HH} \right) > 0$
\[ W_{I\psi} = -p(1-\mu)(1-\beta)e'_{\psi}\nu'(e)u_{xx} < 0 \]
\[ \text{sgn}(\partial T^*/\partial \psi) = \text{sgn}\left(-W_{T\psi}W_{Ii} + W_{I\psi}W_{IT}\right) \]
\[ = p\left(\theta^2\beta\left(N(e) + \psi e'_{\psi}\nu'(e)\right)W_{II}u_{HH} - (1-p)(1-\beta)e'_{\psi}\nu'(e)u_{xx}v_{\xi\xi}\right) \]
\[ \text{sgn}(\partial I^*/\partial \psi) = \text{sgn}\left(-W_{I\psi}W_{TT} + W_{T\psi}W_{IT}\right) \]
\[ = p^2(1-\mu)\theta^2\left[\beta N(e) + (1-\beta(1-\psi))e'_{\psi}\nu'(e)\right]u_{xx}u_{HH} \]

**Effect of \( \beta \)**

\[ W_{T\beta} = -p\left(\theta^2\psi\left(N(e) + \beta e'_{\beta}\nu'(e)\right)u_{HH} + \left(N(e) - (1-\beta)e'_{\beta}\nu'(e)\right)u_{xx}\right) \]
\[ W_{I\beta} = p(1-\mu)\left(N(e) - (1-\beta)e'_{\beta}\nu'(e)\right)u_{xx} < 0 \]
\[ \text{sgn}(\partial T^*/\partial \beta) = \text{sgn}\left(-W_{T\beta}W_{II} + W_{I\beta}W_{IT}\right) \]
\[ = p^2(1-\mu)^2\theta^2\psi\left(N(e) + \beta e'_{\beta}\nu'(e)\right)u_{HH}u_{xx} - (1-p)\mu^2v_{\xi\xi}W_{T\beta} \]
\[ \text{sgn}(\partial I^*/\partial \beta) = \text{sgn}\left(-W_{I\beta}W_{TT} + W_{T\beta}W_{IT}\right) \]
\[ = p^2(1-\mu)\theta^2\left[(\psi - 1)N(e) + (1-\beta + \psi e'_{\beta}\nu(e)\right]u_{xx}u_{HH} \]

**Effect of \( \tau \)**

\[ W_{T\tau} = -p\left[\theta_0(1-\theta\gamma_H)u_H + \left((1-\beta)A_x - \theta\psi\beta A_H\right)e'_{\tau}\nu'(e)u_x\right] \geq 0, \]
where \( \gamma_H = -H\frac{u_{HH}}{u_H} \) is the RRA coefficient (w.r.t. bequest), \( A_x = -\frac{u_{xx}}{u_x} \) is absolute risk aversion for wealth and \( A_H = -\frac{u_{HH}}{u_H} \) is absolute risk aversion for the bequest.

\[ W_{I\tau} = 0 \]
\[ \text{sgn}(\partial T^*/\partial \tau) = \text{sgn}(W_{T\tau}) = -p\left[\theta u_H(1-\theta\gamma_H) + \left((1-\beta)A_x - \theta\psi\beta A_H\right)e'_{\tau}\nu'(e)u_x\right] \]
\[ \text{sgn}(\partial I^*/\partial \tau) = \text{sgn}(W_{T\tau}W_{IT}) \]
\[ = -p\left[\theta u_H(1-\theta\gamma_H) + \left((1-\beta)A_x - \theta\psi\beta A_H\right)e'_{\tau}\nu'(e)u_x\right]W_{IT} \]
References


